RADIAL B-SPLINES FOR OPTIMAL DETECTION IN IMAGES

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ABSTRACT

We present a two-step pipeline for the detection of patterns of interest in an image. An optimal detector is learned from a single template, and this detector is then used to retrieve the unknown location and orientation of all occurrences of the pattern in the image. Our approach relies on continuous-domain additive image model, where the analysed image is the sum of the template and a background signal with self-similar isotropic power-spectrum. We are then able to compute the optimal filter for the detection tasks, which maximizes the SNR criterion for this image model.

In [3], we thoroughly describe the underlying image model and detection procedure. In this work, we focus on the use of splines in the construction of the detector, and on how it provides the right framework to approximate the initial template. Cardinal B-splines are well-known for the capability of approximating continuous-domain functions based on discrete measurements [2]. We define radial splines to be radial functions $\hat{f}(r) \in L_2(\mathbb{R}^2)$ defined for a radius $r \geq 0$ as $\hat{f}(r) = \sum_{k\in\mathbb{Z}} \frac{c[k]}{r_0} \beta_M(r/r_0 - k)$, with $r_0 > 0$ the discretization step and $c[k]$ the spline coefficients of $\hat{f}$, and β_M is the polynomial B-spline of order $M \geq 1$ [2]. Our formulation relies on the adaptation of the classical spline expansion to $2D$ radial functions. The expression of the optimal detector given a single template T is obtained in Fourier domain and polar coordinates as

$$\hat{f}(r,\theta) = r^{2\gamma} \sum_{|n|\leq N} \widehat{T}_n(r) \mathrm{e}^{\mathrm{j}n\theta} \quad (1)$$

$$= r^{2\gamma} \sum_{|n|\leq N} \sum_{k\in\mathbb{Z}} \frac{c_n[k]}{r_0} \beta_M \left(\frac{r}{r_0} - k\right) \mathrm{e}^{\mathrm{j}n\theta}, \quad (2)$$

where N is the maximal angular frequency of the detector, $\gamma \geq 0$ is the power-spectrum rate of the background, and $\widehat{T}_n$ are the Fourier domain spline radial profiles of the template T with spline coefficients $c_n[k]$. Its angular dependency is controlled via the use of circular harmonics $\mathrm{e}^{\mathrm{j}n\theta}$, and its radial structure is captured with radial splines.

This construction has three crucial consequences for practical implementation: first, any rotated versions of the detector can be readily computed in closed-form exploiting the frequency restriction [1]. Second, the spline coefficients of the detector are easily obtained through efficient spline fitting algorithms. Last, our model is rich enough to include background models with various power-spectrum through the self-similarity parameter γ. When correctly estimated, this significantly improves the detection performance. Formulating the construction of the detector in a spline setting allows us to benefit from the powerful approximation properties of these functions. Any radial function can indeed be efficiently approximated by radial B-splines up to an arbitrary, user-defined level of precision. As a consequence, we can represent any $2D$ template with arbitrary precision. Finally, we demonstrate the practical usefulness of our method on a variety of template approximation and pattern detection experiments.

1. REFERENCES

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