Short-term hydro scheduling of a variable speed pumped storage hydropower plant considering head loss in a shared penstock

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Abstract. In this paper we present in detail how to handle the nonlinear and state-dependent elements in the power consumption function (PCF) of a variable speed pump (VP). The function is formulated by piecewise linear approximations with dynamically specified breakpoints. In the proposed method, the head variation resulting from the change in the water level of the upstream and downstream reservoirs and head loss caused by the friction of water on the penstock wall are both considered. Furthermore, we put forward a heuristic to incorporate the head loss in a penstock shared by multiple VPs in a short-term hydro scheduling tool used for daily operation in the real world. The case study is taken from the Limmern pumped storage hydropower plant with 4×250 MW reversible Francis pump-turbines with variable speed technology. Each penstock is shared by two of the four units. The numerical results demonstrate that the appropriate formulation of the PCF and the accurate representation of the head loss in a shared penstock are crucial for obtaining realistic and optimal scheduling for VPs.

1. Introduction

With the increasing generation from intermittent renewable energies, pumped hydroelectric energy storage is widely adopted as a way to accommodate the power fluctuations of the system. In comparison with conventional fixed speed pumps (FPs), variable speed pumps (VPs) enlarge the operational flexibility of pumped storage hydropower plant (PSHPs) by providing power regulation in pumping mode. That is, given a certain head, the rate of pumped water flow for a FP is fixed and hence the consumed power is fixed, whereas a VP can vary its power consumption within the rotation speed limits.

The research on short-term hydro scheduling (STHS) of FPs is widely conducted [1]. However, in most papers, though head variation is taken into consideration in generating mode, it is neglected in pumping mode [2, 3]. Head variation is caused by the fact that more power consumption is needed with the increase of the water level difference between the upper and lower reservoirs. Reference [4] first considered the head variation for a FP in pumping mode and emphasized the importance of including head variation in scheduling results to avoid significant discrepancy from a real situation. By contrast, STHS of VPs in competitive electricity markets has received little attention in the technical literature [1]. The existing literature concentrates mainly on the theoretical analysis of profitability and feasibility of variable speed technology [5, 6], while few focuses on the practical problems occurred in the operational scheduling of VPs.

Since VPs can be operated within certain range in both generating and pumping modes, in fact, a plant with VPs faces the same challenges in STHS as a pure power generating plant confronts. Operators must consider not only constraints due to complex cascaded watercourses and multipurpose use, but also technical constraints, such as head-dependent working limits, and various market constraints. The electric power consumed by a VP is a nonlinear function of the water pumped, the net head and the turbine/generator efficiency. The net head is defined as the sum of the gross head and the penstock head loss. The unit efficiency is also a nonlinear function of the net head and the water pumped. The appropriate formulation of the power consumption function (PCF) of the VP is crucial to attain realistic and optimal scheduling. However, to our knowledge, neither head variation nor penstock head loss for VPs is mentioned in the existing papers.

In this paper, we approximate the nonlinear PCF with a piecewise linear input-output (I/O) curve for one specific VP. Different from the existing literature concerning the approximation of the hydropower production function of the generating unit [7-9], each breakpoint of the I/O curve is dynamically calculated for each period. Then instead of representing the PCF as a family of curves, each one for a specified value/interval of the head, only one I/O curve will be built and the net head at each breakpoint of the curve is distinct.

Furthermore, the penstock head loss, as one of the important issues that cannot be ignored in real-world operation, is typically a nonlinear element in the PCF of a VP. Similar as the penstock head loss for a generating unit [10], penstock head loss for a VP is associated with the friction of water on the penstock wall and can be represented as a quadratic function of the pumped water flow. The penstock head loss (in meter) can be transformed into the penstock power loss (in megawatt) as a cubic function of the flow in that tunnel [11]. Note that "head/power loss" for a generating unit means the reduction of the head and hence less production of the power generated for a given water discharge; whereas, for a pumping unit, it implies the increase of the head and consequently more consumption of the energy for a given pumped flow.

In this paper, a shared penstock refers to a penstock that feeds more than one unit, i.e. turbine, pump or reversible pump-turbine. The head loss in a shared penstock depends on the operating situation of all the units that are connected to this penstock. However, whether and at what level the units should run, depends on how large the loss is. This makes it practically difficult to accurately model the penstock loss since the dispatch of the units is unknown prior to optimization. We propose a heuristic to solve the problem: when building the I/O curve for one VP, we firstly do not include penstock head loss. It leads to the underrated power consumption for the given pumped water flow. We then include the sum of power loss for each pump in the plant energy balance constraint. In order to incorporate the nonlinear function into a mixed integer linear programming (MILP) formulation framework, the power loss in the penstock is approximated by a convex piecewise linear function of the total flow in the penstock.

The main goals and contributions of this paper are: 1) to convert the nonlinear PCF of a VP into a piecewise linear unit I/O curve; 2) to explicitly formulate the head loss in a shared penstock for a VP; 3) to propose the heuristic that can (nearly) optimally solve the nonlinearity and state-dependency. The remainder of the paper is organized as follows. In Section 2, firstly the main characteristics of a PSHP with 4 reversible VPs are presented. A brief introduction is then given to an operational STHS tool where the proposed modelling of power loss for VPs is implemented. In Section 3, we present the mathematical formulation for the scheduling model, with special emphasis on the approximation of the PCF and the modelling of power loss in a shared penstock. The presented method of determining the unit I/O curve and the heuristic of incorporating power loss in STHS are then computationally evaluated in Section 4. The conclusions and suggestion for future research are summarized in Section 5.

2. Problem description

In this section, we first give the background information of the Limmern PSHP that is later used in the case study. Then, we describe the solution strategy for an operational STHS tool in which the presented method and heuristic are implemented.

2.1 Limmern PSHP

The Limmern PSHP is part of one of the most important hydro expansion projects in Switzerland. The addition of a 1,000 megawatt (MW) pumped storage is realized by exploiting the hydraulic head of around 630 meter (m) between the reservoirs Limmernsee and Muttsee (Figure 1). The project was finalized in 2017. The total costs were around 2.1 billion CHF [12].

The lower reservoir Limmernsee is at an elevation of 1,857 m above sea level (masl) while the upper reservoir Muttsee is situated at 2,474 masl. Limmernsee can store up to 92 million m^3 (Mm^3) of water as a storage for the Limmern PSHP as well as a seasonal storage for water inflows and the power plants located further down the water cascade. Muttsee has a storage capacity of 9 Mm^3 which is currently being expanded to 23 Mm^3 for the new PSHP. Muttsee does not have notable water inflows. The new Limmern PSHP is equipped with 4×250 MW reversible Francis pump-turbines with variable speed technology, enabled by a doubly-fed asynchronous induction machine. The nominal discharge in generating mode for each unit is 47 m^3/s and in pumping mode it is 40 m^3/s [13]. The pump-turbine in the production mode can be operated roughly between 30% to 100% and in the consumption mode it can be operated between 70% to 100%. This will allow water to be shifted between both lakes to produce pumped-storage hydroelectricity and provide ancillary services for grid stabilization.

As it can be seen in Figure 2, the machines in the Limmern PSHP are fed by two penstocks. That is, each penstock is shared by two of the four units. The diameter of each penstock is about 4.2 m and length is 1,054 m. Together with the water rate of round 40 m^3/s, there will be non-negligible head loss present due to friction. Therefore, the Limmern PSHP serves as an ideal case for performing analysis of power loss in a shared penstock for reversible VPs.

Figure 1. Cutaway graphics of the Limmern PSHP (source:[13]).

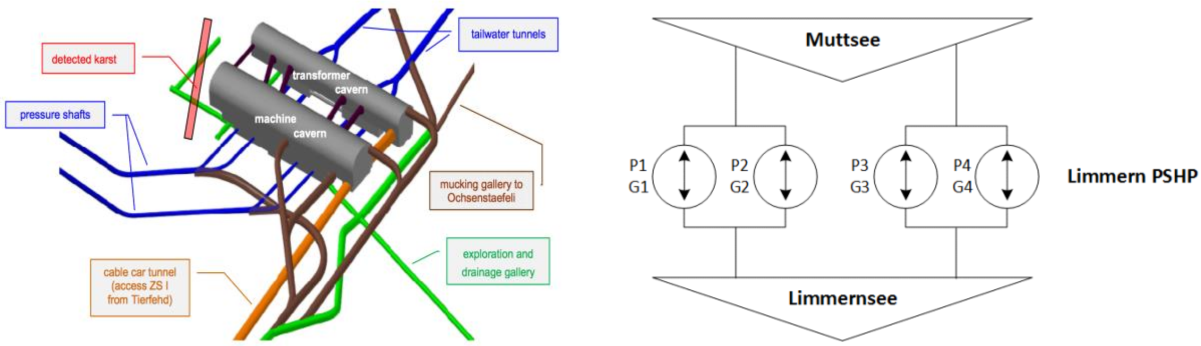

Figure 2. Illustrations of the hydraulic system in the Limmern PSHP (source for first graph: [12]).

2.2 Short-term hydro optimization program (SHOP)

The presented method to determine the I/O curve of VPs and the heuristic to calculate the power loss in a shared penstock is implemented in SHOP, an operational scheduling tool that is used by many large hydropower producers in Scandinavia as well as in Switzerland, Austria and Chile [14]. In order to precisely deal with the head variation and head loss, the overall solution strategy of SHOP involves two modelling modes, i.e. Unit commitment (UC) mode and Close-in mode. Further, an iterative procedure is employed to refine nonlinearities and state-dependencies. In the first mode, a MILP model representing the watercourse topology and hydraulic system is established. The volume and water level of the reservoirs will be updated after each iteration. After 2 or 3 iterations, the convergence criterion will usually be met. Based on the fixed UC plan found in the UC mode, a dispatch plan regarding exact generation/consumption level for each unit is obtained in the Close-in mode.

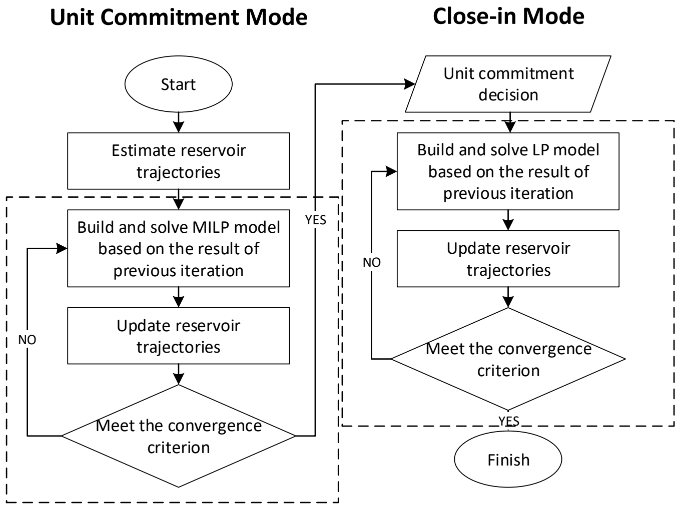

Figure 3. Solution strategy in SHOP.

3. Mathematical formulation

In this section, we mainly present the two main issues addressed in the paper: 1) how the nonlinear PCF is incorporated with a MILP framework; 2) how the power loss in a shared penstock is formulated, piecewise linearized and added to the energy balance constraint. We only focus on the core constraints associated with the power consumption of the VPs and the power balance. Other physical constraints regarding to the reservoir management and strategic constraints reflecting producer's regulation choice are skipped, since they can be found in most papers dealing with STHS problem [15]. Moreover, we assume that the reversible units in the plant can only be operated in the same mode, either generating or pumping. In other words, there is no simultaneous use of both turbine and pump, i.e. the so-called hydraulic short-circuit operation. The reader interested in the formulation of the power compensation in a shared penstock caused by the hydraulic short-circuit operation is referred to [16].

3.1 Sets and Indices

$T = \{1, \dots, \overline{t}\}$	Set of time periods, index $t \in T$.
$N = \{1, \dots, \overline{n}\}$	Set of penstocks in the plant, index $n \in N$.
$I = \{1, \dots, \overline{i}\}$	Set of turbines in the plant, index $i \in I$.
$J = \{1, \dots, \overline{j}\}$	Set of pumps in the plant, index $j \in J$.
$J_n = \{1, \dots, \overline{j}_n\}$	Set of pumps that are connected to the same penstock n, index $j \in J_n$.
$S = \{0, \dots, \overline{s}\}$	Set of breakpoints/segments for piecewise linear approximation of the PCF, index $s \in S$.
$K = \{0, \dots, \overline{k}\}$	Set of breakpoints/segments for piecewise linear approximation of the power loss function, index $k \in K$.

3.2 Parameters

ΔT	Time resolution per time period (h).
$M_t^{\text{SELL}}, M_t^{\text{BUY}}$	Forecasted market selling and buying price of electricity in period t (€/MWh).
H_t^{GROSS}	Gross head of the plant in period t (m).
α_n	Loss factor of penstock n in the plant, which depends on the penstock characteristics (s^2/m^5).
G	Conversion constant taking into account the gravity acceleration, water density and makes the appropriate unit conversions from (m) and (m^3/s) to (MW), default setting is $10^{-3} \cdot 9.81$.
$P_j^{\text{MIN}}, P_j^{\text{MAX}}$	Minimum and maximum power consumption of pump j (MW).
$\overline{Q}_{s,j,t}$	Water flow pumped at breakpoint s by pump j in period t, $s = 0, \dots, \overline{s}$ (m^3/s).
$\overline{P}_{s,j,t}$	Power consumption at breakpoint s by pump j in period t, $s = 0, \dots, \overline{s}$ (MW).
$\overline{H}_{s,j,t}^{\text{NET}}$	Net head at breakpoint s of pump j in period t (m).
$\hat{P}_{j,t}^{\text{MIN}}, \hat{P}_{j,t}^{\text{MAX}}$	Constrained minimum and maximum power consumption of pump i in period t
$\hat{Q}_{j,t}^{\text{MIN}}, \hat{Q}_{j,t}^{\text{MAX}}$	Constrained minimum and maximum flow pumped by pump j in period t (m^3/s). (MW).
$\overline{Q}_{k,n,t}$	Water flow pumped at breakpoint k in penstock n, $k = 0, \dots, \overline{k}$ (m^3/s).
$\Delta\overline{P}_{k,n,t}$	Consumption power loss at breakpoint k in penstock n, $k = 0, \dots, \overline{k}$ (MW).

3.3 Head-dependent parameters

$\eta_j\left(h_{j,t}^{\text{NET}}, q_{j,t}\right)$	Turbine efficiency of pump j as a function of the net head and flow pumped by the unit in period t (%).
$Q_{j,t}^{\text{MIN}}\left(h_{j,t}^{\text{NET}}\right), Q_{j,t}^{\text{MAX}}\left(h_{j,t}^{\text{NET}}\right)$	Minimum and maximum flow pumped by pump j as a function of the net head in period t (m^3/s).
$Q_{j,t}^{\text{BEST}}\left(h_{j,t}^{\text{NET}}\right)$	Best efficiency point of water pumped by pump j as a function of the net head in period t (m^3/s).

3.4 Variables

$\omega_{j,t} \in \{0,1\}$	Status of pump j in period t (1 on, 0 off).
$h_{j,t}^{\text{NET}}$	Net head of pump j in period t (m).
$p_t^{\text{SELL}}, p_t^{\text{BUY}}$	Power sold to and bought from the market in period t (MW).
$p_{i,t}^{\text{PROD}}$	Power produced by turbine i in period t (MW).
$p_{j,t}^{\text{CONS}}$	Power consumed by pump j in period t (MW).
$\Delta p_{j,t}^{\text{CONS}}$	Consumption power loss for pump j in period t (MW).
$\Delta p_{n,t}^{\text{CONS}}$	Consumption power loss in penstock n in period t (MW).
$\Delta p_{n,t}^{\text{PROD}}$	Production power loss in penstock n in period t (MW).
$q_{j,t}$	Water flow pumped by pump j in period t (m^3/s).
$q_{s,j,t}$	Water flow pumped in segment s by pump j in period t, $s = 1, \dots, \overline{s}$ (m^3/s).
$q_{k,n,t}$	Water flow pumped in segment k in penstock n in period t, $k = 1, \dots, \overline{k}$ (m^3/s).

3.5 Objective function

$$Max \sum_{t \in T} \left(M_t^{\text{SELL}} \cdot \Delta T \cdot p_t^{\text{SELL}} - M_t^{\text{BUY}} \cdot \Delta T \cdot p_t^{\text{BUY}}\right) \tag{1}$$

The goal of the model is to maximize the profit during the scheduling horizon, partially constituted by the production revenue minus the power consumption cost as expressed in equation (1). In theory, unit selling and buying prices in the spot market have the same value and can be forecasted. However, in order to avoid making money machines that gives the opportunity to buy power and sell it within the same period, the unit selling price M_t^{SELL} should always be set marginally lower than the buying price M_t^{BUY}.

3.6 Core constraints associated with power consumption and power balance

$$p_{j,t}^{\mathrm{CONS}} = G \cdot \frac{1}{\eta_j\left(h_{j,t}^{\mathrm{NET}}, q_{j,t}\right)} \cdot h_{j,t}^{\mathrm{NET}} \cdot q_{j,t}, \qquad \forall j \in J, t \in T. \tag{2}$$

$$Q_{j,t}^{\mathrm{MIN}}\left(h_{j,t}^{\mathrm{NET}}\right) \cdot \omega_{j,t} \le q_{j,t} \le Q_{j,t}^{\mathrm{MAX}}\left(h_{j,t}^{\mathrm{NET}}\right) \cdot \omega_{j,t}, \qquad \forall j \in J, t \in T. \tag{3}$$

$$P_j^{\mathrm{MIN}} \cdot \omega_{j,t} \le p_{j,t}^{\mathrm{CONS}} \le P_j^{\mathrm{MAX}} \cdot \omega_{j,t}, \qquad \forall j \in J, t \in T. \tag{4}$$

$$h_{j,t}^{\mathrm{NET}} = H_t^{\mathrm{GROSS}} + \alpha_n \cdot \left(q_{j,t} + \sum_{j' \in J_n \setminus \{j\}} q_{j',t}\right)^2, \qquad \forall j \in J_n, n \in N, t \in T. \tag{5}$$

$$\sum_{i \in I} p_{i,t}^{\mathrm{PROD}} - \sum_{j \in j} p_{j,t}^{\mathrm{CONS}} = p_t^{\mathrm{SELL}} - p_t^{\mathrm{BUY}}, \qquad t \in T. \tag{6}$$

Without loss of generality, the reversible VP is separately modelled as a turbine unit and a pump unit. In the same modelling manner for the FPs as in [3], logic constraints using binary variables will be introduced to ensure that for each unit at most one mode can be chosen to operate and all the units in the plant can only operate in the same mode, either generating or pumping.

For a specific VP in pumping mode, the PCF, as expressed in equation (2), depends on the net hydraulic head and the flow processed by that unit. It also relies on the head-dependent turbine efficiency. Equation (3) corresponds to the head-dependent permissible pumping range, whereas equation (4) determines the lower and upper consumption limits of the unit. Equation (5) accounts for that, if pump j is fed by a shared penstock n, the net head of the unit is calculated based on not only the flow processed by the unit $q_{j,t}$ but also the total flow of all the other units that are connected to the same penstock $\sum_{j' \in J_n \setminus \{j\}} q_{j',t}$. The power production of the unit in generating mode $p_{i,t}^{\mathrm{PROD}}$ and corresponding operation constraints can be analogously formulated as in [17] and are omitted here. The power produced by turbines can be sold to the market, and vice versa, the energy needed by pumps can be bought from the market. This relationship can be formulated by the energy balance constraint for the plant in equation (6).

As it can be seen in the equations mentioned above, there are two main modelling challenges. First, the PCF (2) is a complex nonlinear equation with the characteristic of head-dependency. In the next subsection, we will discuss how to convert it into a piecewise linear I/O curve while considering all the bounds restricted by equations (4) and (3). Second, the nonlinear head loss in a shared penstock (5) is involved with the running statue of all the units supported by the same penstock. To solve this state-dependent problem, in subsection 3.8, we will quantify the loss with detailed mathematical formulation and propose a heuristic to incorporate the loss into the MILP model.

3.7 Convert the nonlinear PCF to the piecewise linear I/O curve

A full description of the determination of the unit I/O curve for a generating unit is given in [17] (the first quadrant in Figure 4). In this paper, we only summarize the main process of building I/O curve for pump j in a specific period t, which is illustrated in the third quadrant in Figure 4. Note that only three main breakpoints (the minimum, best efficiency and maximum water flow in pumping mode) are determined. The number of breakpoints $\bar{s} + 1$ in the I/O curve can be defined according to the needs of the user.

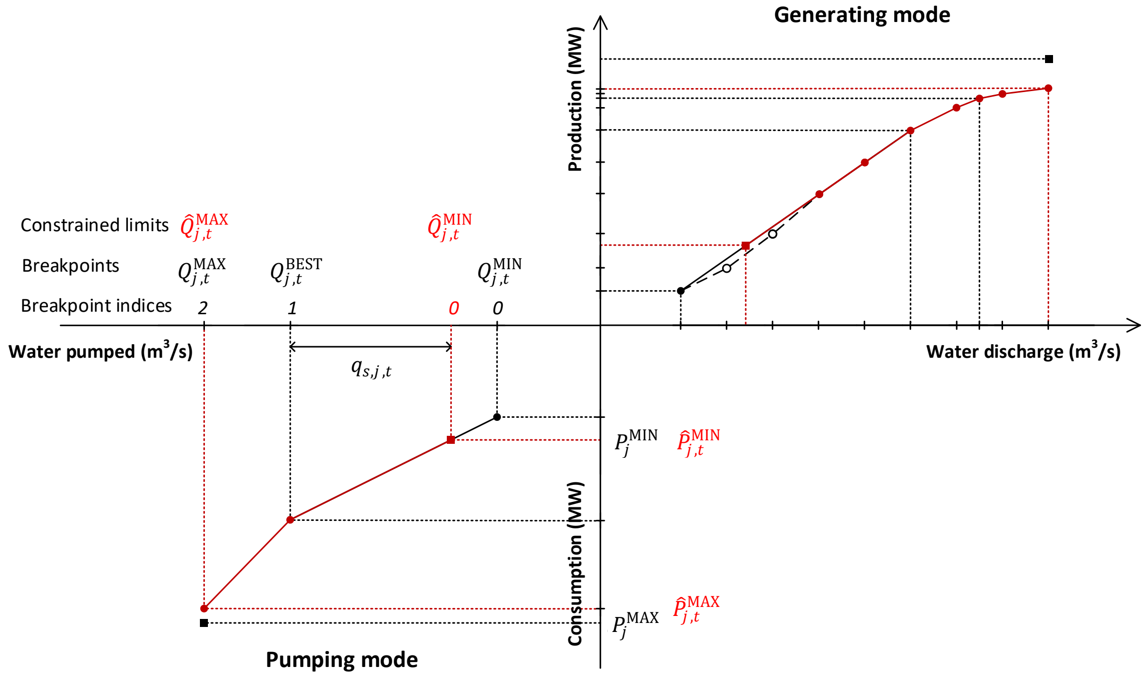

Figure 4. Illustration of the piecewise linear I/O curve for a unit in pumping and generating modes.

Step 1: Update the trajectory of the reservoirs and calculate the gross head of the plant.
The gross head of the plant is the elevation difference between the water level of the upstream and downstream reservoirs. As mentioned in subsection 2.2, SHOP adopts an iterative procedure to update the trajectory of the reservoir. In the 1st iteration, the initial value of the reservoirs is used for the entire scheduling horizon. After solving the problem, the volume and water level of the reservoirs for each period are updated, and therefore, the gross head of the plant H_t^{GROSS} can be obtained and taken as known parameter for the next iteration.

Step 2: Determinate the head-dependent minimum flow $Q_{j,t}^{MIN}$, best efficiency point $Q_{j,t}^{BEST}$ and maximum flow $Q_{j,t}^{MAX}$ of the unit.
The turbine efficiency as a function of the net head and the water flow can be taken from the Hill chart of the unit [18]. Depending on the design and operating conditions of the VP, the minimum, best efficiency and maximum water flow in pumping mode (the three main breakpoints) may change with the head. An iterative approach as proposed in [17] should be used to stabilize the net head at different breakpoints. Then the water flow of each breakpoint can be linearly interpolated for the net head. That is, $\overline{Q}_{0,j,t} = Q_{j,t}^{MIN}$ when $\overline{H}_{0,j,t}^{\text{NET}}$, $\overline{Q}_{1,j,t} = Q_{j,t}^{\text{BEST}}$ when $\overline{H}_{1,j,t}^{\text{NET}}$, and $\overline{Q}_{2,j,t} = Q_{j,t}^{\text{MAX}}$ when $\overline{H}_{2,j,t}^{\text{NET}}$.

Step 3: Calculate the corresponding power consumption of each breakpoint.
Given a point $\overline{Q}_{s,j,t}$ and $\overline{H}_{s,j,t}^{\text{NET}}$ obtained from *Steps 2*, we use bilinear interpolation to get the turbine efficiency $\eta_j\left(\overline{H}_{s,j,t}^{\text{NET}}, \overline{Q}_{s,j,t}\right)$. The corresponding power output $\overline{P}_{s,j,t}$ can be reckoned by equation (2).

Step 4: Define the final operating limits.
Now, the first and last breakpoints reflect the permissible pumping range of the unit, i.e. equation (3). We should also include other limits, i.e. consumption bounds in equation (4). We hence define

$$\hat{P}_{j,t}^{\text{MIN}} = \text{MAX}\{\overline{P}_{0,j,t}, P_j^{MIN}\}, \qquad \forall j \in J, t \in T. \tag{7}$$

$$\hat{P}_{j,t}^{\text{MAX}} = \text{MIN}\{\overline{P}_{2,j,t}, P_j^{MAX}\}, \qquad \forall j \in J, t \in T. \tag{8}$$

The corresponding $\hat{Q}_{j,t}^{\text{MIN}}, \hat{Q}_{j,t}^{\text{MAX}}$ are linearly interpolated. Note that if the limits are changed, the relevant breakpoint indices and values should also be updated. Take the unit I/O curve in pumping mode in Figure 4 as example. The minimum operating limit of the unit in period t is constrained by the minimum consumption, whereas the maximum operating limit is restricted by the maximum permissible pumped flow.

Step 5: Replace the PCF by a piecewise linear approximation

Finally, the nonlinear PCF is represented by a piecewise linear I/O curve with the chosen breakpoints. Equations (2) – (4) are hence replaced by the following constraints

$$p_{j,t}^{\text{CONS}} = \hat{P}_{j,t}^{\text{MIN}} \cdot \omega_{j,t} + \sum_{s=1,\dots,\overline{s}} \frac{\overline{P}_{s,j,t} - \overline{P}_{s-1,j,t}}{\overline{Q}_{s,j,t} - \overline{Q}_{s-1,j,t}} \cdot q_{s,j,t}, \qquad \forall j \in J, t \in T. \tag{9}$$

$$p_{j,t}^{\text{CONS}} \le \hat{P}_{j,t}^{\text{MAX}} \cdot \omega_{j,t}, \qquad \forall j \in J, t \in T. \tag{10}$$

$$0 \le q_{s,j,t} \le \overline{Q}_{s,j,t} - \overline{Q}_{s-1,j,t}, \qquad \forall s = 1, \dots, \overline{s}, \qquad \forall j \in J, t \in T. \tag{11}$$

$$q_{j,t} = \hat{Q}_{j,t}^{\text{MIN}} \cdot \omega_{j,t} + \sum_{s=1,\dots,\overline{s}} q_{s,j,t}, \qquad \forall j \in J, t \in T. \tag{12}$$

3.8 Formulate the consumption power loss in a shared penstock

As shown in equation (5), the net head available at pump j mainly depends on the gross head of the plant and the head loss associated with the total flow pumped through the penstock. However, the determination of the unit I/O curve precedes the optimization. In the UC mode of SHOP, the operating status of other units remains unsolved. We hence propose a heuristic to handle this puzzle.

To begin with, we exclude the penstock head loss from the I/O curve and the power consumption at each breakpoint becomes

$$\overline{P}_{s,j,t} = G \cdot \frac{1}{\eta_j\left(H_t^{\text{GROSS}}, \overline{Q}_{s,j,t}\right)} \cdot H_t^{\text{GROSS}} \cdot \overline{Q}_{s,j,t}, \qquad \forall s = 0, \dots, \overline{s}, j \in J, t \in T. \tag{13}$$

It implies that the power consumption for the unit is only associated with its own flow rate. However, the power consumption $p_{j,t}^{\text{CONS}}$ in equation (9) will hereby be underestimated for the given water flow $q_{j,t}$. To make up for the gap, the sum of consumption power loss in shared penstocks must be added to the plant energy balance constraint.

The power loss of pump j can be defined as

$$\Delta p_{j,t}^{\text{CONS}} = G \cdot \frac{1}{\eta} \cdot \alpha_n \cdot \left(q_{j,t} + \sum_{j' \in J_n \setminus \{j\}} q_{j',t}\right)^2 \cdot q_{j,t}, \qquad \forall j \in J, t \in T. \tag{14}$$

To keep the formulation tractable, in the equations related to the power loss in a shared penstock we assume that the turbine efficiency is a constant number η, rather than the head- and flow-dependent $\eta_j\left(h_{j,t}^{\text{NET}}, q_{j,t}\right)$. Summing up the power loss for all the units that are fed by the same penstock n, we can get the total consumption power loss in the shared penstock, which is a cubic function of the total flow pumped through the penstock.

$$\Delta p_{n,t}^{\text{CONS}} = \sum_{j \in J_n} \Delta p_{j,t}^{\text{CONS}} = G \cdot \frac{1}{\eta} \cdot \alpha_n \cdot \left(\sum_{j \in J_n} q_{j,t}\right)^3, \qquad \forall n \in N, t \in T. \tag{15}$$

Secondly, we use the piecewise linear approximation to incorporate the nonlinear function (15) into the MILP problem [9]. The maximum total flow that can be pumped through penstock n is the sum of maximum water flow pumped by each unit that is fed by this penstock, i.e. $\sum_{j\in J_n} Q_{j,t}^{\mathrm{MAX}}$. The interval $\left[0,\ \sum_{j\in J_n} Q_{j,t}^{\mathrm{MAX}}\right]$ can be equally divided into $\overline{k}$ segments and the breakpoints can be denoted as follows:

$$\overline{Q}_{k,n,t} = k \cdot \frac{\sum_{j\in J_n} Q_{j,t}^{\mathrm{MAX}}}{\overline{k}}, \qquad \forall k = 0, \dots, \overline{k}, n \in N, t \in T. \tag{16}$$

$$\Delta\overline{P}_{k,n,t} = G \cdot \frac{1}{\eta} \cdot \alpha_n \cdot \left(\overline{Q}_{k,n,t}\right)^3, \qquad \forall k = 0, \dots, \overline{k}, n \in N, t \in T. \tag{17}$$

Then equation (15) can be approximated by the piecewise linear function as expressed in (18). Equation (19) gives the lower and upper bounds for water flow at each segment. Equation (20) ensures that the water flow in the penstock should be the same as the sum of the flow of the units connected to the same penstock.

$$\Delta p_{n,t}^{\mathrm{CONS}} = \sum_{k=1\dots\overline{k}} \frac{\Delta\overline{P}_{k,n,t} - \Delta\overline{P}_{k-1,n,t}}{\overline{Q}_{k,n,t} - \overline{Q}_{k-1,n,t}} \cdot q_{k,n,t}, \qquad \forall k = 1, \dots, \overline{k},\ \ n \in N, t \in T. \tag{18}$$

$$0 \le q_{k,n,t} \le \frac{\sum_{j\in J_n} Q_{j,t}^{\mathrm{MAX}}}{\overline{k}}, \qquad \forall k = 1, \dots, \overline{k}, n \in N, t \in T. \tag{19}$$

$$\sum_{k=1\dots\overline{k}} q_{k,n,t} = \sum_{j\in J_n} q_{j,t}, \qquad \forall n \in N, t \in T. \tag{20}$$

In a similar matter as determining the I/O curve for a unit in generating mode, the power produced from the turbine will be overrated and the difference is the production power loss. Therefore, the total production power loss (can be obtained under the analogous analysis) must be deducted from the plant energy balance constraint (6), and the total consumption power loss must be added in equation (6). Then equation (6) is updated as

$$\left(\sum_{i\in I} p_{i,t}^{\mathrm{PROD}} - \sum_{n\in N} \Delta p_{n,t}^{\mathrm{PROD}}\right) - \left(\sum_{j\in j} p_{j,t}^{\mathrm{CONS}} + \sum_{n\in N} \Delta p_{n,t}^{\mathrm{CONS}}\right) = p_t^{\mathrm{SELL}} - p_t^{\mathrm{BUY}}, \qquad t \in T. \tag{21}$$

Finally, in implementing equations (18) – (21) and the counterpart equations for turbines in SHOP, the power loss in a shared penstock can be effectively handled in the UC mode.

In the Close-in mode of SHOP, the operating status of the units (generating, pumping, or standing still) are fixed as the result of the UC mode. The optimization problem becomes a LP model and only the committed units will be considered. The production/consumption level of all the units in the Close-in mode are nearly resolved. There is only minor modification between iterations. Therefore, when calculating the net head for the breakpoints in the unit I/O curve, we directly use the optimal results from the previous iteration as the current flow for other units that are fed by the same penstock.

4. Case study

In this section, the determination of the I/O curve of the VP in pumping mode is presented. The effects of the proposed heuristic to calculate the power loss in a shared penstock is quantified. At last, the importance of precise representation of the shared penstock is revealed.

The numerical results are based on the hydraulic system in Limmern PSHP introduced in subsection 2.1. Some data have been modified for confidentiality reasons. The main characteristic parameters of the reservoirs and the four identical VPs in Limmern PSHP are summarized in Table 1. The relationship of the volume and water level of the reservoirs are assumed to be linear. There is no inflow during the study period. The loss factors for both penstocks are assumed to be $0.003\ \mathrm{s^2/m^5}$. The simplified head- and flow-dependent turbine efficiency of the unit in both generating and pumping modes is given in Figure 5. To make the presentation easy to understand, we assume that the minimum, best efficiency and maximum points does not change with the head. The turbine efficiency when formulating the power loss in a shared penstock is assumed to be 90%.

Table 1. Characteristic parameters of the reservoirs and VPs

Upper Reservoir Muttsee		Lower Reservoir Limmernsee		4×VPs in Limmern PSHP	
Volume limits (Mm^3)	[0, 13]	Volume limits (Mm^3)	[0, 92]	Power production limits (MW)	[75, 250]
Water level limits (masl)	[2400, 2474]	Water level limits (masl)	[1750, 1857]	Power consumption limits (MW)	[175, 250]

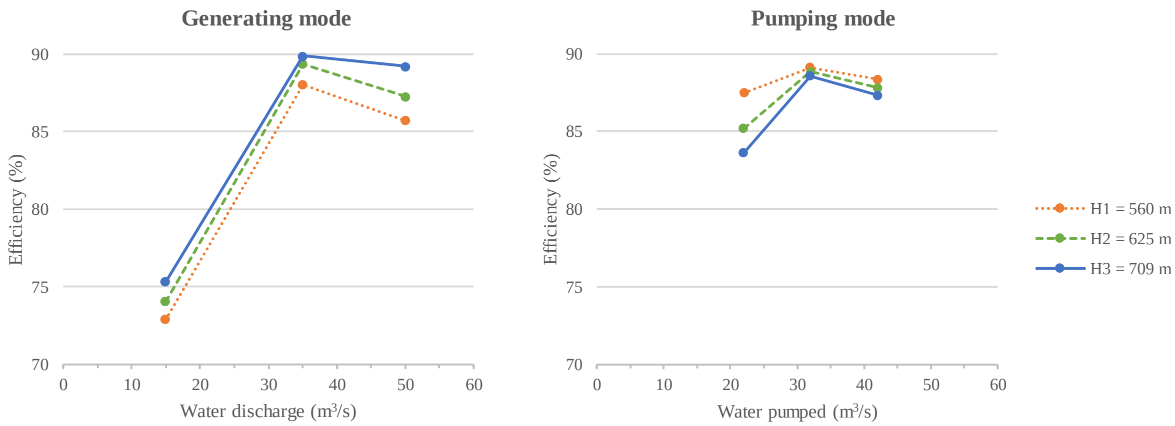

Figure 5. Head- and flow-dependent turbine efficiency in both generating mode and pumping mode.

The scheduling horizon is one day with hourly time resolution. The historical price for electricity is depicted as the red line in Figure 8. It refers to the periods from 8 pm on Saturday to 8 pm on Sunday in the summer of central Europe. All the tests are run by SHOP on an Intel Core i7-6600U processor with 16 GB of RAM. CPLEX 12.6.3 is used as the solver of the mathematical programs. Computational time for each iteration in the UC mode is around 0.1 second and in the Close-in mode it is less than 0.01 second.

4.1 The determination of the I/O curve of the VP in pumping mode

We determine the I/O curve of a VP in pumping mode according to the process mentioned in subsection 3.7. We first run the test under Situation A in which the upper reservoir is 10% full and the lower reservoir is 90% full. It implies the gross head is at a very low level. The I/O curve of unit P1 in period 1 (orange line), built in the first iteration in the UC mode, is illustrated in Figure 6. For the sake of clarity, we flip the I/O curve from the third quadrant to the first quadrant.

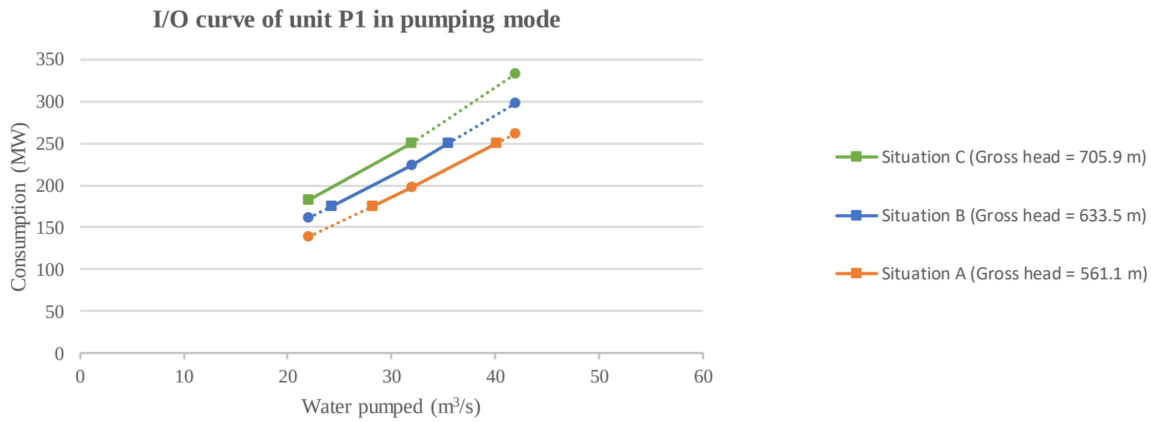

Figure 6. I/O curve of unit P1 in pumping mode built under three different gross head.

The main breakpoints are determined by minimum flow 22 m^3/s, best efficiency 32 m^3/s and maximum flow 42 m^3/s, consistent with the figures in Figure 5. The corresponding power consumption of each breakpoint is calculated based on the gross head, i.e., without considering the penstock loss. Then, the minimum operating limit is constrained by the minimum consumption 175 MW and the maximum operating limit is restricted by the maximum consumption 250 MW. The solid orange line is the final piecewise linear I/O curve of P1.

We also test the other two situations: Situation B (blue line) where the initial volume of the upper and lower reservoirs is both 50% full, and Situation C (green line) where the initial volume of the upper reservoir is 90% full and the lower reservoir is 10% full. As it can be seen in Figure 6, with the increase of the gross head, more energy is needed to pump up the water at the same rate. In other words, less water can be pumped up given the same energy consumption. Note that, in Situation C, the minimum operating limit is constrained by the minimum permissible pumped flow (22 m^3/s, 182.04 MW).

4.2 The effects of the proposed heuristic to calculate the power loss in a shared penstock

We first use the results under Situation B to analyse the effects of the heuristic proposed in subsection 3.8. The results are obtained after 3 iterations in the UC mode of SHOP. We have pointed out in subsection 3.8 that, if the I/O curve is determined based on the gross head instead of the net head, the power production under the MILP solution will be overestimated and the power consumption under the MILP solution will be underestimated. Nevertheless, the actual power production/consumption can be recomputed by equation (2) with the water discharged/pumped under the MILP solution. Then we can calculate the power difference between the MILP solution and the actual value. In theory, this difference is the power loss that we first exclude in the determination of the unit I/O curve and then include in the plant energy balance constraint. The comparisons between the power difference and the power loss under the MILP solution are given in the left graph of Figure 7.

It is found that there is unbalance between the power difference and the power loss, in both generating and pump modes. The unbalance is caused by the fact that the power loss in a shared penstock is calculated by a constant turbine efficiency ($\eta = 90\%$) while the power generated/consumed is based on the head- and flow-dependent turbine efficiency $\eta_j\left(h_{j,t}^{\text{NET}}, q_{j,t}\right)$. If we assume that the unit turbine efficiency in both generating and pumping modes keeps constant as 90% and run the test again, then there is no unbalance (right graph of Figure 7). Therefore, the unbalance is the trade-off between the precision of the formulation and the solvability of the MILP problem. Since the purpose of running the model in the UC mode is to decide which unit should be committed and the exact production/consumption level will be furthermore decided in the Close-in mode, the unbalance will be mostly diminished in the Close-in mode.

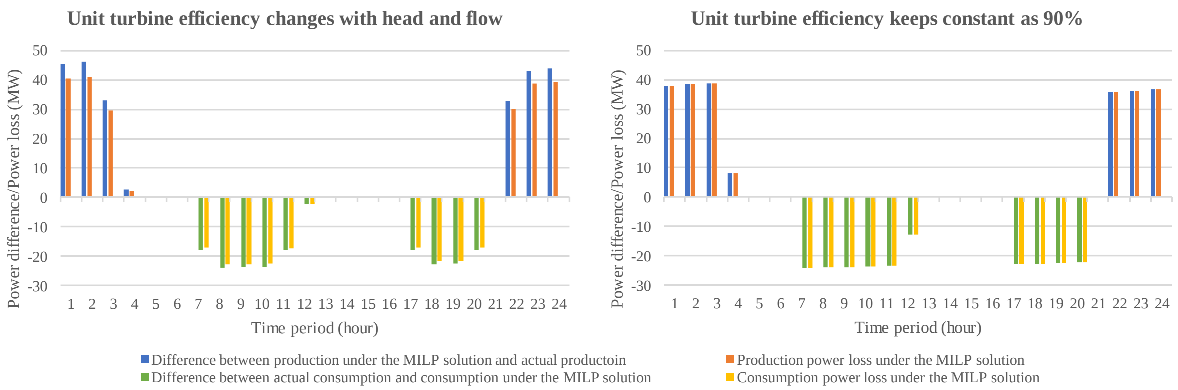

Figure 7. Comparisons of power difference and power loss under two settings of turbine efficiency

4.3 The importance of precise representation of the shared penstock

Now we study the importance of accurate modelling of the shared penstock in solution quality. Two tests are carried out: 1) Case I: There is no shared penstock in the plant. That is, each VP is connected to individual penstock; 2) Case II: It represents the real-world situation. Each penstock is shared by two units. In both cases, the initial volume of the upper and lower reservoirs is both 50% full (Situation B). The scheduling results after 3 iterations in the UC mode and 3 iterations in the Close-in mode are contrastingly shown in Figure 8.

In general, the operation follows the trend of the forecasted electricity price. During periods of low price (in periods 7–12 and 17–20), water is pumped from Limmernsee to Muttsee. When the price is high (in periods 1–4 and 22–24), the water is released down to the power station for generation. The operation range of the VPs varies between best efficiency and maximum limit according to the price.

However, since the penstock head loss is quadratic with the total water flow in the penstock, given the same discharged/pumped flow of two units, the sum of the power loss in two individual penstocks will be lower than that in a shared penstock. Therefore, in Case I, the plant is willing to produce more power than it actually can when the price is high, and to pump more water than it eventually should when the price is low. It will also lead to the possibility that more units would be committed than necessary. The most noticeable periods are in periods 7 and 17 when more water is pumped in Case I than in Case II. In period 12, 4 units are committed to pump in Case I rather than 2 units in Case II. If we disregard the configuration of the shared penstocks and model them as individual penstocks, as assumed in Case I, it will result in an inferior solution. This demonstrates how important it is to have a correct modeling for shared penstocks to guarantee an optimal dispatch schedule for the daily operation.

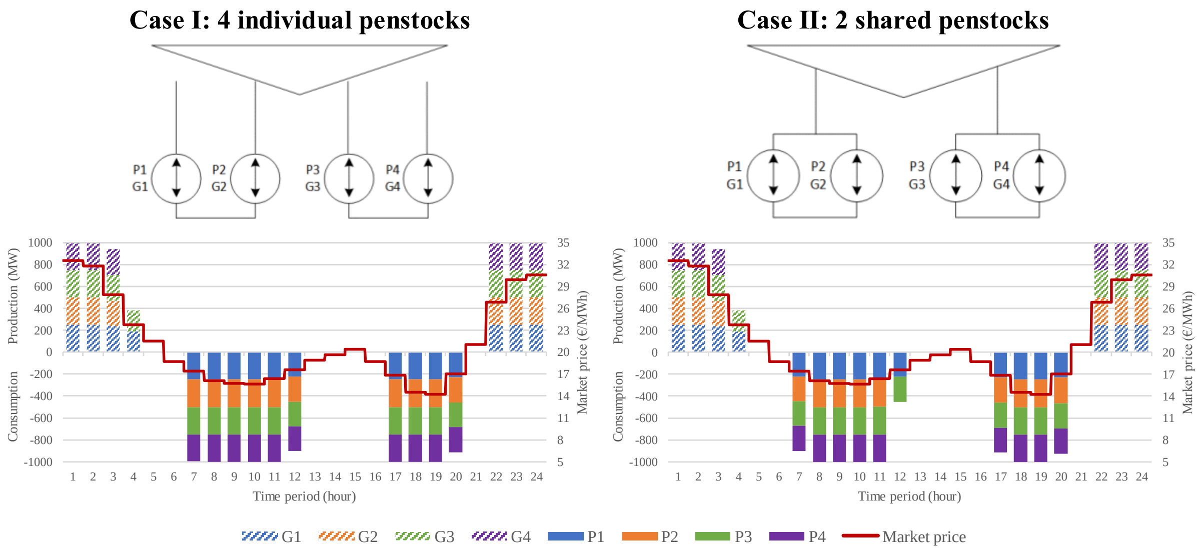

Figure 8. Final scheduling results under two representations of the hydraulic system.

5. Conclusion

In this paper, we present the method to deal with the nonlinear and state-dependent elements in the PCF of a VPs. The function is formulated by piecewise linear approximations with dynamically specified breakpoints. In addition, we propose a heuristic to include the power loss in a shared penstock in the STHS problem. Accounting for the power loss in a shared penstock involves coupling variables. That is, the net head for a VP depends not only on the flow processed by the unit but also on the operating situation of all the units that are connected to the same penstock. The heuristic can solve the state-dependent problem and convert the nonlinear loss function into a MILP formulation framework. Future developments will focus on refining the state-dependent feature of the power loss on unit level and including it in the unit energy balance constraints. For the PSHPs that participate in both energy and reserve markets, it is critical to know the exact capacity available for each unit.

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