The control of selective attention and emerging mathematical cognition: Beyond unidirectional influences

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Abstract

Individual differences in the ability to select information important to our current behavioural goals (i.e., the control of selective attention, henceforth "selective attention" for brevity) are related to individual differences in achievement in mathematics. In this chapter, we discuss, first, the overlap of "selective attention" with other commonly used terms, such as "executive functions" and "cognitive control" in the context of the developmental and mathematical cognition literature. We then consider potential mechanisms underlying these relationships and explore how the control of selective attention and other correlated constructs may play a role in developing mathematical skills. We conclude that assessing the importance of selective attention for learning mathematics requires further longitudinal research and experimental manipulations designed to tease apart the reciprocal interactions between attention and mathematics. Specifically, selective attention not only influences the selection of information to be encoded into memory, but prior knowledge stored in memory also influences the control of attention. We propose that this mutual interplay between attention, memory, and learning contributes to emerging mathematical cognition in early childhood, and as such should be more carefully considered in numerical cognition research.

Keywords: selective attention, executive functions, cognitive control, numerical cognition, development, cognitive training

It has been well established that the control of attention is correlated with mathematics ability, but the causal direction of the relationship remains unclear. In this chapter, we explore how the control of attention plays a role in emerging numerical cognition. First, we provide a brief operationalisation of the constructs encompassed by the term “selective attention”, and their overlap with “executive functions” and “cognitive control”, as they are studied in the context of mathematical development. Following from this, we summarise the correlational evidence for relationships between the control of attention and the development of mathematical abilities. We then critically review research exploring causal mechanisms underlying this relationship and highlight directions for future research. We argue that executive control of attention operates in conjunction with relevant mathematics-specific knowledge (e.g., Amso & Scerif, 2015; Johnson, 2011) to support the acquisition of mathematical skills.

Defining the control of attention: Operationalising construct overlap and differences

Different terms are used throughout the literature to describe the control of attention. The idea of “attentional control”, or a “central executive” is a crucial component of both prominent models of attention (e.g., Desimone & Duncan, 1995; Petersen & Posner, 2012; Posner & Petersen, 1990) and memory (e.g., Baddeley & Hitch, 1974; Baddeley, 2000). One of the most popular models of attention in adults (Posner & Petersen, 1990) and of its development (Rueda et al., 2004) operationalises the control of attention (termed “executive attention”) as the ability to resolve the conflict between competing stimuli and responses (as classically measured by the flanker task, Eriksen & Eriksen, 1974). This is one of the three key processes encompassing attention, alongside orienting of attention in space, and alerting of attention to particular moments in time (see Petersen & Posner, 2012 for an updated overview of this model). In this context, the control of attention encompasses the bias imposed on incoming information to prioritize task relevant stimuli or responses, and suppress or ignore what is not task relevant (Desimone & Duncan, 1995).

The supramodal "Central Executive" within the working memory model (Baddeley & Hitch, 1974) and executive functions (EFs) have been likened to a chief executive officer, as they are responsible for directing and monitoring all other cognitive processes, especially important in situations involving processing unfamiliar stimuli (Goldberg, 2002). There is some consensus that executive function consists of several domains, including complex reasoning and problem solving, working memory (WM), attentional control, cognitive flexibility, self-monitoring, and regulation of cognition, emotion, and behaviour (Miyake et al., 2000). However, there have been many challenges for defining and measuring cognitive control and EFs. As such there currently is no universally accepted model. One popular model is Miyake and colleagues' (2000) model of EFs, which proposes three major EF processes: updating, shifting, and inhibition. *Shifting* refers to flexibility of attention and ability to switch between tasks, *updating* is short for monitoring and updating information in WM, and *inhibition* is defined as the ability to inhibit automatic responses.

Throughout this chapter, we focus on discussing the evidence for the role of these control functions on emerging mathematical cognition. We note that we will use the term "control of attention" to refer to the broader construct that encompasses attentional skills such as executive control, executive attention, selective and sustained attention (e.g., Posner & Petersen, 1990). We shall refer to the model by Miyake and colleagues (Miyake et al., 2000) when discussing EFs. We note that attention control and executive functions differentiate (see the section on cognitive control within this volume), but they also overlap. For example, models of the mechanisms of selective attention have explicitly emphasised the role of representations held in working memory as the source of attentional bias on visual selection (Duncan & Desimone, 1995).

Correlational evidence for relationships between the control of attention and early numeracy

Correlational relationships between the control of attention and mathematics have been found both concurrently and longitudinally in school-aged children (e.g. Hassinger-Das, Jordan, Glutting, Irwin, & Dyson, 2014; Lefevre et al., 2013; LeFevre et al., 2010) as well as in preschoolers (e.g. Steele, Karmiloff-Smith, Cornish, & Scerif, 2012) (for reviews see Bull & Lee, 2014; Cragg & Gilmore, 2014). For example, in one study modelling relationships between multiple domain-general and domain-specific contributors to the development of numeracy in young children, 4- and 5-year-old children completed behavioural assessments of linguistic skills, numeracy skills, and spatial attention, and were subsequently assessed on mathematics achievement two years later (LeFevre et al., 2010). Results supported the hypothesized relationships between linguistic skills and symbolic mathematics, as well as between early quantitative skills and numerical magnitude processing. Crucially, spatial selective attention was related to both symbolic and non-symbolic number skills, both concurrently as well as longitudinally.

Similarly, teacher's classroom observations support the importance of domain-general cognitive control processes for mathematics education. A qualitative study of teachers' perceptions of the role of EFs in mathematics found that most teachers believed that skills that could be classified as EFs were important for learning maths, despite the fact that only 20% of the participating teachers were familiar with the term "EF" (Gilmore & Cragg, 2014). Teachers rated the EF skills almost as highly as the maths specific skills in terms of importance for successful development of mathematics skills. Somewhat surprisingly, shifting and inhibition were rated more highly than updating skills, which empirical studies have found to be *de facto* most strongly related to maths success (e.g. Alloway & Alloway, 2010; Bull & Scerif, 2001; Bull et al., 2008; Bull & Lee, 2014). Notably, both child-based assessments and teacher reports of children's' EFs (Fuhs, Farran, & Nesbitt, 2015) were significantly predictive of gains in mathematic skills over 8 months in a preschool program.

As mathematics and control of attention and EFs are multi-componential, some studies have aimed to tease apart relationships between specific processes across these domains. For example, one study found that inhibition was related to all components of early numeracy in 3-5-year-old children, whereas shifting was related specifically to digit identification and cardinal knowledge of number symbols (Purpura, Schmitt, & Ganley, 2017). Similarly, in older children, inhibition has been repeatedly found to be related to multiple components of mathematics ability (Gilmore et al., 2013; Gilmore, Keeble, Richardson, & Cragg, 2015; Robinson & Dubé, 2013). Notably, inhibition has been specifically associated with performance on nonsymbolic magnitude comparison tasks (e.g. Clayton & Gilmore, 2014), with this in turn often considered a key marker task of emerging numerical cognition (Halberda, Mazzocco, & Feigenson, 2008). These tasks require participants to choose the more numerous of two simultaneously presented arrays. The number of objects in an array is sometimes in conflict with visual cues from the continuous perceptual dimensions of the array (such as the size of objects, the total area they subtend) and inhibitory control plays a role in resolving this conflict (Clayton & Gilmore, 2014). Resolving this conflict also requires having a good understanding of what discrete number is, and young children seem to struggle with this (Merkley et al., 2016; Rousselle & Noël, 2008). In fact, a recent review challenged the notion that the sense of number is innate and proposed instead that separation of discrete numerosity and continuous quantity is something that children have to consciously learn (Leibovich, Katzin, Harel, & Henik, 2016). However, it remains unclear exactly how children learn to select number as the relevant dimension on tasks such as nonsymbolic magnitude comparison. In other words, it is unclear how selective attention bolsters and is bolstered by an increasing awareness of number in the context (and in conflict with) other dimensions representing quantity (Merkley, Scerif & Ansari, in press).

Thus, there is substantial evidence that the control of attention is important for effectively deploying knowledge when doing mathematics (especially as indexed by the role of individual differences in EFs in this case), but it could additionally be that it plays an essential role in the acquisition of basic numerical knowledge. While correlational evidence supports the importance of the control of attention for performing numerical operations, it cannot verify whether and, if so, how exactly selective attention matters for the *learning* of numerical skills. One study found that executive attention predicted growth in arithmetic fluency in 8-10-year-old children (LeFevre et al., 2013), which suggests that the control of attention played a role in the acquisition of these skills.

Similarly, Clark et al. (2013) found that preschool executive control at 3-years of age predicted performance on mathematics assessments at 5-years of age, and an earlier study demonstrated the role of EF prior to school entry as a predictor of growth in both mathematics and reading over the first two years of primary school (Bull et al., 2008).

These findings support the importance of cognitive control also for the learning of mathematical skills, rather than just their mere implementation. At the same time, they are insufficient as support for the causality of this influence. Equally, these studies provide no insights as to the potential mechanisms whereby cognitive, goal-based attentional control could shape maths skills acquisition. It is likely that executive control is necessary for effectively deploying knowledge when doing mathematics, but it could additionally be that executive control plays an essential role in the acquisition of basic numerical knowledge. Cognitive control facilitates the preferential allocation of attention to information that relevant to learning, relative to other, learning-irrelevant information, and maintenance of such focus for prolonged periods of time, so that the selected information can be encoded into long-term memory (Posner & Rothbart, 2007). As such, theoretically, cognitive control should be particularly relevant to educational achievement. However, currently the existing causal evidence supporting this hypothesis encompasses longitudinal data, rather than explicit experimental manipulations. However suggestive, the former are always liable to alternative variables driving significant correlations between early attentional control skills and growth in maths (for example, correlated differences in the environment children experience and are able to learn more from). We therefore move onto discussing causal evidence emerging from direct manipulations of attention control skills.

Does attention play a causal role in learning mathematics?

Investigating whether the control of attention plays a role in the acquisition of basic numerical competencies requires experimental intervention, such as, for example, design and systematic evaluation of preschool curricula (Diamond, Barnett, Thomas, & Munro, 2008). One type of intervention that has gained a substantial amount of attention during the last decade is training of domain-general functions, such as attention. It usually takes the form of a computerized regime designed to improve a specific cognitive function, such as updating in working memory. These interventions are indeed based on the idea that the adaptive nature of the training task, i.e., increasing difficulty with increasing on-task performance, will lead to, first, long-term improvements in the trained functions (tested with untrained tasks, i.e., near transfer), but also to improvements in untrained functions, skills or behaviours (i.e., far transfer). The latter assumption is based on the discussed and similar, correlational evidence reviewed in the previous section. That is, if the control of attention and EFs are malleable and causally related to mathematics achievement, then training-related improvements them should, in principle, lead to higher math performance or growth in math skills.

Indeed, early training studies suggested strongly that attention (e.g., Rueda et al., 2005) and EFs (e.g., Klingberg et al., 2002) can be improved by computerized interventions training the construct of interest (see Diamond, 2012 for a review), and some have even shown that training working memory (updating) can lead to improvements in mathematics (e.g. Kroesbergen, van ’t Noordende, & Kolkman, 2014).

However, verifying causal links between cognitive control and maths skills through this direct intervention route remains difficult for several reasons. First, the degree to which attention or executive training leads to far transfer effects to maths skills varies greatly across studies, which is likely linked to the methodological limitations that characterise the majority of existing, published studies on computerised training of domain-general functions such as working memory (Melby-Lervåg & Hulme et al., 2017). Second, the majority of existing studies in the area have been motivated by application rather than theory. By this we mean, more specifically, that training could be used with the goal of improving outcomes in populations with learning difficulties, or used to investigate causal relationships between cognitive processes. It can be challenging or even unacceptable for a study to try accomplishing both aims (Jolles & Crone, 2012; Wass, Scerif, & Johnson, 2012).

In this section, we will first discuss critically the early studies on far transfer of attention or EF training, to then present the existing evidence for transfer specifically to mathematical abilities. We will then place these findings within the broader context of current evidence for far transfer of domain-general training, and delineate the pre-requisite conditions for a sound experimental study design to test for causal links between cognitive control and mathematical abilities.

The majority of existing domain-general training studies were aimed at improving educational outcomes in children who struggle with both attention and learning, such as children with attention deficit hyperactivity disorder (ADHD) (e.g., Klingberg, Forssberg, & Westerberg, 2002; Green et al. 2012). The initial, highly promising studies demonstrated that training on a mixed regime of tasks targeting EFs led to improvements on the trained tasks, as well as on untrained tasks of visuo-spatial updating and nonverbal reasoning (Klingberg et al., 2002) or conflict resolution (Klingberg et al. 2005).

Crucially, when the same training protocol was tested on a larger, more heterogeneous sample of ADHD children and using more rigorous control of confounding factors across intervention and control groups (e.g., contingent reinforcement, time-on-task with computer training, parent-child interactions, supportive coaching), no difference was found in parental and teacher ratings of behaviour across groups (Chacko et al., 2014). Thus, the efficacy of domain-general training in generating transfer to classroom-relevant behaviour might have been overestimated (Melby-Lervåg et al., 2017). Given the controversy surrounding far transfer of EF training, it is necessary to critically evaluate the evidence for it for numeracy.

Fewer studies have investigated transfer from cognitive control training to academic achievement, including mathematics outcomes. At the same time, many of the existing studies in this area share both the theoretical and methodological limitations characterising the early ADHD training studies. In a study evaluating a computerized WM training program, eight weeks of training did not transfer to standardized mathematics assessments in a sample of typically developing 5-8-year old children (St Clair-Thompson, Stevens, Hunt, & Bolder, 2010). Witt (2011) found that 9-10-year old children who completed a WM training intervention did show greater improvements in mental arithmetic. However, the training program consisted of a variety of games, including practicing a backwards digit recall task, inhibiting distractors, and verbal rehearsal, thus making it difficult to determine which of the trained tasks led to the observed improvements. This is especially pertinent as the study included only a passive, but no active, control comparison group. In contrast, a study in 6-year-olds from low (socio-economic status) SES areas showed that children who played adaptive games targeting updating, planning, and inhibition improved at school measures of language and math compared to an active control group who played less cognitively demanding computer games (Goldin et al., 2014). However, the study lacked a passive control group, and therefore it is not possible to rule out the possibility that improvements were due to simple practice effects or change over time on tests administered before and after training, rather than to training benefits exclusively.

It has been postulated that cognitive training in general may be more effective in younger children, possibly due to heightened general plasticity of their brains and/or lack of differentiation of their brain networks, including those involved in attentional control (Jolles & Crone, 2012; Wass et al., 2012). However, to date, both domain-general and domain-specific computerized interventions aiming to improve numeracy skills in preschool have revealed inconsistent results. For example, Kroesbergen and colleagues (2014) compared training effects across a group of 51 low-performing 5-6-year-old children, where some completed domain-general WM training, others completed a WM training with number-specific stimuli, and others completed no training at all. Children in both training groups showed improvements in post-training measures of WM as well as on a standardised early numeracy skills assessment. Notably, only the domain-specific WM training group showed significant improvements on a nonsymbolic magnitude comparison task, suggesting that domain-specific training more strongly influences mathematical abilities. This contrasts with the results of Passolunghi and Costa (2014), who also compared WM and numeracy training interventions in a group of preschoolers. They demonstrated that training numeracy led to improvements in math skills, but not in WM, whereas training WM led to transfer in both WM and numeracy. However, low sample sizes (~50 children in total) as well as a lack of a passive (Passolunghi & Costa, 2014) or an active control group prevents drawing strong conclusions from both studies. Another EF intervention study found that the training group performed better than the control group on an early numeracy assessment following training, but did not measure numeracy at baseline (Blakey & Carroll, 2015). Another study that used very similar interventions in typically developing pre-schoolers found that domain-specific counting training was more effective at improving mathematics performance than a combination of working memory and counting training (Kyttälä, Kanerva, & Kroesbergen, 2015). The evidence for a causal role of control functions in early mathematics learning is at best mixed. However, it is possible that domain-specific trainings may act by improving inhibitory functions. In one training study aimed at improving nonsymbolic magnitude comparison in preschool-aged children from low income homes, such improvements were driven by performance on trials on which number was in conflict with continuous quantity (Fuhs & McNeil, 2015).

To summarize, currently it remains unclear whether training the control of attention reliably leads to improvements in educationally-relevant mathematics outcomes. First of all, studies on low-SES or low-achieving populations (Fuhs & McNeil, 2015; Goldin et al., 2014; Kroesbergen et al., 2012) have focused on applied goals and therefore are, in terms of experimental design, less appropriate for addressing theoretical questions pertaining to mechanisms underlying transfer. Specifically, the lack of double-blind experimental procedures that frequently characterise the applied studies may contribute to the overestimation of true effects of training cognitive control on far transfer (Sonuga-Barke et al. 2013. This notwithstanding, the current evidence for transfer from training

EFs to math is mixed. Notably, it shares many of the limitations characterising other studies on cognitive training and far transfer. A lack of appropriate control group(s) renders impossible to distinguish effects driven by training compared to placebo effects (no active control) or regular development (no passive control). In turn, small sample sizes may lead to overestimations of wider transfer effects. This was demonstrated by a recent meta-analysis of 87 publications with 145 experimental comparisons by Melby-Lervåg et al. (2017) who concluded that wider transfer of the effects of training of cognitive control, such as WM, to other cognitive processes, including maths skills, is not convincingly supported by existing data. Notably, the authors suggest that even in the more rigorous studies, which used rigorous control groups and have sufficient statistical power, the effects of wide transfer are only statically small in size (Cohen's d ~0.25; e.g., Sonuga-Barke et al. 2013). Crucially, this effect size estimate is likely exaggerated, as many researchers finding null results on the far transfer of EFs, on behavioural outcomes and/or educational achievements, fail to publish their findings (i.e., publication bias). This problem may be alleviated by the greater emphasis on pre-registration. More research is presently necessary in order to clarify whether training of attentional control reliably leads to improvements in educationally-relevant mathematics outcomes.

To summarise the points we have made in this section, an ideal training attention / EF intervention study should: a) randomly assign participants to groups and have assessors blind to their assignment (double-blindness), b) have a sample size that is sufficiently large to afford high statistical power, c) include a group that does not receive any training as well as an active control group that does some form of intervention that is not hypothesized to lead to the same improvements as the experimental training regime, and also d) use multiple measures of the construct being trained as well as constructs to which far transfer is hypothesized (Gathercole et al., 2012; Rabipour & Raz, 2012; Shipstead et al., 2012).

Bi-directional relationships between attention and math: Expertise influences the deployment of attention

Most previous studies exploring relationships between the control of attention and math have aimed to test the unidirectional causal hypothesis that attention is important for selecting information relevant to learning. However, bidirectional relationships between EFs and mathematics were found in one longitudinal study (Van der Ven, Kroesbergen, Boom, & Leseman, 2012) and another showed that math achievement predicted EFs, but that EFs did not predict mathematics over and above early math measures (Watts et al., 2015). Therefore, improvements in mathematics could in turn contribute to improvements in attentional control and EFs. Further research is needed to determine the direction of causality between the two cognitive capacities (Clements et al., 2016). We propose that the control of attention and domain-specific mathematics knowledge dynamically interact over the course of the development of numerical cognition. Specifically, attentional control processes enable children to select information relevant to math to be encoded into short- and then long-term memory. At the same time, previously learned knowledge guides how strongly attentional resources are allocated to task-relevant stimuli in the environment, which in turn improves as knowledge accrues (e.g. Amso & Scerif, 2015; Johnson, 2011).

One learning challenge in the domain of mathematics is acquiring the meaning of numerical symbols (words and Arabic digits). Children learn the meaning of the number words one to four sequentially and there is some evidence that their prior knowledge influences subsequent learning (Huang, Spelke, & Snedeker, 2010). Specifically, a group of three-year old children were taught the meaning of a number word they had not yet learned (Huang et al., 2010). Children who knew ‘two’, as demonstrated by the fact that they could reliably give an experiment two objects when asked, were trained on the word ‘three’. Children who knew ‘three’ were trained on the word ‘four’. The training involved an experimenter showing children cards depicting different numbers of objects and verbally indicating whether the card did or did not have the target number of objects. Results showed that children who learned ‘four’ were more likely to generalize the newly acquired knowledge and apply the word in novel contexts compared to children who learned ‘three’. These results could be interpreted as evidence suggesting that children who have more prior number word knowledge are more likely to select abstract numerosity as the relevant referent when learning a new number word.

Higher proficiency with numerical symbols is also associated with better learning of a novel abstract symbol set (Merkley, 2015). In a series of studies, adults, older children (10-year-olds) and young children (6-year-olds) were taught to associate a set of abstract symbols with numerical meaning. Following the learning phase, they performed a magnitude comparison task with the learned symbols. During the learning phase, the symbols were paired with nonsymbolic arrays of dots and participants learned to associate the symbols with the approximate magnitude of the corresponding arrays. Half of the participants were also taught the order of the symbols from smallest to largest. Adults and older children performed equally well on the comparison task regardless of whether they had been explicitly cued to the order of the symbols, which suggests that they could infer the order of the symbols based on magnitude information alone. However, the younger children who were not cued to order failed to perform above chance on the comparison task. Thus, it appears that greater experience with real numbers is associated with more efficient formation of novel symbolic representations. In particular, it seems that having more experience with number exerts a top-down guidance on the ability to extract numerosity from nonsymbolic arrays, a process that in turn is likely supported by goal-based attentional control.

In addition to learning the meaning of numerical symbols, young children must also learn to selectively attend to discrete number when faced with competing cues associated with continuous magnitude (Leibovich et al., 2016). In one study, children who did not know the cardinal meaning of number words failed to perform above chance on a nonsymbolic magnitude comparison task when numerosity conflicted with continuous quantity (Negen & Sarnecka, 2015). This suggests that having a better understanding of the cardinal meaning of number symbols may facilitate attention to numerosity, but the direction of this relationship is still debated. Uncovering the causal mechanisms underlying the relationship between number knowledge and attention to number requires testing bi-directional hypotheses and moving beyond correlational, cross-sectional experimental designs (Merkley, et al., in press).

Conclusion

Relationships between the control of attention and mathematics abilities have been demonstrated in many correlational studies. However, the causal nature of this relationship remains unclear. Some executive function training interventions have been associated with far transfer to mathematics, yet others have failed to find similar results. Further, more rigorous investigations are necessary to determine whether cognitive control interventions are indeed effective at improving mathematical outcomes, and which populations stand to benefit most. Moreover, bi-directional relationships between selective attention and the development of numerical cognition should be more systematically investigated. Not only does selective attention influence which information is filtered into memory, but prior knowledge also guides the deployment of attention. This interplay between the control of selective attention and emerging numeracy likely plays a role in learning early mathematics.

References

Alloway, T. P., & Alloway, R. G. (2010). Investigating the predictive roles of working memory and IQ in academic attainment. *Journal of Experimental Child Psychology*, *106*(1), 20–9. http://doi.org/10.1016/j.jecp.2009.11.003

Amso, D. & Scerif, G. (2015) The attentive brain: Insights from developmental cognitive neuroscience. *Nature Reviews Neuroscience* 16(10):606–19.

Baddeley, A. (2000). The episodic buffer: a new component of working memory? *Trends in Cognitive Sciences*, *4*(11), 417–423. http://doi.org/10.1016/S1364-6613(00)01538-2

Baddeley, A. D., & Hitch, G. (1974). Working memory. *Psychology of learning and motivation*, *8*, 47-89.

Best, J. R., Miller, P. H., & Naglieri, J. a. (2011). Relations between Executive Function and Academic Achievement from Ages 5 to 17 in a Large, Representative National Sample. *Learning and Individual Differences*, *21*(4), 327–336. http://doi.org/10.1016/j.lindif.2011.01.007

Blakey, E., & Carroll, D. J. (2015). A short executive function training program improves preschoolers' working memory. *Frontiers in Psychology*, *6*(NOV), 1–8. http://doi.org/10.3389/fpsyg.2015.01827

Bull, R. (2009). NIH Public Access. *Office*, *33*(3). http://doi.org/10.1080/87565640801982312.Short-Term

Bull, R., Espy, K. A., Wiebe, S. a, Sheffield, T. D., & Nelson, J. M. (2011). Using confirmatory factor analysis to understand executive control in preschool children: sources of variation in emergent mathematic achievement. *Developmental Science*, *14*(4), 679–92. http://doi.org/10.1111/j.1467-7687.2010.01012.x

Bull, R., & Lee, K. (2014). Executive Functioning and Mathematics Achievement. *Child Development Perspectives*, *8*(1), 36–41. http://doi.org/10.1111/cdep.12059

Bull, R., & Scerif, G. (2001). Executive functioning as a predictor of children's mathematics ability: inhibition, switching, and working memory. *Developmental Neuropsychology*, *19*(3), 273–93. http://doi.org/10.1207/S15326942DN1903_3

Chacko, A., Bedard, A. C., Marks, D. J., Feirsen, N., Uderman, J. Z., Chimiklis, A., ... & Ramon, M. (2014). A randomized clinical trial of Cogmed working memory training in school-age children with ADHD: a replication in a diverse sample using a control condition. *Journal of Child Psychology and Psychiatry*, *55*(3), 247-255.

Clark, C. a C., Pritchard, V. E., & Woodward, L. J. (2010). Preschool executive functioning abilities predict early mathematics achievement. *Developmental Psychology*, *46*(5), 1176–91. http://doi.org/10.1037/a0019672

Clayton, S., & Gilmore, C. (2014). Inhibition in dot comparison tasks. *Zdm*. http://doi.org/10.1007/s11858-014-0655-2

Clements, D. H., Sarama, J., & Germeroth, C. (2016). Learning executive function and early mathematics: Directions of causal relations. *Early Childhood Research Quarterly*, *36*, 79-90.

Cragg, L., & Gilmore, C. (n.d.). Skills underlying mathematics : The role of executive function in the development of mathematics proficiency, 1–16.

Desimone, R., & Duncan, J. (1995). Neural mechanisms of selective visual attention. *Annual review of neuroscience*, *18*(1), 193-222.

Diamond, A., Barnett, W. S., Thomas, J., & Munro, S. (2008). NIH Public Access, *318*(5855), 1387–1388.

Diamond, A. (2012). Activities and programs that improve children's executive functions. *Current directions in psychological science*, *21*(5), 335-341.

Eriksen, B. A., & Eriksen, C. W. Effects of noise letters upon the identification of a target letter in a nonsearch task. Perception & Psychophysics, 1974, 16, 143-149.

Fuhs, M. W., Farran, D. C., & Nesbitt, K. T. (2015). Prekindergarten children's executive functioning skills and achievement gains: The utility of direct assessments and teacher ratings. *Journal of Educational Psychology*, *107*(1), 207–221. http://doi.org/http://dx.doi.org/10.1037/a0037366

Fuhs, M. W., & McNeil, N. M. (2015). The Role of Non-Numerical Stimulus Features in Approximate Number System Training in Preschoolers from Low-Income Homes Mary. *Statewide Agricultural Land Use Baseline 2015*, *1*, 1–58. http://doi.org/10.1017/CBO9781107415324.004

Gathercole, S. E., Dunning, D. L., & Holmes, J. (2012). Cogmed training: Let's be realistic about intervention research. *Journal of Applied Research in Memory and Cognition*, *1*(3), 201–203. doi:10.1016/j.jarmac.2012.07.007

Gilmore, C., Attridge, N., Clayton, S., Cragg, L., Johnson, S., Marlow, N., … Inglis, M. (2013). Individual differences in inhibitory control, not non-verbal number acuity, correlate with mathematics achievement. *PloS One*, *8*(6), e67374. http://doi.org/10.1371/journal.pone.0067374

Gilmore, C., & Cragg, L. (2014). Teachers ' understanding of the role of executive functions in maths, 37474.

Gilmore, C., Keeble, S., Richardson, S., & Cragg, L. (2015). The role of cognitive inhibition in different components of arithmetic. *Zdm*, (2010). http://doi.org/10.1007/s11858-014-0659-y

Goldberg, E. (2002). *The executive brain: Frontal lobes and the civilized mind*. Oxford University Press, USA.

Goldin, A. P., Hermida, M. J., Shalom, D. E., Elias Costa, M., Lopez-Rosenfeld, M., Segretin, M. S., … Sigman, M. (2014). Far transfer to language and math of a short software-based gaming intervention. *Proceedings of the National Academy of Sciences of the United States of America*, 1–6. doi:10.1073/pnas.1320217111

Green, C. T., Long, D. L., Green, D., Iosif, A. M., Dixon, J. F., Miller, M. R., ... & Schweitzer, J. B. (2012). Will working memory training generalize to improve off-task behavior in children with attention-deficit/hyperactivity disorder?. *Neurotherapeutics*, *9*(3), 639-648.

Halberda, J., Mazzocco, M. M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature*, *455*(7213), 665–8. http://doi.org/10.1038/nature07246

Hassinger-Das, B., Jordan, N. C., Glutting, J., Irwin, C., & Dyson, N. (2014). Domain-general mediators of the relation between kindergarten number sense and first-grade mathematics achievement. *Journal of Experimental Child Psychology*, *118*, 78–92. http://doi.org/10.1016/j.jecp.2013.09.008

Huang, Y. T., Spelke, E., & Snedeker, J. (2010). When is four far more than three? Children’s generalization of newly acquired number words. *Psychological Science*, *21*(4), 600–6. http://doi.org/10.1177/0956797610363552

Johnson, M. H. (2011) Interactive specialization: A domain-general framework for human functional brain development? *Developmental Cognitive Neuroscience* 1(1):7–21.

Jolles, D. D., & Crone, E. A. (2012). Training the developing brain: a neurocognitive perspective. *Frontiers in human neuroscience*, *6*.

Klingberg, T., Forssberg, H., & Westerberg, H. (2002). Training of working memory in children with ADHD. *Journal of Clinical and Experimental Neuropsychology*, *24*(6), 781–91. http://doi.org/10.1076/jcen.24.6.781.8395

Klingberg, T., Fernell, E., Olesen, P. J., Johnson, M., Gustafsson, P., Dahlström, K., ... & Westerberg, H. (2005). Computerized training of working memory in children with ADHD-a randomized, controlled trial. *Journal of the American Academy of Child & Adolescent Psychiatry*, *44*(2), 177-186.

Kroesbergen, E. H., van ’t Noordende, J. E., & Kolkman, M. E. (2012). Training working memory in kindergarten children: Effects on working memory and early numeracy. *Child Neuropsychology : A Journal on Normal and Abnormal Development in Childhood and Adolescence*, (January), 37–41. http://doi.org/10.1080/09297049.2012.736483

Kyttälä, M., Kanerva, K., & Kroesbergen, E. (2015). Training counting skills and working memory in preschool. *Scandinavian Journal of Psychology*, n/a-n/a. http://doi.org/10.1111/sjop.12221

Lefevre, J.-A., Berrigan, L., Vendetti, C., Kamawar, D., Bisanz, J., Skwarchuk, S.-L., & Smith-Chant, B. L. (2013). The role of executive attention in the acquisition of mathematical skills for children in Grades 2 through 4. *Journal of Experimental Child Psychology*, *114*(2), 243–61. http://doi.org/10.1016/j.jecp.2012.10.005

LeFevre, J.-A., Fast, L., Skwarchuk, S.-L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: longitudinal predictors of performance. *Child Development*, *81*(6), 1753–67. http://doi.org/10.1111/j.1467-8624.2010.01508.x

Leibovich, T., Katzin, N., Harel, M., & Henik, A. (2016). From “sense of number” to “sense of magnitude” – The role of continuous magnitudes in numerical cognition. *Behavioral and Brain Sciences*, (1), 1–62. http://doi.org/10.1017/S0140525X16000960

Melby-Lervåg, M., Redick, T. S., & Hulme, C. (2016). Working memory training does not improve performance on measures of intelligence or other measures of “far transfer” evidence from a meta-analytic review. *Perspectives on Psychological Science*, *11*(4), 512-534.

Merkley, R. (2015). Beyond number sense: contributions of domain general processes to the development of numeracy in early childhood. Doctor of Philosophy thesis, University of Oxford, Oxford, UK.

Merkley, R., Thompson, J., & Scerif, G. (2016). Of huge mice and tiny elephants: Exploring the relationship between inhibitory processes and preschool math skills. *Frontiers in Psychology*, *6*(JAN). http://doi.org/10.3389/fpsyg.2015.01903

Merkley, R., Scerif, G., & Ansari, D. (in press). What is the precise role of cognitive control in the development of a sense of number? (Commentary on Leibovich et al.). *Behavioral and Brain Sciences*

Miyake, a, Friedman, N. P., Emerson, M. J., Witzki, a H., Howerter, a, & Wager, T. D. (2000). The unity and diversity of executive functions and their contributions to complex "Frontal Lobe" tasks: a latent variable analysis. *Cognitive Psychology*, *41*(1), 49–100. http://doi.org/10.1006/cogp.1999.0734

Negen, J., & Sarnecka, B. W. (2014). Is there really a link between exact-number knowledge and approximate number system acuity in young children? *The British Journal of Developmental Psychology*, 1–14. http://doi.org/10.1111/bjdp.12071

Passolunghi, M. C., & Costa, H. M. (2014). Working memory and early numeracy training in preschool children. *Child Neuropsychology : A Journal on Normal and Abnormal Development in Childhood and Adolescence*, (November), 1–18. http://doi.org/10.1080/09297049.2014.971726

Petersen, S. E., & Posner, M. I. (2012). The attention system of the human brain: 20 years after. *Annual review of neuroscience*, *35*, 73-89.

Posner, M. I., & Petersen, S. E. (1990). The attention system of the human brain. *Annual Review of Neuroscience*, *13*, 25–42.

http://doi.org/10.1146/annurev.ne.13.030190.000325

Posner, M. I., & Rothbart, M. K. (2007). Research on attention networks as a model for the integration of psychological science. *Annu. Rev. Psychol.*, *58*, 1-23.

Purpura, D. J., Schmitt, S. A., & Ganley, C. M. (2017). Foundations of mathematics and literacy: The role of executive functioning components. *Journal of Experimental Child Psychology*, *153*, 15–34. http://doi.org/10.1016/j.jecp.2016.08.010

Rabipour, S., & Raz, A. (2012). Training the brain: Fact and fad in cognitive and behavioral remediation. *Brain and Cognition*, *79*(2), 159-179.

Robinson, K. M., & Dubé, A. K. (2013). Children's additive concepts: Promoting understanding and the role of inhibition. *Learning and Individual Differences*, *23*, 101–107. http://doi.org/10.1016/j.lindif.2012.07.016

Rousselle, L., & Noël, M.-P. (2008). The development of automatic numerosity processing in preschoolers: evidence for numerosity-perceptual interference. *Developmental Psychology*, *44*(2), 544–60. http://doi.org/10.1037/0012-1649.44.2.544

Rueda, M. R., Posner, M. I., & Rothbart, M. K. (2005). The development of executive attention: Contributions to the emergence of self-regulation. *Developmental neuropsychology*, *28*(2), 573-594.

Shipstead, Z., Redick, T. S., & Engle, R. W. (2012). Is working memory training effective? *Psychological Bulletin*, *138*(4), 628–54. doi:10.1037/a0027473

Sonuga-Barke, E. J., Brandeis, D., Cortese, S., Daley, D., Ferrin, M., Holtmann, M., ... & Dittmann, R. W. (2013). Nonpharmacological interventions for ADHD: systematic review and meta-analyses of randomized controlled trials of dietary and psychological treatments. *American Journal of Psychiatry*, *170*(3), 275-289.

St Clair-Thompson, H. L., & Gathercole, S. E. (2006). Executive functions and achievements in school: Shifting, updating, inhibition, and working memory.

Quarterly Journal of Experimental Psychology (2006), *59*(4), 745–59. http://doi.org/10.1080/17470210500162854

St Clair-Thompson, H., Stevens, R., Hunt, A., & Bolder, E. (2010). Improving children's working memory and classroom performance. *Educational Psychology*, *30*(2), 203–219. http://doi.org/10.1080/01443410903509259

Steele, A., Karmiloff-Smith, A., Cornish, K., & Scerif, G. (2012). The multiple subfunctions of attention: differential developmental gateways to literacy and numeracy. *Child Development*, *83*(6), 2028–41. http://doi.org/10.1111/j.1467-8624.2012.01809.x

Van der Ven, S. H. G., Kroesbergen, E. H., Boom, J., & Leseman, P. P. M. (2012). The development of executive functions and early mathematics: a dynamic relationship. *The British Journal of Educational Psychology*, *82*(Pt 1), 100–19. http://doi.org/10.1111/j.2044-8279.2011.02035.x

Wass, S. V., Scerif, G., & Johnson, M. H. (2012). Training attentional control and working memory–Is younger, better?. *Developmental Review*, *32*(4), 360-387.

Watts, T. W., Duncan, G. J., Chen, M., Claessens, A., Davis-Kean, P. E., Duckworth, K., ... & Susperreguy, M. I. (2015). The role of mediators in the development of longitudinal mathematics achievement associations. *Child development*, *86*(6), 1892-1907.

Witt, M. (2011). School based working memory training : Preliminary finding of improvement in children ' s mathematical performance. *Advances in Cognitive Psychology, 7*(0), 7–15.