Stochastic scheduling for a price-maker hydro producer considering forward trading

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Abstract—**This paper presents a short- and medium-term scheduling for a price-maker pumped storage power plant. Considered are stochastic hourly and seasonal water inflows, stochastic prices and detailed operational constraints. The model can choose between production of energy in the own turbines and pumps or bidding energy in the day-ahead market or in the Forward market. Proposed is a multistage stochastic program with a quadratic recourse problem, which is dynamically solved. The results of such an optimization, water values and optimal Forward bids, can be used as decision support in daily operation. A simulation of this operation throughout a year illustrates the use of the optimization on a realistic setting.**

Index Terms—**hydro power, scheduling, short-term planning, medium-term planning, stochastic programming, Forward contracts, pool market, price-maker.**

I. INTRODUCTION

If a hydro power plant is operated in a portfolio with thermal power plants, a scheduling optimization typically deploys the hydro plant in order to minimize the thermal production costs. For generation companies which operate only hydro storage power plants this is not possible. In this case the future profit, which can be achieved by the revenue of sold energy out of stored water on different markets as well as zero or constant operating costs, is estimated. These opportunity costs - the so called *water values* - depend not only on the current amount of stored water but also on the future water inflows as well as the achievable future profits. Water inflows and future profits are uncertain and not fully known in advance so a stochastic programming problem occurs.

In a deregulated market environment the self-scheduling problem is a bidding problem, where the producer can choose among several market products to bid. This problem is so complex that it is usually solved in different steps. First a medium-term planning for determining the monthly/yearly production strategy is formulated. Then a short-term planning for optimizing the production over the next days is performed, where different market products are considered. For the medium-term planning a coarser model with longer time steps is used. Especially for a hydro producer with pumping capabilities such a medium-term planning neglects the profit which can be achieved on a hourly basis. In previous works [1], [2] we have shown how to integrate a hourly based deterministic optimization within a stochastic dynamic programming scheme considering different market products.

The self-scheduling problem is becoming more complicated if the influence of the generation company on market prices is studied. This influence is important when analyzing Forward contracts. By nature hydro producers are in long position and they try to hedge this position by Forward contracts. Most prominent reasons for this is first due to limited liquidity of market products and second because of their price influence on day-ahead and spot markets. Such problems are typically solved by considering a market clearing process where each market participant either bid their marginal costs or act strategically. Apart from modeling issues this solution concept becomes quickly computationally intractable if there is considered hourly based bidding with some stochastic variables for a time horizon of several months. In this work suggested is therefore an alternative, where by assuming a linear dependency between amount and price of the commodities price influences are modeled. It will be shown how such an approach remains tractable.

Overall the goal of this work is to propose a modeling concept, which is able to solve the bidding problem for a price-maker hydro power plant, with pumping capabilities and seasonal water inflows, considering forward contracts and stochastic water inflows for a time horizon of one year. It will be shown, how such an optimization could be used as decision support. Further simulations illustrate this use on a realistic setting.

Proposed is a multi-stage stochastic program. For each stage, the optimal amount of a Forward contract is found, taken into account hourly recourse actions. For these recourse actions stochastic inflows are considered and the algorithm estimates the profits which can be achieved within the respective stages by taking into account both a day-ahead market and the physical constraints of the power plant. Hourly priced forward curves are taken as clearing prices for the day-ahead market. However the algorithm assumes linear dependence on the price depending on the bidding amount. With this approach water values are calculated as well as an optimal bidding strategy for each stage.

Most important contribution of this paper is the consideration of a price-maker in this bidding problem easily applicable to

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a self-scheduling of a hydro power plant operator, considering day-ahead and a Forward market. Second contribution is the simulation of this bidding process for one year in a realistic setting.

For an overview of stochastic programming in the energy sector [3] can be recommended. A review about stochastic dynamic programming in hydro power planning is given in [4].

In [5] an hourly day ahead market and a monthly financial Future market are considered. Stochasticity on the pool prices is introduced via scenarios. Large scale linear programming, which has the advantage of easily incorporating risk measures and minimizations, is used. Because of the well-known "curse of dimensionality" [6], computational tractability is a problem. So in their example they consider only two periods with five price scenarios in the recourse stage and three different forward contracts, which is by far a too simplified view for incorporating hourly and seasonal water inflows as in our case.

In [7] the focus is on constructing bid curves by a large scale mixed integer linear program. A finer model is used on near term and then a coarser one going forward. Whereas the finer model models e.g. non-convex efficiency curves and hourly time resolution, the coarser one is very simplified with price-segments. Again the problem is to be computationally tractable because no decomposition was considered.

The mentioned approaches haven't dealt with limited liquidity in their models. The most interesting ones that does are as follows:

In [8] game theory is used to study a duopolistic case where by applying stochastic dynamic programming they compute the water values of these two utilities. However this approach would be not suitable to consider hourly bidding for yearly time horizon.

In [9], [10] an optimal scheduling of a price-maker pumped hydro storage producers was performed. By using residual demand curves the influence on the pool market prices was modeled. This mixed-integer problem results in short-term bidding strategies as well as mid-term reservoir management. Apart from issues in modeling the competition as a demand curve, Forward contract were considered only as predefined and fixed. The authors in [11] used a similar approach, however they considered stochastic water inflows but no forward contracts. They applied stochastic dual dynamic programming in order to find a long-term operation strategy.

The remainder of the paper is organized as follows: First, the model is explained both conceptually and mathematically. Afterwards the Swiss energy market environment is explained briefly and the data used in the case study is motivated. This case study then illustrates the outcomes of the model and simulates its usage on a realistic setting.

II. MODEL

A. Model characteristics

The overall model is similar to those in [1], [2], [5]. Forward contracts can be traded at the beginning of every time stage, e.g. daily or monthly. Those decisions have to be made without any knowledge about the realization of the random variables, the water inflows in this case. So they are referred as *here-and-now*-decisions. The prices for the forward contracts however are known beforehand. The are modeled as linearly depending on the amount of traded contracts.

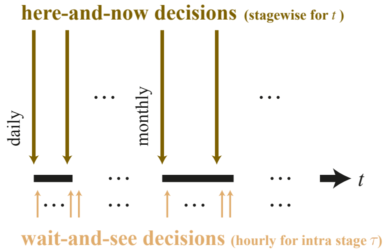

Fig. 1. Multi-stage program with varying time length between stages and therefore also varying amount of hourly wait-and-see decisions.

Within the time stage recourse actions can be performed. Those actions are made with perfect information and the random variables are known, so they are denoted as *wait-and-see*-decisions. In contrast to previous works, there are two decisions to make for each hourly intra-stage time step: production and pool decisions. Production decisions take into account all technical constraints of the power plant whereas pool decisions are about fulfilling the forward contracts obligations (which could have been settled earlier) as well as bidding the production on the day-ahead market. The goal of the recourse action is maximum profit. The prices on the day ahead market depend linearly on the bid. This results in a quadratic problem. The outcome of this intra-stage problem is an estimation of the profit which can be achieved for a given amount of water to discharge from the basins.

It should be noted, that production of energy and pumping is not part of the objective function. The idea is that the algorithm can choose between generating energy with water stored in the basins or buy it on the day-ahead market.

Fig. 1 shows the decision framework. The time length between the stages will vary depending on available Forward contracts. Therefore also the problem size of the recourse action will change. Because the cumulated profit will be strictly increasing in time, a decomposition is meaningful. By also discretizing the state and decision space the well-known stochastic dynamic programing scheme can by applied (introduced in [12] and [13], first for hydro power planning problems in [14]).

B. Mathematical model

Table I gives an overview and explanation of the used variables. Let $\theta_{t,x}$ be the expected future profit for state x and time stage t, also referred as *profit-to-go*. This profit-to-go can be recursively calculated:

$$\theta_{t,x} = \max\ \frac{1}{2} y_{t,x}^{T} \cdot \mathrm{H} \cdot y_{t,x} + y_{t,x} \cdot c_t + \mathop{\mathrm{E}}_{\xi \in \Xi_t} \left[Q_{t,x,\xi} + \theta_{t+1,x} \right] \quad (1)$$

TABLE I
VARIABLES

Variable	Explanation
$t \in \mathrm{T}$	time stage
$x \in \mathrm{X}$	state variable (discretized basin filling [m^3])
$\xi \in \Xi_t$	random data (water inflows [m^3])
$\theta_{t,x}$	profit-to-go [€]
$y_{t,x}$	here-and-now decision (Forward contract bid [MW])
H, c_t	quadratic return for $y_{t,x}$
$Q_{t,x,\xi}$	optimal value of recourse actions [€]
$\tau \in \mathcal{T}$	hourly intra-stage time steps [h]
$\chi_{t,x,\xi}$	hourly recourse decision (production / pool / basin)
G, f_t	quadratic return for $\chi_{t,x,\xi}$
$A_{t,x}, b_{t,x,\xi}$	equality constraints (technical and financial)
lb_t, ub_t	lower / upper bound

with $\underset{\xi \in \Xi_t}{\mathrm{E}}$ being the expected value considering random data ξ for this stage. Note, that H, c_t are predefined because power Futures prices, which depend linear on $y_{t,x}$, are known beforehand. This results in a quadratic objective function. $Q_{t,x,\xi}$ is the optimal value of a deterministic maximization problem depending on time, state and realized random data. This problem has a quadratic objective function with linear equality constraints:

$$Q_{t,x,\xi} = \max \frac{1}{2}\chi_{t,x,\xi}^T \cdot \mathrm{G} \cdot \chi_{t,x,\xi} + f_t^T \chi_{t,x,\xi} \tag{2}$$

$$\begin{aligned} \mathrm{A}_{t,x} \cdot \chi_{t,x,\xi} &= b_{t,x,\xi} \\ lb_t \leq \chi_{t,x,\xi} &\leq ub_t \end{aligned}$$

$\chi_{t,x,\xi}$ is a vector containing both state variables (if needed) and decision variables. The right hand side of the constraints, $b_{t,x,\xi}$, depends on realized random data while $\mathrm{A}_{t,x}$ is not. So it is assumed that the actual realization of the random data is known. Therefore this is a deterministic problem. Constraints to fulfill are first technical constraints like assuring a certain amount of water to discharge and others associated to the plant topology. Secondly also financial constraints like satisfying the contracted energy are part of the equality constraints. The variable bounds lb_t, ub_t depend on time t since turbine and pump capacities may vary e.g. because of scheduled maintenance.

By discretizing both state variable (basin filling) as well as the amount of water to discharge the problem reduces to a shortest path problem (Fig. 2). The algorithm has to find the optimal amount of water to discharge for each basin filling which is found by solving the recourse problem. Similarly also the optimal amount of a Forward contract is found for each basin filling. This procedure has to be repeated for every time step t recursively.

Outcome at the end is the expected profit-to-go depending on time stage and basin filling. The derivation of it results in the opportunity costs, the water values. A forward simulation, which applies the optimal decisions for given starting basin fillings, finds the optimal amount of Forward contracts to sell for each time stage.

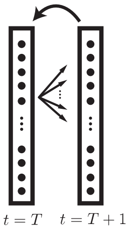

Fig. 2. Discretization of the state and decision variables. The problem reduces to a shortest path problem which can be solved recursively.

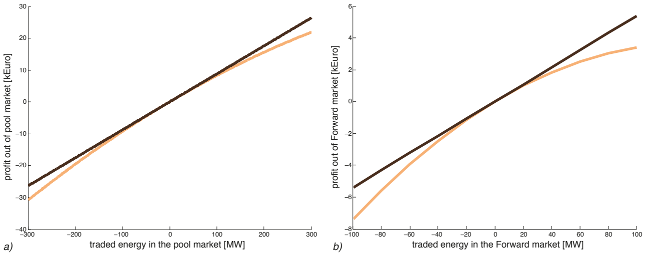

Fig. 3. Linear dependent prices for a) pool and b) Forward prices (orange) and constant price (brown). This results in profit quadratically depending on traded amount.

III. DATA REQUIREMENTS AND ACQUISITION

The model is applied to a realistic example of a hydro pumped storage power plant located in the Swiss Alps. The power plant is built up of two stages. The first stage consists of an upper basin, which acts as seasonal storage and a lower much smaller basin. In between two turbines and two pumps are able to move water up or down. In the second stage the water out of the smaller basin can be used in two turbines. Both basins are fed by water inflows from precipitation and from glaciers. Total installed generation capacity is 240 MW, pumping capacity 46 MW and total storage is about 200 GWh or 100 millions of m^3.

The length of the time stages $t \in \mathrm{T}$ as well as of the intra-stage time steps $\tau \in \mathcal{T}$ are given by the market products: daily, weekly and monthly steps for the length of t depending on available Forward products and hourly steps for τ because of the hourly pool market. Because the available Forward products are overlapping in time there is taken first daily products for one month and then available weekly and monthly products.

The bigger seasonal basin is chosen as state variable x in the here-and-now decision problem since it collects all necessary past information. The smaller basin acts as state variable in the wait-and-see decision problems allowing to formulate technical constraints. In both reservoirs stochastic water inflows are considered, estimated out of historical data.

The power plant is operated within a deregulated market environment with several market products available. Most importantly are day-ahead market, over-the-counter (OTC) and ancillary services market. Forward contracts, which are traded OTC, are mostly based on standard power Future contracts

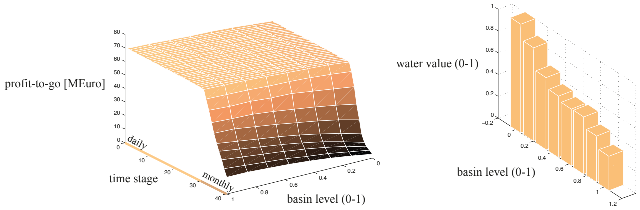

Fig. 4. a) Profit-to-go for all basin levels and time stages. Note, that the first stages have daily step size and therefore the profit relatively does not change much. b) Water values out of the profit-to-go function at the first time stage. They can be used directly for comparison with the day-ahead market prices.

with financial settlement. Since the power plant is usually operated in peak hours, peak Futures make the most sense to consider. So EEX Phelix German peak Future prices are taken as estimation of a Forward contract bid price. The day-ahead price is estimated by an hourly priced forward curve (HPFC) which is constructed by the industry partner and which is arbitrage-free to the Future products.[1]

Fig. 3 shows the profit for given pool and Future prices depending on the bidding amount. Because of the modeled linear dependencies between prices and amount the profit depends quadratically on the bidding amount. The linear dependency has to be estimated by experience or historical data. For the case study this influence was made constant throughout all time stages although it would make sense to adjust the influence depending on time. Note, if no price dependency would be modeled, there would be not much affinity present to trade Forward contracts from a profit maximization point of view.

As optimization start, 1st April 2009 was chosen in order to have enough historical data available. The water inflows for hydro power plants located in the Alps have great seasonality with high inflows in late string and summer and no inflows in winter. This means the optimization starts with zero amount of water stored in the basins but also that the residual water in the basins at the end of the optimization one year later is given no value.

All optimizations were done in Matlab R2012a with CPLEX 12.4 as solver for the quadratic program. The stochastic dynamic program problem can be formulated embarrassingly parallel, so that one optimization run took no longer than 4 minutes with a standard computer, with a quad-core 2.3 GHz Intel Core i7 processor and 8 GB of RAM. However, if the special structure of the dynamic program is not exploited in the algorithm, it can easily take hours to solve.

IV. CASE STUDY

The case study is divided into two parts. First, the results out of one optimization run is revealed. These results are then used for simulating the operation of the hydro plant for actual realized data which is explained in the second part.

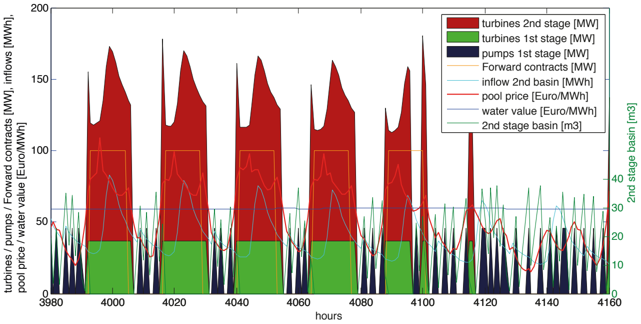

Fig. 5. Snapshot of one week of the forward simulation. Production depends on current water value and pool price. However 2nd stage basin limits generation and pumping capabilities by a large extend.

A. Optimization results

Fig. 4 a) shows the expected profit-to-go for all time stages and upper basin fillings. Important is the strictly increasing profit over time as well as the concavity within a stage in respect to the basin filling. Since the first few time stages correspond to daily stages the profit won't change much. For these stages the derivation of the profit-to-go function with respect to the basin filling is of more importance. These water-values are shown in Fig. 4 b) for the first stage. The water values express how much profit is expected for an additional amount of water stored in the upper basin. They can be used directly for comparison with the day-ahead market prices.

Another outcome of the optimization is the optimal amount of Forward contracts to buy or sell depending both on time stage and basin level. Important are only the values at the actual basin levels. Those are unknown therefore a forward simulation is used to find expected basin levels. This simulation is based on the found water values, expected day-ahead prices and optimal Forward bids. It mimics the operation of the power plant and expected basin levels over time are found. By iterating over all time stages the following simulation is proposed:

1) Find actual reservoir level, water value and positions.
2) For all hourly intra-stage time steps: Heuristic on how to deploy turbines and pumps.

At the beginning the actual reservoir level is taken as input and the corresponding water value is calculated. The positions are for the financial balance, specifying the Forward contracts obligations per time stage. Afterwards for each hourly intra-stage time step a heuristic deploys turbines and pumps. For this the estimated water values are compared with the day-ahead market prices: If the water value is lower than the market price, turbines are fully deployed, and if they are higher pumps are fully deployed. Of course this deployment is not always possible since it has to take into account all technical details like correct efficiencies, expected water inflows and basin balances. Good insights in daily practice of the power plant is needed in order to accomplish this heuristic. At the end the financial balance has to be evaluated, considering

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[1] Note, that the arbitrage-free condition may get lost depending on how the price influence was modeled. However this may be desired.

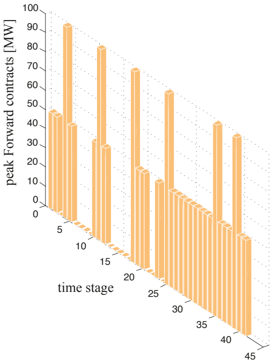

Fig. 6. Example of optimal peak power Forward bids for expected basin levels. Note that since the hydro power plant is in a long position, no bids are bought. However this could change for later optimization runs, if already sold bids are bought back.

Forward obligations and produced/used energy on one side and bidding on the day-ahead market on the other side. For the day-ahead prices again the linear dependency on traded amount is assumed.

Fig. 5 shows the results of one week for the forward simulation. For this week 100 MW of peak Forward contracts were sold. For better readability not all results are shown and turbines and pumps are combined stage-wise. As one can see, the fully deployment of turbines and pumps is not always possible. One reason for this is the 2nd stage basin, which is relatively small and because of that limits turbine and pump capabilities in both stages depending on actual filling and water inflows. Interesting is also the fifth day, where the obligation of forward contracts are not met and therefore relatively cheap energy are bought back in the day-ahead market.

Note that the meaning of the forward simulation is not a hourly bidding strategy for the whole year, which would be unrealistically. The results are expected basin fillings and with these values the optimal bids for the Forward market can be evaluated. Fig. 6 shows an example of these optimal bids for expected basin levels. These values can be used as decision support for the trading group in the respective company.

B. Simulation of the bidding process

Fig. 7 shows a flow chart, which describes how the operation of the power plant was mimicked. Remember that the proposed scheduling considers only one snapshot of the whole bidding problem, that is what would be the present most optimal bidding strategy without taken into account that this strategy could be changed afterwards. In order to evaluate how the strategy would change over time, if new information are available the following procedure is proposed:

1) Find expected water values and optimal power Forward bids for all discretized basin levels (1st stage basin) and time stages. Inputs are stochastic spot prices (out of HPFC), stochastic water inflows (from historical data) and actual available power Future prices.

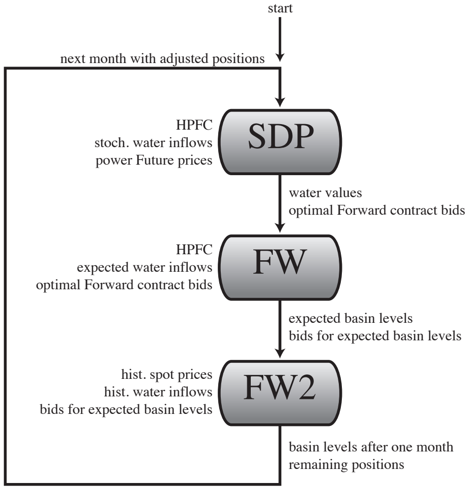

Fig. 7. For every month one optimization is done with updated data. The first forward simulation is for determining expected basin levels for optimal bidding, the second forward simulation is for simulating the production for one month with actual data.

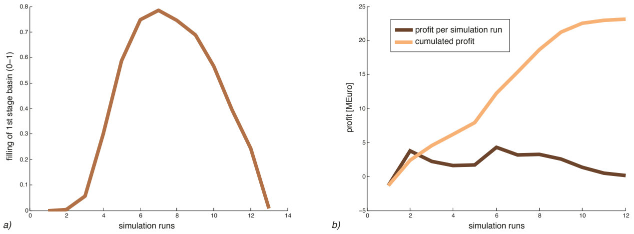

Fig. 8. Results out of bidding process simulation: a) Filling of 1st stage basin: The basin is used up to 80% of its maximum capacity. b) (Cumulated) profit for each simulation run.

2) Find expected basin levels and power Forward bids. Inputs are expected spot prices and water inflows as well as calculated water values and optimal Forward bids from the first step. Note that in this forward simulation no stochasticity has to be taken into account.

3) The second forward simulation mimics the application of the found optimal peak power Forward bids and water values. For this simulation actual historical data is used both for spot prices and water inflows. The outcome of this simulation are resulting basin levels of the 1st stage as well as remaining financial positions and achieved profit.

This procedure is repeated, in our case monthly, with a receding horizon. Beginning is again 1st April 2009 and the end is one year later. The time horizon of the optimization and the forward simulations are therefore one month up to one year. Each Forward contract amount were discretized into 11 values. Total computation time for the whole simulation was 35 minutes.

Fig. 8 shows the overall results of the bidding process simulation. On one side the filling of the 1st stage basin, the upper bigger one and also state variable, is shown normalized

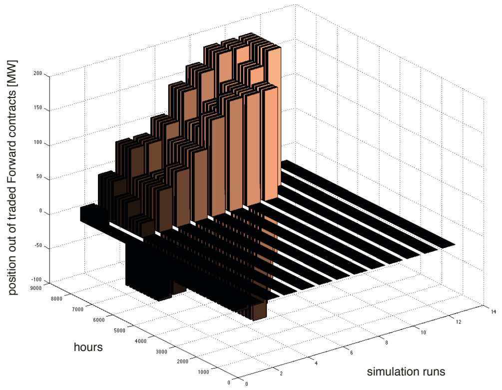

Fig. 9. Positions out of traded Forward contracts. Note, that traded contracts are only in the peaks hours and that the higher the simulation run the less hours are able to trade. One can see an increase of short positions.

to its capacity. Such a filling was expected, since the basin is empty at the beginning and inflows occurs only in the first months. So the basin acts as seasonal storage in order to be able to produce energy throughout the whole year. Fig. 8 b) outlines the profit which is achieved in the day-ahead market and by trading Forward contracts. At the beginning the profit is negative, since Forward contracts were bought so the power plant went even more in a long position.[2] However afterwards profit is achieved throughout the year with a tendency to more profit in the summer than in winter time which was also expected because of non-storable water inflows.

If no trading in the Forward market was allowed (not shown in figures) total profit sum up to 20 millions of Euro compared to 24 millions in the former case. So the simulation shows that trading in the Forward market results to an increase of profit of about 20%. Note, that historical achieved revenue out of sold energy were about 20 to 30 millions of Euro, making the simulated ones realistic estimations.

Fig. 9 finally shows the positions of Forward contracts. At the beginning there are even long positions but the less time left until settlement the more the positions are in short. This is of course reasonable since the storage power plant is long by nature and short positions are needed in order to hedge its operation.

V. CONCLUSION

This paper presented as first contribution a modeling concept which is able to solve the bidding problem for a price-maker power plant. The concept was applied to a pumped storage hydro plant with seasonal and hourly operational constraints in a deregulated market environment. Time horizon was one year and considered were stochastic water inflows and day-ahead prices as well as Forward contracts.

Proposed was a multistage stochastic program with a quadratic recourse problem, which was dynamically solved. It was shown how the results of such an optimization, water values and optimal Forward bids, can be used as decision support. Further as second main contribution a simulation revealed this use on a realistic setting.

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[2] Note, that financial and operational risks were not considered explicitly but were taken care by the risk-neutral stochastic optimization.