Modeling Coopetition Dynamics Using Agent-Based Approaches

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Abstract

This paper presents an agent-based model that reproduces the results of previous work on coopetition in supply chains by Scherrer et al., titled ***Towards a Symbiotic Mutualism Through External Horizontal Supply Chain Integration***[1]. The model simulates interactions between competing companies, defined as exchanges of market shares. The original results were successfully reproduced, and an enhanced version of the model was implemented. Additionally, a steady state analysis is provided that highlights how temporary steady states can be achieved between competing companies through collaboration.

Keywords

Agent-based models, Supply chain, Coopetition, Equilibrium states

1. Introduction

Coopetition in supply chains is an emerging concept where competing companies collaborate for mutual benefit. The paper by Scherrer et al. [1] explores this concept by adapting the well-known prey-predator model also known as the Lotka-Volterra model [8, 9] to economic competition and coopetition in order to analyze how coopetition influences market dynamics. By modifying the Lotka-Volterra equations to suit the supply chain coopetition context, the study introduces a cooperation factor that represents how collaborating companies benefit by gaining market shares from the rest of the market. The findings from Scherrer et al. [1] include a vector field analysis that demonstrates the influence of cooperation rates on the stability and coexistence of competing firms, providing insights into the complex interplay between competition and cooperation in the market.

This work aims to reproduce the results presented by Scherrer et al. [1] through a simulation-based approach. An agent-based model (ABM) is implemented to explore the cooperative behavior between companies and the market, with the goal of gaining deeper insights into coopetition dynamics. This serves as a first step to investigate the subject in further details, which can be facilitated by an ABM approach more easily than by extending the Lotka-Volterra mathematical model.

Supply chains consist of a set of entities with local objectives and operate under various constraints. Agent-Based Models (ABMs) are well-suited for modeling these independent entities, as they provide a realistic representation of the supply chain network structure and can detect emerging behaviors due to their flexibility.

Consequently, many studies have investigated supply chain modelling and simulation using ABM approaches, addressing topics such as supply chain reactivity, risk management, configuration, and other aspects of supply chain management. For instance, the paper by Akanle and Zhang [2] presents an agent-based model that optimizes supply chain configurations by dynamically adapting to time-dependent demand changes through an iterative bidding system and clustering of resource combinations. Um et al. [3] developed an agent-based system using JADE [4] to enhance supply chain interactions, introducing a new negotiation algorithm that outperforms KASBAH [5], emphasizing trade-offs and multi-criteria decision-making in dynamic environments to achieve optimal outcomes for both sellers and buyers. Mizgier et al. [6] addressed the collective dynamics of a supply chain using a system of suppliers, manufacturers and retailers. Recent challenges for supply chain has also been studied using ABMs, such as the paper by Towfique Rahman et al. [7], in which, the authors addressed the production strategies to face the supply chain disruptions and meet the demands during COVID-19 pandemic, where the face-masks supply chain was considered in the study.

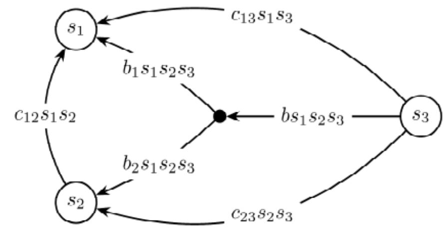

Figure 1: Interaction model including bilateral flows of market shares [1]

This paper is organized as follows: Section 2 presents the structure and behavior of the agent-based model. Section 3 describes the simulation experiments conducted, along with the results and steady states analysis. Key results from the experiments are discussed in Section 4 followed by a discussion on the original model in Section 5. Finally, an outlook on future work is provided in Section 6

2. The agent-based model

In their work, Scherrer et al. [1] modified the Lotka and Volterra [8, 9] model to represent the interactions between companies under a coopetition setting. Figure 1 illustrates the interactions of competitors with each other and with the rest of the market,where equations (1) and (2) hold:

- S_i: The market shares of company i
- c_{ij}:The bilateral interaction rate between companies i and j, where $c_{ij} > 0$ ($i \neq j$) means company i takes market shares from j.
- b:cooperation rate.

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- b_i:The benefit of company i from the cooperation.
- e_i:The efficiency of cooperation for company i, $e_i \in [0, 1]$.

$$b_i = e_i \cdot b, i \in 1, 2 \tag{1}$$

$$\sum_{i=1}^{3} S_i = 1 \tag{2}$$

To represent the coopetition settings adopted in the original work, we implement an agent-based model where each company is represented by an agent that interacts with other agents. Three agents were used, one for each company (company 1 and 2) and a third to represent the rest of the market. The latter is treated as a company that gains and loses market shares that we refer to it in the rest of the paper as *company 3*.

Each interaction between the agents results in an exchange of market shares according to the interaction matrix C, where $(c_{ij} < 0)$ indicates that company i loses market shares to company j. This model retains the same characteristics as the mathematical model presented in the paper by Scherrer et al. [1] and is designed with a focus on exploring market dynamics starting from an initial distribution of market shares. The study of how companies evolve from zero market shares to these initial conditions is beyond the scope of our work and also the model we aim to reproduce. Our primary goal is to analyze the effects of coopetition on market dynamics from the given starting point.

To adopt the same aspects as presented in the mathematical model we consider the following:

- The interaction coefficients c_{ij} represent how many market shares are derived/ lost by company i from/ to company j.
- The cooperation rate b represents how much market shares *company 1* and *2* derive from the rest of the market (*company 3*) through the cooperation.
- The sum of all market shares is always 100.
- We introduce the collaboration frequency $R \in [0, 1]$ which represents how frequently *company 1* and *2* collaborate, this parameter allows for the simulation of different scenarios.

The behavior of agents in the model is illustrated by Algorithm 1

Algorithm 1 Agent-Based Model Process

```
Inputs:
Initialize model parameters S_i, C, b, e_i, R
Outputs:
The final market shares distribution among the companies.
Behavior:
while There is more than one agent with S_i > 0 do
    Exchange c_ij market shares between two randomly selected companies i and j (i ≠ j)
    if All agents have S_i > 0 then
        Calculate the probability of collaboration P.
        if P ≥ R then
            Take b market shares from the rest of the market (company 3).
            Compute benefit b_i for agent i ∈ 1, 2 using equation (1).
        end if
    end if
end while
```

3. Simulation experiments and results

The interactions in the original work [1] (the base model) are given by the matrix C_1:

$$C_1 = \begin{pmatrix} 0 & 1 & -2 \\ -1 & 0 & -2 \\ 2 & 2 & 0 \end{pmatrix}$$

Matrix C_1 implies that company 1 always takes market shares from *company 2* and *company 1* and *2* always lose market shares outside the collaboration to *company 3*. This leads every time to the dominance of *company 1* over *2*. Therefore, in matrix C_2 we adjusted the coefficient c_{21} to -1 to allow *Company 2* to derive the same amount of market shares from *Company 1* as *Company 1* does from *Company 2*. This adjustment aims to represent full competition between these two companies, which we consider more representative of competitive interactions. Nevertheless, it represents an alternative to the one presented in the original study.

$$C_2 = \begin{pmatrix} 0 & -1 & -2 \\ -1 & 0 & -2 \\ 2 & 2 & 0 \end{pmatrix}$$

The model was implemented using the NetLogo (v6.4.0) [10] framework and an experiment that consists of multiple simulation runs was conducted using both model interaction matrices (C_1, C_2). Each simulation ends when one company dominates the whole market (100% of market shares), and the following measures are recorded:

- The market shares of all companies at the end of each simulation.
- The duration of the simulation in steps, where a step refers to one cycle (competition + collaboration) in the simulation.
- The lifespan of the rest of the market (*company 3*) is measured in steps.

The experiment involved varying the following variables and simulating all possible combinations:

- Market shares of Company 1: Values ranged from 5 to 50 with increments of 5.
- Market shares of Company 2: Values ranged from 5 to 45 with increments of 5.
- Collaboration frequency: Values ranged from 0.1 to 1 with increments of 0.1.
- Cooperation Rate (b): Values ranged from 1 to the total market shares of company 3 with increments of 1.
- For each unique combination of parameters, the simulation is repeated:
  - 20 Times in the case of matrix C_1, which gives a total number of simulation runs of 1,710,000.
  - 10 Times in the case of matrix C_2, which gives a total number of simulation runs of 855,000.

Table 1
Number of times each company takes over the market

Company	Market dominance (Times)	Ratio
Company 1	1,565,528	91.55%
Company 2	0	0%
Company 3	144,472	8.45%

Table 2
Average steps with respect to the collaboration frequency

Collaboration frequency (R)	Percentage of company 3 dominations	Average steps
10%	36.08%	52.38
20%	17.24%	53.84%

Note that we kept the same value in the paper for the efficiency coefficient $e_i = 0.5$. This parameter refers in the simulation model to the percentage of market shares that company i takes from the total market shares derived from the rest of the market through collaboration. The coefficient value is kept constant in all the simulations as we believe that this factor will only determine the winner between *company 1* or *2* and not the *company 3*, moreover, the goal of the study is to analyse the effects on the rest of the market and not between the collaborating companies.

3.1. Adaptation of the Base Model Using Matrix C_1

Through the simulations we observed that only two potential states were obtained, where always one company of the three that dominates the other two, giving a final distribution of one of the following: company 3 dominates (0, 0, 100%) or company 1 dominates (100%, 0, 0), which aligns with the results reported in the paper [1] (see Table 1). Additionally, we observed that Company 2 was never dominant in the market, which confirms our observations regarding the unidirectional flow of market shares between companies in matrix C_1.

The trivial states in which either company 1 wins or the rest of the market wins, were always the outcome. This is mainly due to the simulation model itself being based on pure competition and collaboration, and relies in its core on the distribution of wealth between agents. In addition, the collaboration frequency between partners affects the longevity of the market competition (Figure 2: 1) and the lifespan of company 3 (Figure 2: 2). In Figure 2, unlike plot 2, where the behavior is expected with the lifespan of company 3 decreasing as the cooperation rate increases, in plot 1, we observe that there are two regions: the first is when the collaboration frequency is under 20%, and the second is when the collaboration frequency is higher than 20%. For the first half ($R \leq 0.2$), when the collaboration frequency is 10%, the average number of steps is lower than in the case of 20%.

To understand this behavior, we repeated the first experiment, but we limited the collaboration frequency to 0.1 and 0.2 only, each combination of parameters has been simulated 100 times which gives in total 1,710,000 simulation runs (Table 2).

The results from Table 2 illustrate that Company 3 dominates the market more significantly at a 10% collaboration frequency compared to a 20% collaboration frequency. We observed that when Company 3 wins, the average duration of the competition is shorter, at 44.24 steps, compared to 53.34 steps when it loses. This is because the simulation stops when Company 3 wins after the depletion of the market shares of the other remaining companies 1 and 2, while simulation continues when Company 3 loses, allowing other companies to compete for market dominance. In general, whether Company 3 wins or loses, its lifespan tends to be shorter than that of other companies. When it wins, it wins quickly, and when it loses, it loses quickly. This pattern might explain why the duration of the simulation is shorter with a 10% collaboration frequency than with a 20% collaboration frequency. Additionally, plot 2 (Figure 2) supports the hypothesis that as the collaborative effort increases, the average market lifespan of Company 3 decreases in a pseudo-exponential decay, while the competition between the remaining companies decreases quasi-linearly. In cases where the collaboration frequency exceeds 20%, Company 3 tends to dominate less and loses quickly to the competition due to strong collaboration (more frequent collaboration). This results in a shorter average lifespan for Company 3, leading to a decrease in the average simulation time.

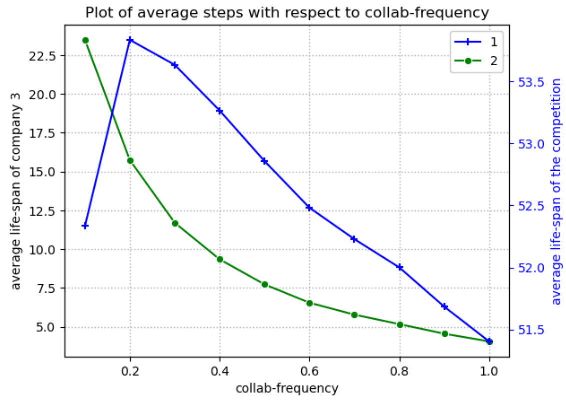

Figure 2: The effect of the collaboration frequency on the lifespan of the market competition, plot 1: lifespan of the competition, plot 2: lifespan of company 3

The cooperation rate b, represented here by the number of market shares the collaboration derives from the rest of the market, also plays a critical role. In Figure 3, we observe that as b increases, the average lifespan of the competition decays

Table 3
Cooperation rate with respect to average steps

Cooperation rate	Percentage of times company 3 dominates	Average steps
1	99.9%	48.16
2	79.5%	73.28

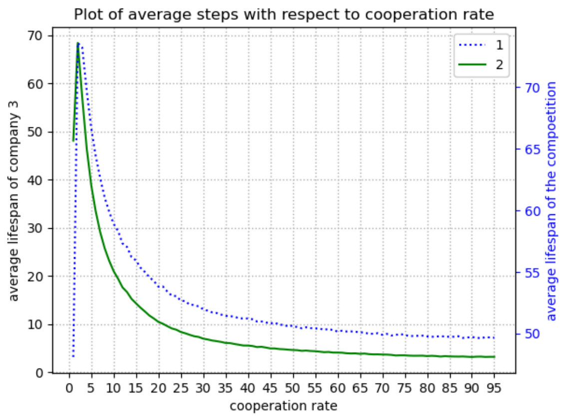

Figure 3: The effect of the cooperation rate on the lifespan of the rest of the market, plot1: average lifespan of competition, plot2: average lifespan of the rest of the market (company 3)

pseudo-logarithmically. However, there is an interesting trend between values 1 and 2, where the duration increases from 1 to 2 before beginning to decay after peaking at 2. The correlation between the cooperation rate and the life span of company 3 measured by Kendall correlation coefficient is given by: -0.98, this means that there is a strong negative correlation between these parameters. To learn more about the $b = 1$ and $b = 2$ cases, we conducted the same initial experiment while limiting the cooperation rate to 1 and 2 and varied all other parameters. Each combination of parameters is simulated 100 times.

The results in Table 3 indicate that at a cooperation rate of 1, Company 3 consistently dominates the market compared to a cooperation rate of 2. Consequently, the average duration of the simulation is equal to the average lifespan of Company 3 (the rest of the market), which explains the increase in the average simulation time in the case of $b = 2$. This is illustrated in Figure 3 ($b = 1$), where the average duration of the simulation and the average lifespan of Company 3 are nearly identical. The results show similar behavior to the case of varying collaboration frequency. Specifically, in cases where $b = 1$ and $b = 2$, have a strong effect on market dynamics. These experiments suggest that there are thresholds for collaboration and cooperation rates that significantly influence the outcome of the competition.

To investigate the relationship between the collaboration and cooperation rates, we recorded from the initial experiment the minimum values of the cooperation rates at which company 1 dominates the market. Additionally, we recorded the maximum values of the cooperation rate at which company 3 consistently dominates the market as illustrated in Figure 4 where:

- **Plot 1**: Represents the maximum values of b where company 3 always won the competition.
- **Plot 2**: Represents the minimum values of b where company 3 always lost the competition.
- **The scatter plot**: Represents combination of parameters (100 point per collaboration frequency) that lead both company 3 and company 1 to dominate the market. The frequency is defined for each unique combination of parameters by the number of times company 3 dominates over the total number of simulations. The darker the hue the more frequently company 3 dominates the market.

We observe in Figure 4 that there is a critical cooperation rate threshold, depending on the collaboration frequency, which determines whether Company 3 dominates or not. In scenarios where Company 3 consistently dominates, the maximal values' threshold line decreases in an approximately pseudo-exponential decay. Similarly, the threshold minimal values, where Company 3 always loses, follow a similar trend. These two lines are close to each other and overlap in some cases, suggesting that there are distinct regions where Company 3 either always dominates the market or always loses. This trend illustrates also that the more often companies collaborate the less market shares are required to derive from the rest of the market in each collaboration to dominate the market. However, the winning and losing outcomes for company 3 can occur on both sides of these lines. In other words, combinations of parameters below the threshold can lead to company 3 losing, and similarly, some combinations above the threshold can lead to company 3 winning. This suggests that the decisive factor for determining whether company 3 wins or loses is the combination of all parameters, including the initial market shares of companies 1 and 2. To illustrate these results, we scattered randomly selected combinations that lead to both sides of the competition to win (either company 1 or the rest of the market), we observe that there are two directions to analyze the data:

- 1- Increasing collaboration: As the collaboration increases the frequency of company 3 winning decreases above the threshold line (Plot 1), we notice also that there is less domination by the rest of the market.
- 2- Increasing cooperation rate: We observe that below the threshold (Plot 1) company 3 dominates the market more frequently compared to combinations with cooperation rates greater than the threshold.

Therefore, the final observation is that there exist parameter combinations that consistently lead to company 3 losing, others that consistently lead to company 3 winning, and combinations where the outcome can vary.

Now that we understand that it's not solely the collaboration and cooperation rates that determine the dynamics of the market, we investigate the relationship between the initial market shares distribution and Company 3's winning frequency with respect to the cooperation rate and collaboration frequency. Because the relationship between Company

1 and Company 2 is asymmetrical—meaning that Company 1 always takes market shares from Company 2—we calculate the gap between their initial market shares, defined as Company 1's initial market shares minus Company 2's initial market shares, to see if this influences market domination. We define also the frequency by the number of times company 3 dominates the market over the number of simulations of the same unique parameter combination. Figure 5 illustrates the relationship between the initial market shares' distribution and the frequency by which Company 3 wins the competition across different intervals of b rates. We observe that certain b values, such as $[10, 20]$, $[20, 50]$ and $[50, 95]$, exhibit a relatively stable evolution. In contrast, cooperation rates 1 and 2 show a relatively overall increasing winning frequency for Company 3 as the gap between the market shares increases. Specifically, when Company 2 has more market shares than Company 1 (gap < 0), the frequency of Company 3 winning is higher compared to when Company 2 has fewer market shares than Company 1 (gap $\geq$ 0).

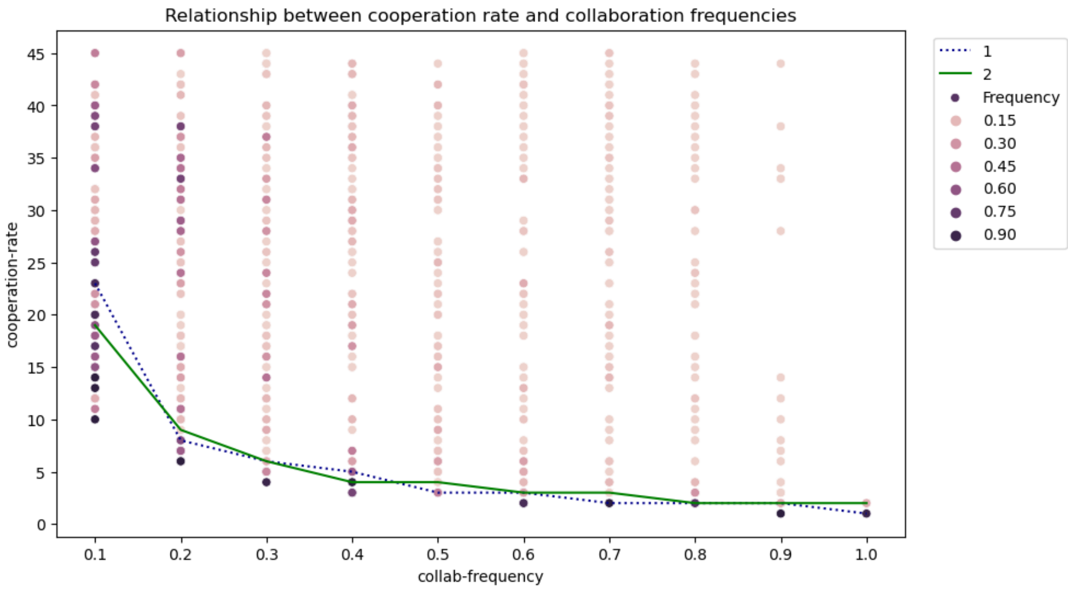

Figure 4: Relationship between cooperation rate and collaboration frequency, plot1: Threshold where company 1 dominates, plot2: threshold where the rest of the market dominates

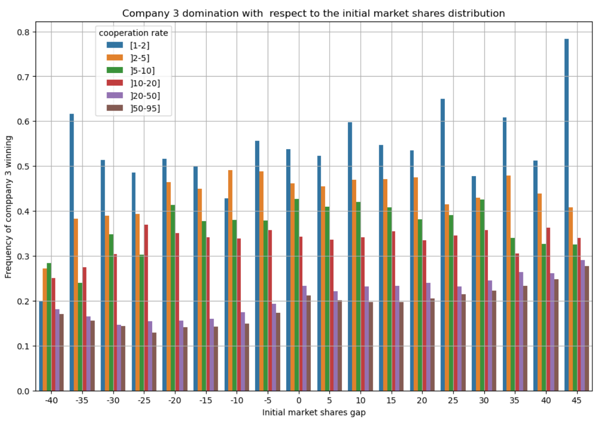

Figure 5: Effects of the initial market shares distribution and cooperation rates on the dynamics of the market

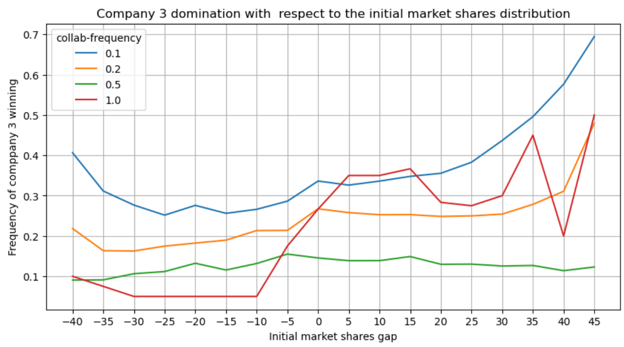

Figure 6: Effects of the initial market shares distribution and collaboration frequency on the dynamics of the market

The collaboration frequency has also an effect on the company 3 domination with respect to the initial market shares distribution, Figure 6 illustrates this relationship with the most interesting collaboration frequencies: 10%, 20%, 50% and 100%.

We observe in Figure 6 that the scale ranges from -40 to +45 market shares. For negative gap values, specifically between -40 and -25, there is a decrease in the winning frequency for Company 3. The frequency then starts to increase moderately throughout the remaining negative values. However, for positive gap values, the winning frequency increases quickly, with a particularly significant rise at the highest positive values of 40 and 45. This trend is evident for collaboration frequency of 10% and 20% although the overall frequency is lower for the 20% compared to the 10% collaboration frequency. Given that Company 2 always loses market shares to Company 1, these observations suggest that when Company 1 has significantly more market shares than Company 2, Company 1 depletes Company 2 quickly. Additionally, since Company 1 always loses market shares to Company 3, this puts Company 3 in a winning situation. Thus, as soon as Company 2 is depleted and collaboration stops, Company 3 dominates Company 1. This might explain the increase in winning frequency for Company 3 at high positive gap values. The collaboration frequency acts as attenuating factors for this effect. Overall, we observe that the winning frequency for Company 3 is lower at a collaboration frequency of 50% compared to rates of 20% and 10%. For the case of 100% collaboration, the winning frequency for Company 3 exhibits specific patterns across different gap values. Initially, the frequency is very low for gap values ranging from -40 to -10. Then, there is a sudden increase in frequency within the range of $[-5, 5]$. Following this, the frequency shows a relatively constant evolution until around 30, with some fluctuations. Beyond 30, there is another notable increase in the frequency. This latter behavior can be attributed to the dynamics between Company 1, Company 2, and Company 3, influenced by their initial market share gaps. When Company 2 initially holds significantly more market shares than Company 1 and collaborates heavily with Company 1, they can dominate the market, despite Company 2 always losing market shares to Company 1. Conversely, when the gap values are similar or favor Company 1 (meaning Company 1 depletes Company 2 quickly due to its larger or equal market shares), Company 3 benefits. After collaboration ceases, Company 3 capitalizes on its advantageous position against Company 1, increasing its winning frequency.

3.1.1. Steady states analysis

To assess the stability of market competition, we define a steady state where the distribution of market shares among competitors shows minimal change or slight variations over a specified time interval. This is characterized by a slope α between the bounds of an interval to be less than ϵ, where ϵ is a significantly small positive number ($\alpha \in [-\epsilon, \epsilon]$). To analyze the presence of steady states in our simulations' results, we smoothened the market shares distributions using a moving average applied over equal intervals of $T = 10steps$. Subsequently, we calculated the slope for each segment of the resulting data to identify any discernible trends.

We define a steady state as a state where the slope values of all market shares of competitors $\alpha \in [-0.05, 0.05]$ over an interval T of 10 steps. And we define also, a short-term steady state as a state that spans on one T interval, and a long-term one that spans across multiple consecutive T intervals.

Out of 85,500 unique simulations, a total of 128 cases exhibited steady states. Among these cases, we identified two distinct categories:

- The slope $\alpha = 0$.
- The slope $\alpha \in [-0.05, 0.05]$ and $\alpha \neq 0$.

For the first category two combinations of parameters exhibit steady states (see Table 4).

Figures 7 and 8 illustrate the smoothened market shares evolution, and the steady states that are highlighted at their start and end by the vertical lines.

For the second category, where $\alpha \in [-0.05, 0.05]$, a total of 126 cases were recorded. Table 5 illustrates the characteristics of the parameter combinations in these cases. It is evident from Table 5 that steady states occur in scenarios

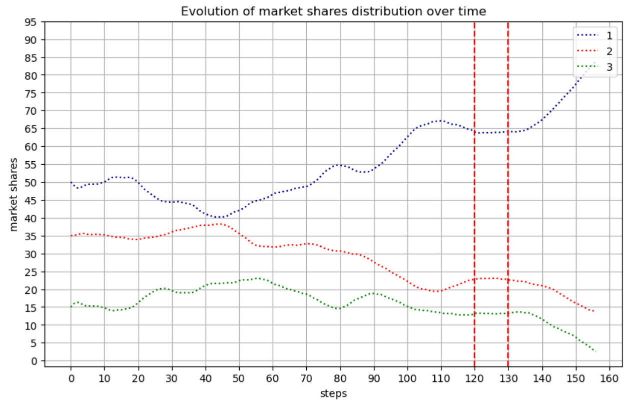

Figure 7: Steady sates case C_1 n° 1, plot 1: market shares of company 1, plot 2: market shares of company 2, plot 3: market shares of the rest of the market, vertical lines: steady state bounds.

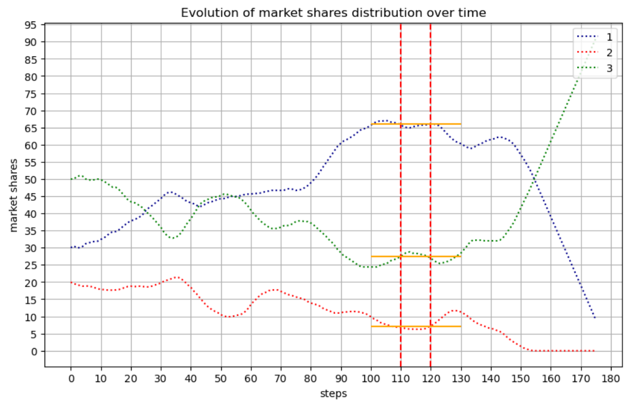

Figure 8: Steady sates case C_1 n° 2, plot 1: market shares of company 1, plot 2: market shares of company 2, plot 3: market shares of the rest of the market, horizontal lines: slopes, vertical red lines: steady state bounds

where company 1 dominates the market as well as in those where it does not.

In Figure 9, the evolution of market shares in a case from the second category is depicted. The red lines indicate the start and end of steady states. It is evident that multiple steady states can exist, influenced by the parameter T used. For lower values of T, more steady states may be identified, whereas higher values of T tend to exhibit fewer steady states. By adjusting this parameter along with the moving average interval, it may become possible to identify larger steady states, as marked by the yellow lines.

To learn more about how much the duration of steady states represent in a simulation, we calculated for each simulation that exhibited steady states, the ratio of the total duration of steady states to the total duration of the simulation.

$$\text{ratio} = \frac{\sum \text{steady states durations}}{\text{total steps of the simulation}} \tag{3}$$

Table 6 illustrates the characteristics of the distribution of the steady states' ratios in the simulation. The results of this analysis highlighted that a limited number of cases exhibited a steady state, and the steady state duration in

Table 4
Combination of parameters with slope $\alpha = 0$, case C_1

Case n°	Company 1 initial market shares	Company 2 initial market shares	e_i	b	Collaboration-frequency	Total steps	Steady state at step
1	30	20	0.5	4	0.4	175	120
2	50	35	0.5	2	0.7	156	110

Table 5
Parameter characteristics for steady states with $\alpha \neq 0$, using C_1

Parameter	Min	Max
Company 1 initial market shares	5	50
Company 2 initial market shares	5	45
The rest of the market final market shares (company 3)	0	100
Cooperation rate (b)	1	62
Collaboration frequency	10%	100%

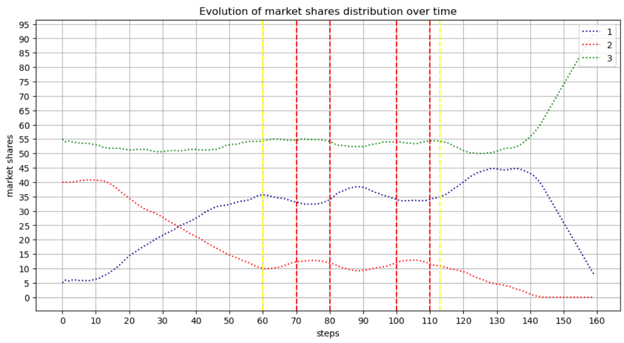

Figure 9: plot 1: market shares of company 1, plot 2: market shares of company 2, plot 3: market shares of the rest of the market. red vertical line steady states' bounds, yellow vertical line represent bounds of a possible steady state.

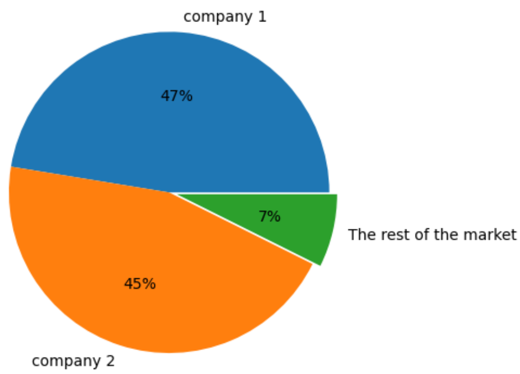

Figure 10: Percentage of times of market dominance by companies (855,000 simulation runs using C_2)

these cases is short compared to the life span of the competition. We observe that the distribution of ratios is right skewed meaning that cases with higher number of steady states are rare. These findings underscore the dynamics of coopetition, where certain companies may hold advantages over others, few short-term steady states can be achieved. In such coopetitive environments, competitive pressures and collaborative dynamics intertwine, creating conditions where temporary equilibria can emerge, albeit transiently.

3.2. Modified Base Model: Analysis with Matrix C_2

The introduction of the ability of company 2 to derive market shares from company 1 allowed company 2 to dominate the market approximately the same number of times as company 1 and this was due to the equal competition from both sides (Figure 10).

Figure 11 illustrates the relationship between the collaboration frequency and the average lifespan of Company 3 (plot 2), as well as the overall competition duration (plot 1). We note that as the collaboration frequency increases, the average lifespan of Company 3 decreases. Conversely, the average duration of the simulation increases. This can be explained by the fact that at higher collaboration frequencies, the rest of the market tends to lose more, compared to lower collaboration frequencies (see Figure 12). Knowing that the average duration is shorter when Company 3 wins, a higher collaboration frequency results in fewer low-duration simulations due to the frequent losses of Company 3. Consequently, because there are fewer short-duration simulations, the overall average duration becomes higher. Additionally, because Companies 1 and 2 are in equivalent competition, the determining factor for the end of the simulation is largely the random sequence of interactions between these two companies.

The same behavior can be seen in the effect of cooperation rate on the duration of the competition and the lifespan of the rest of the market (see Figure 13), which could be due to the same cause as in the collaboration frequency case (see Figure 14). We also observe in Figure 13 that the lifespan of company 3 follows the same trend as in the case of interaction matrix C_1 with the same interesting values $b = 1$ and $b = 2$.

To investigate the relationship between the collaboration and cooperation rates, as in the previous case with matrix C_1, we recorded from the initial experiment (with C_2) the minimum values of the cooperation rates at which either company 1 or company 2 consistently dominates the market. Additionally, we recorded the maximum values of the cooperation rate at which company 3 consistently dominates the market as illustrated in Figure 15 where:

Table 6
Steady states proportion distribution case C_1

	Minimum	Maximum	Mean	Median	Mode
Ratio	9.43%	18.87%	9.98%	9.43%	9.43%

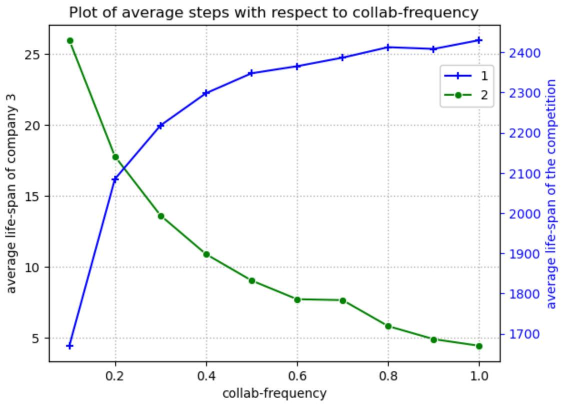

Figure 11: The effect of the collab-frequency on the duration of the competition (C_2),plot 1: the duration of the competition, plot 2: the lifespan of the company 3

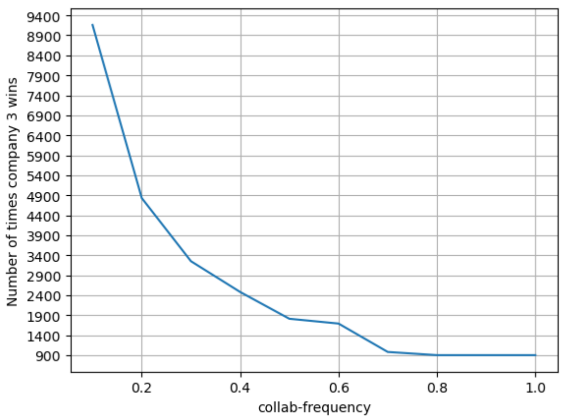

Figure 12: Number of wins of company 3 with respect to the collab-frequency (C_2)

- **Plot 1**: represents the maximum value of b where company 3 always won the competition.
- **Plot 2**: Represents the minimum values of b where company 3 always lost the competition.
- **The scatter plot**: Represents combination (100 point per collaboration frequency) of parameters that lead both company 3 and company 1 or 2 to dominate the market. The frequency is defined for each unique combination of parameters by the number of times company 3 dominates over the total number of simulations. The darker the hue the more frequently company 3 dominates the market.

As in the case of the interaction matrix C_1 we observe the same trend in thresholds, except in this case we have less parameter combinations in which company 3 won more, in higher values of collaboration frequency. We also see that in the collaboration rate 0.1 there is a significant difference

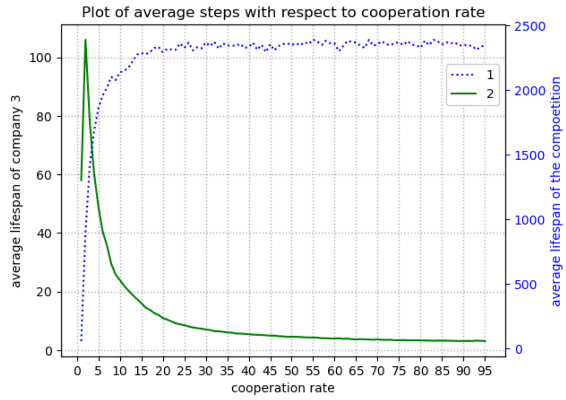

Figure 13: The effect of the cooperation rate on the duration of the competition (C_2), plot 1: the duration of the competition, plot 2: the lifespan of company 3

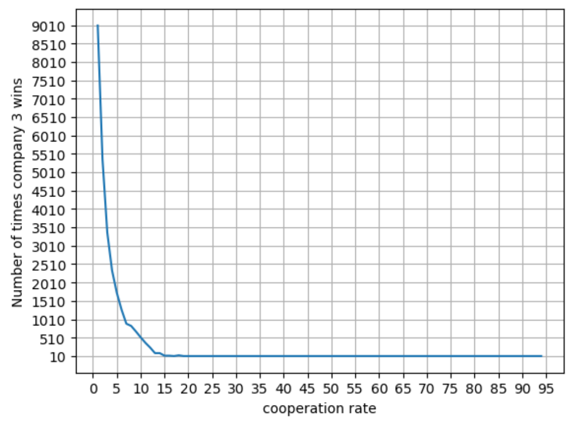

Figure 14: Number of wins of company 3 with respect to the cooperation rate (C_2)

compared to the case C_1.

As for the effect of the initial market shares distribution, we conducted the same simulation as in the case of C_1 where we simulated the effect of the gap between company 1 and 2 initial market shares distribution.

The effect of the gap is relatively similar across all classes of cooperation rates, with much lower frequency of company 3 dominations (see Figure 16). Whereas for the effect of the gap on the frequency with respect to the different values of the collaboration frequency (Figure 17). We observed that the overall frequency is lower than the case of C_1, and the different values of cooperation frequency have relatively the same trend with lower overall frequency. We notice also that the collaboration frequency of 0.1 the company 3 winning frequency tend to achieve its peek values at the extremes of gap values, this might suggest that the significant difference between the companies' market shares

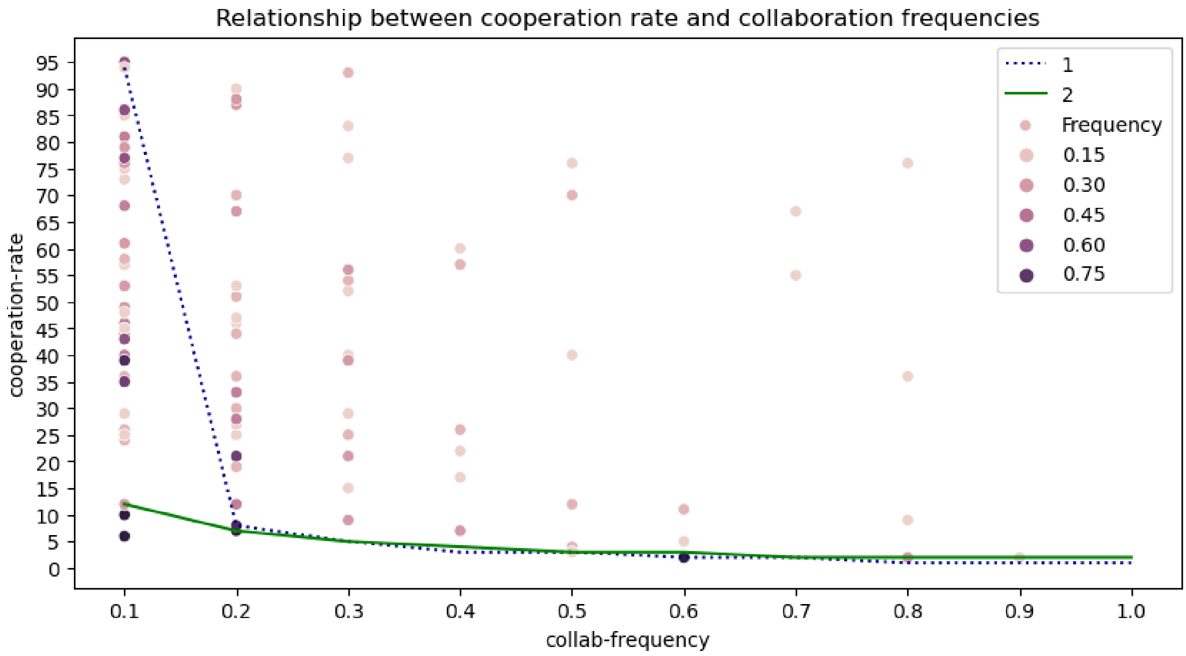

Figure 15: Relationship between cooperation rate and collaboration frequency (C_2)

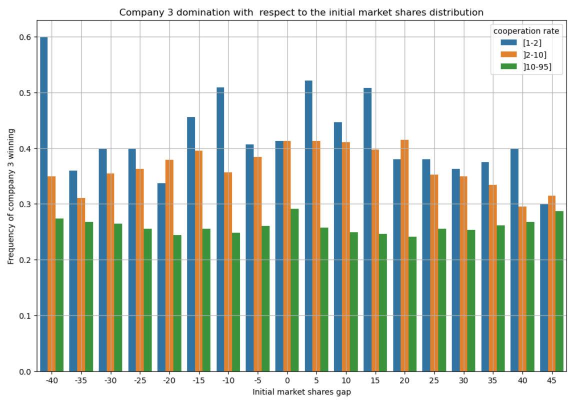

Figure 16: : Effects of the initial market shares distribution and cooperation rates on the dynamics of the market (C_2)

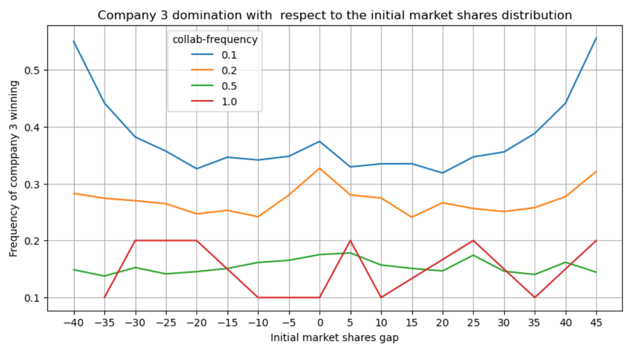

Figure 17: Effects of the initial market shares distribution and collab-frequency on the dynamics of the market (C_2)

might give slightly advantage for company 3 over company 1 and 2.

3.2.1. Steady states analysis

We conducted the same analysis as in the case of interaction matrix C_1, the results in the case of C_2 exhibited more steady states compared to the first case (427 steady state out of 85,500 unique simulation). This might suggest that equal competition between company 1 and 2 might lead to more frequent steady states. We distinguish between two categories:

- The slope $\alpha = 0$.
- The slope $\alpha \in [-0.05, 0.05]$ and $\alpha \neq 0$.

In 17 out of 427 simulations, the slope was $\alpha = 0$. This indicates a higher occurrence within this category compared to the case of interaction matrix C_1 which had only two cases (see example Figure 18).

For the second category (e.g Figure 19), where $\alpha \neq 0$, a total of 410 cases were recorded. Table 7 illustrates the characteristics of the parameter combinations in these cases. Similar to the case of matrix C_1 the steady states occur in scenarios where either company 1 or 2 dominate the market as well as in those where they do not.

The analysis of steady states conducted for cases C_1 and C_2 revealed that C_2 demonstrates steady states more frequently than C_1. This suggests that in scenarios of balanced competition among companies, steady states occur more often compared to situations of unilateral interactions. However, we did not observe any instances where steady states persisted longer than the predefined interval T. Thus, under the specific parameters of α and T used in our study, we did not identify any long-term steady states which spans over multiple consecutive intervals T. Furthermore, the analysis of steady states corroborates the findings of the paper on

Table 7
Parameter characteristics for steady states with $\alpha \neq 0$, using C_2

Parameter	Min	Max
Company 1 initial market shares	5	50
Company 2 initial market shares	5	45
The rest of the market final market shares (company 3)	0	100
Cooperation rate (b)	1	70
Collaboration frequency	10%	100%

Table 8
Steady states proportion distribution case C_2

	Minimum	Maximum	Mean	Median	mode
Ratio	5.35%	16.04%	5.85%	5.35%	5.35%

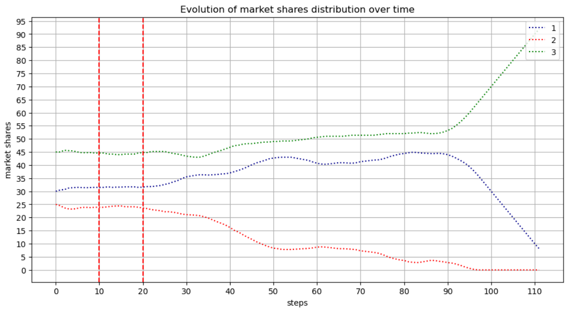

Figure 18: Steady sates case C_2, plot 1: market shares of company 1, plot 2: market shares of company 2, plot 3: market shares of the rest of the market, vertical lines: steady state bounds

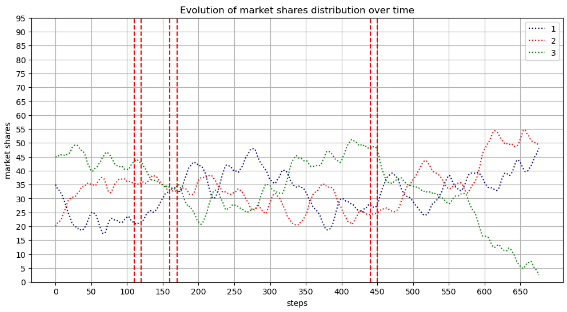

Figure 19: Steady sates case C_2 with $\alpha \neq 0$, plot 1: market shares of company 1, plot 2: market shares of company 2, plot 3: market shares of the rest of the market, vertical lines: steady states' bounds.

globally attractive stable steady states, and the presence of transient ones.

As in the case of C_1 we calculated for each simulation that exhibited steady states, the ratio of the total duration of steady states to the total duration of the simulation (see Table 8).

Similar to the case of the interaction matrix C_1, even though more cases exhibited steady states, the duration of these steady states is very low compared to the total duration of the competition. This leads us to conclude that under equal competition, steady states can occur more often than in the case of uneven competition. However, these steady states remain transient, suggesting that the occurrence of stable steady states might require a competitive environment that favours long-term equilibrium.

4. Key Findings from Simulation Experiments

The simulation experiments conducted in this study yield several significant insights into the dynamics of coopetition and its effects on the market. The main results from the study can be summarized as follows:

- For the first case where C_1 is used there is always the dominance of company 1 over 2.
- All simulations in both cases of C_1 and C_2 end in one of the trivial states where one company dominates the rest, which supports the results of the paper [1] about the global attractor stable steady state.
- Achieving short term stable steady states is possible in C_1 and C_2. This is in line with the results of the paper [1] about the presence of transient steady states.
- The steady states occur more often in an environment with even competition.
- In both C_1 and C_2 cases, the steady states are rare and short-term.
- The cooperation rate b affects the dynamics of the competition.
- The initial distribution of market shares might affect slightly the final state of the competition.
- No matter what the starting conditions are, a company with a market advantage can still overtake the market even if they start with less market shares compared to their competition.
- Collaboration can extend the life span of companies in the market.

5. Discussion

In the paper, the authors modified the Lotka and Volterra equations to describe the coopetition between two companies and the rest of the market. The presented analogy suggests that each interaction will lead to an exchange of market shares, in addition to a collaborative effect that takes market shares from the rest of the market. While this model is an extension of the Lotka and Volterra framework, it introduces a unique dynamic where companies can act as both predator and prey simultaneously, ultimately leading to either one company dominating the other or reaching a certain equilibrium.

With the parameters chosen, particularly the interaction coefficients that gives advantage for company 1 over 2, the model tends to result in one company taking over the other in each interaction. When there are no market shares left for a company, it ceases to exist, akin to a company going out of business in a real-world scenario, which could be interpreted as achieving a monopoly after one company remains. Unlike biological predators that might die out in the absence of prey, in an economic context, the remaining company continues to exist and operate. Thus, we can state that the dependency relationship between the two types of actors, which exist in the biological system, does not exist in the economic context. Which opens the discussion for to what degree the original model form addressing biological systems is transferable to other types of systems, here a market and its actors.

While the presented model captures some aspects of competitive interactions, there may be limitations in fully representing the complexities of economic environments. Future research could explore enhanced models that consider additional factors, such as resource availability or market demand dynamics, to better simulate and understand the dynamics of competitive markets. Such approaches could provide deeper insights into how different economic factors influence the emergence, growth, and sustainability of companies in competitive environments.

6. Conclusion and Outlook

This paper presented an adaptation of the agent-based model from the work by Scherrer et al. [1], utilizing the original study's parameters for simulations. Variations in these parameters were also tested to broaden the understanding of their effects on company behavior in a coopetition setting. The model successfully replicated the original results and provides a foundation for future research.

While the biological model used in the original work captures several aspects of market coopetition, it also has limitations, as discussed above. Thus, its suitability for further more in-depth analysis is limited. Thus, future research could build on the model presented in this paper by incorporating more realistic supply chain dynamics. A comprehensive approach might consider the entire supply chain from suppliers and manufacturers to retailers and consumers, while making necessary assumptions to manage complexity. Including key factors like resource availability, demand disruptions, and supply chain configurations could provide valuable insights into the effects of horizontal collaboration, offering a closer approximation of real-world conditions.

Another promising direction for future research is the incorporation of external, unforeseen effects such as changes in regulations and stochastic market disruptions. Examining whether collaboration can be advantageous during critical times, such as when the market is shrinking, could provide valuable insights into the resilience of coopetition under severe competitive conditions. This approach could also capture challenges that supply chains have faced in recent years, such as pandemics. Again, this work would require a signifincantly more detailed modelled, as these externalities needs to be encaptured.

Additionally, the model could be enhanced by viewing each company as a unique entity, where the internal decision-making processes of individual agents more closely mirror real-world economic behavior. This would allow for a deeper exploration of how internal decisions impact the overall system in a coopetition setting, offering a more detailed understanding of the dynamics at play. Also this can enable research on the motivational settings enabling coopetitive scenarios as mdoel indogenous aspects, which is at the moment given to the model.

Exploring these potential avenues and others could significantly enhance our understanding of horizontal collaboration between competitors, providing valuable insights into how such strategies might be implemented in real-world contexts.

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