

# Stochastic dynamic programming for unified short- and medium-term planning of hydro power considering market products

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**Abstract**—In a deregulated market, the operator of a hydro storage power plant has to consider different market products as well as uncertainties in its prices for optimal scheduling. A suitable mathematical framework for such problems is stochastic dynamic programming. The critical aspect there is modeling, since efficient solution techniques are well known. We present a modeling approach, where the actual available electricity market products act as the basis for the modeling. Advantages of this modeling approach are the natural consideration of power Future products and Hourly Price Forward Curves. Further the model is capable of unifying the short and medium-term optimizations. A two-stage mixed-integer stochastic program with variable time periods is obtained, with the first stage deciding on the amount of power Futures to bid and the second stage as hourly recourse action on the day-ahead market. The problem can be formulated dynamically so that it can be efficiently solved in parallel.

**Index Terms**—Hydro power, scheduling, short-term planning, medium-term planning, stochastic programming, power Future, hourly price forward curve.

## I. INTRODUCTION

If a hydro power plant is operated in a portfolio with thermal power plants, a scheduling optimization typically deploys the hydro plant in order to minimize the thermal production costs. For generation companies with only hydro storage power plants this is not possible. In this case the future profit, which can be achieved with the water, is estimated. These so called *water values* depend not only on the current amount of stored water but also on the future electricity prices and water inflows. Prices and water inflows are uncertain and not fully known in advance.

Finally in a deregulated market environment a price-taker power producer has to decide which products he wants to offer so that a bidding problem arises. This bidding problem for a hydro power producer is so complex that it is usually solved in different steps: e.g. first a medium-term planning for determining the monthly/yearly production strategy and then a short-term planning for optimizing the production over the next days.

This segmentation often leads to unnatural step-lengths, driven by computational and modeling limitations. Although theoretically a planning with great modeling detail for an

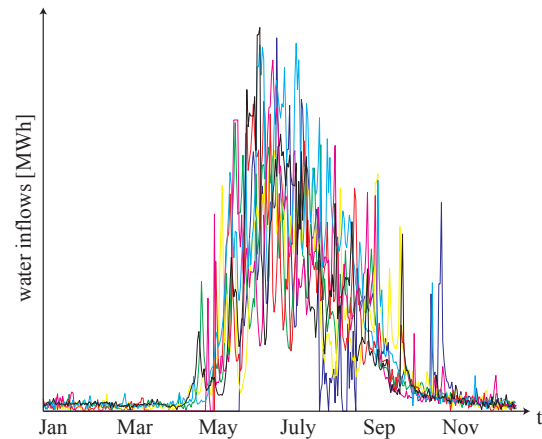


Fig. 1. Water inflows for a storage power plant in the Swiss Alps for some years to illustrate the strong seasonality.

optimization horizon of a year would be feasible, computationally this is not tractable. Further the meaningful parameter modeling, e.g. of hourly market prices for months ahead, is very difficult.

Questions which must be addressed are: what is a sensible segmentation, how long are stages<sup>1</sup>, how many stages are necessary, how must scenarios be created in order to incorporate uncertainties etc. Since efficient solution techniques for such multistage stochastic problems are available these questions indicate that it is the modeling part of the problem which has become the most demanding part, as also shown in [1].

In order to answer these questions this paper proposes a paradigm shift. Instead of choosing an arbitrary segmentation with an afterwards adapted model, the available market products are used directly as modeling guidelines. The proposed model shows that a segmentation into short- and medium term optimization is no longer necessary, which has several advantages.

In the Swiss system, and maybe in many other European,

<sup>1</sup>In this paper the term *stage* is used for denoting both optimization level and time point. There will be no explicit distinction made. However the meaning should be clear in the context.

the following electricity products are available:

- over-the-counter (OTC)
- spot market
- day-ahead market
- physical and financial (German) power Futures
- (German) power options
- ancillary services

In this paper the in-transparent OTC, power options and ancillary services are not considered. The physical and financial power Futures can be further segmented into Future products with delivery periods of a week, a month, a quarter and a year. These are the lengths of the stages which will be used in the model proposed here.

Arguable this segmentation is also arbitrary, but it makes sense from a modeling perspective in several ways: Firstly, the Future products can be used directly as model of a first-stage decision, since their delivery periods are consistent with the stage lengths. Secondly the Hourly Price Forward Curve (HPFC), which is consistent with the Future products (arbitrage-free), can be used as estimation of the Future day ahead market prices. Thirdly, the further ahead products are considered, the more uncertain they are, which is reflected by an increased stage length. Fourthly, this segmentation mimics the actual bidding problem of a power producer.

The optimization is done for a typical Swiss storage power plant with a large hydro head and high power / storage ratio. The water level in the basins have negligible influence on the production and it can be assumed that the power plant is operated as peak power plant. Therefore startup costs, water flow and dynamics, as well as non-linear, non-convex efficiency factors, can be neglected without loss of accuracy. The considered power plant is structured in such a way that it can be aggregated into an upper and lower basin. However, the algorithm can cope with more cascaded power plants.

The water inflows originate mostly from glacierized catchment in the Swiss Alps. Therefore a strong seasonality in inflows is present, and it is common to have a fully depleted upper basin in late-winter (see also Fig. 1). The inflows are therefore modeled deterministically and the optimization horizon ends in late-winter.

Since the power plant model is relatively simple great detail can be considered on the most influential variables: the market prices. Proposed is a two-stage<sup>1</sup> stochastic program to model the bidding problem. In the first stage the algorithm has to decide on the amount of the Future peak product of the next time period to bid without knowing the actual hourly prices. In the second stage the algorithm can make hourly adjustments on the day ahead market using the then known prices. The second stage is modeled as a linear program consisting of hourly stages with price scenarios. The overall optimization is solved by a stochastic dynamic programming (SDP) approach.

For an overview of stochastic programming in the energy sector [2] can be recommended. For SDP in hydro power planning see [3], for a review [4]. Our work can be seen as extensions and/or combinations of concepts appearing in [5], [6] and [7].

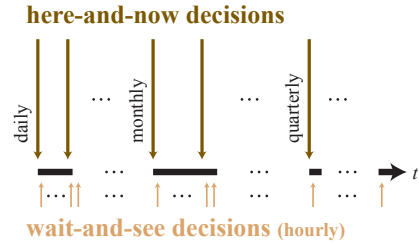


Fig. 2. Two-stage program with varying time length between stages and therefore also varying amount of hourly wait-and-see decisions.

In [5] an hourly day ahead pool market and a financial Future market for a yearly time horizon is considered. They bundle the pool market to 72 different prices. Stochasticity on the pool prices are introduced via scenarios. Large stochastic linear programming, which has the advantage of easily incorporating risk measures and minimizations, is used. Because of the well-known “curse of dimensionality” [8], the model is limited to 72 bundled prices.

In [6] the focus is on constructing bid curves by a large scale mixed integer linear program. A finer model is used on near term and then a coarser one going forward. With 28 stages a time horizon of four to five months is achieved so that it is claimed that the model unifies short and medium-term optimizations.

In [7] SDP is used for finding an operating strategy for a power plant portfolio. However a Lagrange relaxation scheme is used to incorporate long-term guidelines into short-term and vice versa. With this approach they are able to solve the problem for a model case with one thermal and two hydro generating units for 150 stages. It is therefore proposed that this model is capable of unifying short- and medium term optimizations.

This paper is organized as follows: Section II explains the model and its characteristics, mathematical representation and some computational remarks are given. Section III reveal the case study and finally section IV concludes the paper.

## II. THE MODEL

For the operator of a Swiss hydro storage power plant it is typical that the company first decides on the medium-term strategy. This decision is based on deterministic optimizations but builds heavily on experience and tacit knowledge. After this the trading group tries to optimize this medium-term schedule further by taking part in OTC, power Futures, options, day-ahead and spot markets. The trading group is assisted by optimization tools mostly depending on HPFCs and the boundaries given by the medium-term strategy. Since there is no Swiss Futures and options market the HPFCs are constructed using Future products from Germany and France.

### A. Model characteristics

The proposed model follows the in section I outlined procedure in a two-stage<sup>1</sup> manner:

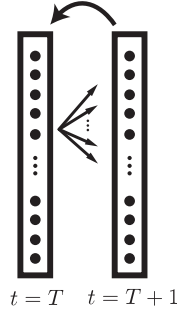


Fig. 3. Discretization of the state and decision variables, the basin filling and the turbined energy. The problem reduces to a shortest path problem which can be solved recursively.

- **Here-and-now decisions:** Choose the optimal amount of peak products with a delivery period of the whole time stage afterwards, priced by the current Future product price for this time period. These decisions are made with respect to the uncertainty in the second stage.
- **Wait-and-see decisions:** Depending on the actual price scenario and the actual here-and-now decision, optimal hourly production for the whole time period is sought, which is also known as recourse action. The price scenarios are constructed using given HPFCs.

Fig. 2 shows a graphical representation of this two-stage program. Until the end of the calendar month the step size is one day. Here-and-now decisions have to be made, daily which means that one has the possibility to hedge against uncertainty in the spot market by buying day-ahead peak products.

The step size is increased to monthly steps for roughly half a year and then finally quarterly step sizes so that a time horizon of a year is obtained. The uncertainty in the HPFC is increased in parallel by assuming more scenarios.

Depending on the start date of the optimization around 30 stages have to be considered with hourly resolution of the second stage. Uncertainty in the HPFC is introduced via scenarios. The amount of scenarios varies between one for the first three days (because the spot prices are then assumed to be known) to 4 scenarios for the quarterly steps.

Because the cumulated profit of a power plant is strictly increasing in time, a decomposition is meaningful. By also discretizing the state and decision space (see Fig. 3) the well-known SDP scheme is attained (introduced in [9] and [10], first applied to hydro power planning problems in [11]).

So for all possible basin levels the most profitable case is searched for recursively, which is similar to a shortest path problem.

### B. Mathematical model

For an overview and explanation of the variables see Table I. As already stated the overall model can be made relatively simple. Let  $\theta_{t,x}$  be the future expected profit for a state  $x$  at time  $t$ , also denoted profit-to-go. Then this profit-to-go can be

TABLE I  
VARIABLES

Variable	Explanation
$t$	Stage
$x$	Discretized basin filling (state variable) [MWh]
$\theta_{t,x}$	Profit-to-go [€]
$bid_{t,x}$	Discretized amount of Future bid [MWh]
$cbid_t$	Estimated Future bid price [€/MWh]
$F_{t,x}$	Valuing function for second stage
$\tau$	Hourly stages within second stage
$cspot_{\tau,t}$	Estimated spot price (HPFC) [€/MWh]
$hpd_{\tau,t,x}$	Hourly production decision [MWh]
$hsd_{\tau,t,x}$	Hourly spill decision [MWh*€/MWh]
$turb_{t,x}$	Discretized amount of available water to discharge [MWh]
$infl_t$	Deterministic water inflows [MWh]

recursively calculated:

$$\theta_{t,x} = \max \left\{ bid_{t,x} \cdot cbid_t + \mathbf{E}_{cspot} [F_{t,x} + \theta_{t+1,x}] \right\} \quad (1)$$

with  $\mathbf{E}_{cspot}$  being the expected value over all spot price scenarios.

$F_{t,x}$  itself is a deterministic maximization problem depending on time, state and also spot price scenario. For each scenario,  $F_{t,x}$  can be stated as:

$$F_{t,x} = \max \{ hpd_{\tau,t,x} \cdot cspot_{\tau,t} - hsd_{\tau,t,x} \} \quad (2)$$

subject to the following equality and bounds to the variable spaces:

$$\begin{aligned} \sum_{\tau} hpd_{\tau,t,x} &= turb_{t,x} + infl_t - bid_{t,x} \\ -bid_{t,x} &\leq hpd_{\tau,t,x}^{peak} \leq turb_{max} - bid_{t,x} \\ 0 &\leq hpd_{\tau,t,x}^{off-peak} \leq turb_{max} \\ hsd &\geq 0 \end{aligned} \quad (3)$$

The discretized amount of available water to discharge  $turb_{t,x}$  depends on the basin level and time stage in order to be meaningful. The algorithm tries to deploy this available water most profitable among the hourly stages within the second stage by buying or selling power. At this point it is important, that the expected HPFC is consistent with the power Futures but modeled with different uncertainty across the hours.

Problem (2) is a mixed-integer linear problem for  $\tau$  stages and  $bid_{t,x}$  as integer variable, which can be efficiently solved. This is done for every basin level in order to get the expected profit-to-go  $\theta_{t,x}$ . The whole procedure is repeated recursively until the first stage is reached.

Out of the profit-to-go function for the first stage  $\theta_{1,x}$  the water values can be constructed through its derivative.

### C. Computational Remarks

All optimizations were done in Matlab R2011a with the standard optimization toolbox. The stochastic dynamic program problem can be formulated embarrassingly parallel, so

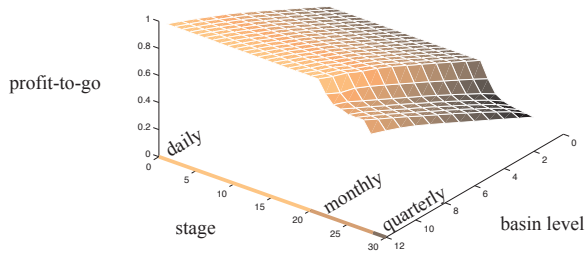


Fig. 4. Profit-to-go for all basin levels and stages. Note, that the first 20 stages have daily step size and therefore the profit relatively does not change much.

that the optimizations took no longer than 15 minutes with a standard computer, with a quad-core 2.3 GHz Intel Core i7 processor and 8 GB of RAM. However, if the special structure of the dynamic program is not exploited in the algorithm, it can easily take hours to solve.

Effective actions for optimizing the algorithm are for instances to make sure, that the number of steps for the basin discretization is a multiple of the available computing units and that the work can be equally distributed across these units.

### III. CASE STUDY

The model is applied to a Swiss hydro storage power plant with installed generation and upper basin capacity of 240 MW and 200 GWh respectively. The hydro inflows are estimated out of historic values for about 6 years. A HPFC, constructed by the industry partner, is used to model the spot price with its uncertainty. The EEX Phelix German peak Future is taken as estimation of the future bid price. This Future with financial settlement is much more liquid than the one with physical settlement, as such it is assumed that the financial one estimates the future prices better.

Since the Phelix power Futures are also used for the construction of the HPFC, the Futures should be consistent with the HPFC without any further modifications.

March 13, 2008 was arbitrarily chosen as optimization start, and it ends on March 31, 2009 with no value given to the residual water in the upper basin. Both the power Futures as well as the HPFC is in respect to the start date.

For the first three days only a single spot price scenario is modeled. For the daily step sizes two scenarios, for monthly three and for quarterly step sizes four scenarios are modeled. This modeling would have to be adapted in real application, if the optimization is done with a receding horizon depending on the start date. The amount of Future bids are discretized in 50 steps. The basin is discretized in twelve steps.

Fig. 4 shows the profit-to-go in respect to all basin levels and stages. One recognizes the concavity of the profit-to-go within each stage as well as the strictly increasing profit with more stages. Since the first roughly twenty days have a daily step size, the profit-to-go relatively will not change much. Also mentionable are the winter months (stages 25 and 26), where

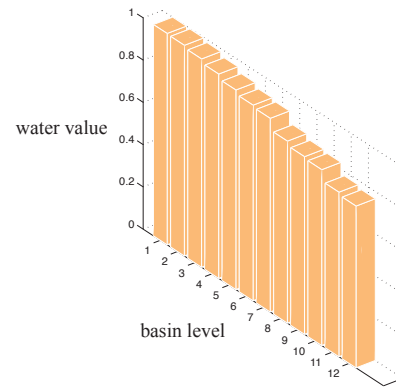


Fig. 5. Water values out of the profit-to-go function at the first stage.

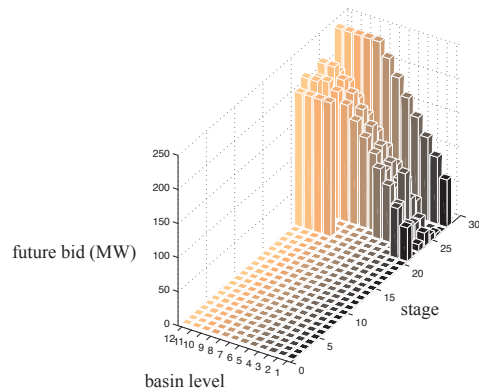


Fig. 6. Optimal Future bids for all stages and basin levels.

because of essentially zero water inflows the profit-to-go stays the same. Nevertheless for the profit-to-go the later bigger stages are more important which would also correspond to a medium-term optimization.

The derivation of the profit-to-go function at the first stage in respect to the basin level results in the main output of this simulation: the water values. Fig. 5 show these water values normalized to its maximum value. If the basin is full the marginal water is roughly one quarter less valuable than it is when it is empty. These values can be used as decision support by the traders in their daily business.

Fig. 6 shows the optimal Future bids. Clearly in the last stage the optimization would like to discharge everything, since the remaining water in the basin has no value anymore. However in the stages before only in few monthly peak power Futures was invested and in no daily ones. This is due to the fact that for flexible hydro power plants it is much more economical to take part in the spot market than in Future markets.

The same effect is also depicted in Fig. 7, where the hypothetical optimal offered power is shown. Hypothetical, because the optimization assumed no already purchased market products. However it also shows, that only in the very peak hours around midday and in the evening maximal power

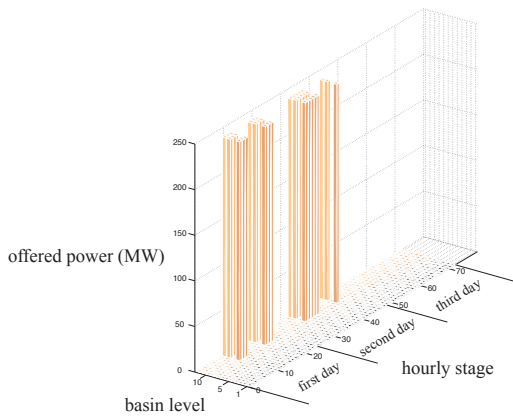


Fig. 7. Optimal (hypothetical) offered power for all basin levels for the first three days.

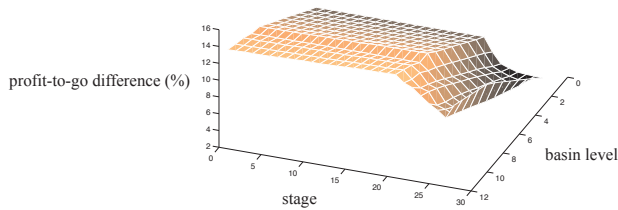


Fig. 8. Profit-to-go difference between a model with and without the possibility to hedge stochastic spot prices. The difference is relative to the maximal profit-to-go value.

is offered and that in the third day, which falls on a Saturday, no power at all is offered because the spot price is too low.

Fig. 8 finally shows the difference between the profit-to-go functions with and without the possibility to hedge stochastic spot prices by Future products. At the first stage this difference is around 10%, which clearly shows the need of considering stochastic spot prices and its hedge by Future products.

#### IV. CONCLUSION

In this paper a stochastic dynamic programming model is proposed, where the discretization of the time is made according to available market products. This choice makes sense not only from a modeling point of view but has also several practical advantages like being able to use market products and Hourly Price Forward Curves directly without further modifications and the possibility to unify short and medium term optimization.

However it is important to note that only a snapshot of the whole market bidding process is made. So the algorithm doesn't take into account, that one could trade the Future products every day with different prices. Further, the results suggest that a simpler model, where the algorithm can only choose between maximal and zero production, might also be satisfactory.

No special technical constraints and no pumping capability was introduced, however the inclusion of these constraints is

not problematic and is point of current work.

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