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Risk-averse medium-term hydro optimization considering provision of spinning reserves

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Abstract—This paper presents two algorithms for solving a medium-term hydro optimization. Considered are risk-averse operation, provision of spinning reserves as well as short-term production flexibility. Proposed is a variant of stochastic dual dynamic programming (SDDP) and stochastic dynamic programming as a benchmark. A risk measure is introduced in both methods. To deal with short-term production flexibility a decomposition of the problem into inter- and intrastage subproblems is performed. The provision of spinning reserves leads to non-convex value functions. To deal with it in SDDP a method based on Lagrangian relaxation was used which was further enhanced by locally valid cuts in order to find realistic water values.

Index Terms—hydro power scheduling, medium-term planning, stochastic programming, spinning reserves, risk measures, intrastage subproblems, stochastic dual dynamic programming, stochastic dynamic programming, convexification.

I. INTRODUCTION

In a deregulated market environment the self-scheduling problem for a hydro storage power plant is a bidding problem, where the producer can choose among several market products to bid. Due to its complexity the problem is usually solved in different steps: First a medium-term planning for determining the monthly/yearly production strategy is formulated followed by a short-term planning for optimizing the next days. For the medium-term planning a coarser model with longer time steps is used.

Such a medium-term planning neglects the profit which can be achieved on a hourly basis. In previous works [1], [2] we have shown how to tackle this problem. This was made possible by decomposing the problem into inter- and intrastage subproblems (term introduced in [3]). In [4] we analyze and evaluate different techniques in this perspective.

In practice however operators are not only interested in highest profit on average but also achieving it under acceptable profit risk. It is therefore important to consider this production-risk awareness in the estimation of the water values, i.e. such that reservoirs are depleted earlier.

Note, that theoretically the risk could be managed solely by using power derivatives, because under some mild assumptions the production planning can be done independently from hedging [5]. However in practice many operators do both: (often implicit) risk-averse production planning as well as hedging by financial instruments.

Therefore in this paper we are interested in profit-risk by operational decisions without taking into account a possible later hedging with financial products.

The typical method, which allows for risk-constrained multi-stage stochastic programs for hydro power plants, is stochastic dual dynamic programming (SDDP) [6]. However such algorithms rely on the convexity of value functions. This condition is not fulfilled in our case, particularly because a market for spinning reserves is considered. However as it was shown in [1], the remuneration of such markets contributes to a substantial part of the revenue of hydro power plants (in Switzerland) and is therefore important to consider.

For only a few interconnected reservoirs another possible solution method in this context is stochastic dynamic programming (SDP) where the discussed non-convexities can be handled. Time-consistent and coherent risk measures as discussed in [7] for SDDP are not directly applicable in SDP. However, recent works [8], [9] suggest using a construction out of single period risk measures for SDDP. We will show in this paper that this construction fits also well in a SDP formulation.

In summary the problem to approach is a risk-constrained medium-term hydro storage operation optimization with hourly production flexibility and non-convex value functions. In this paper we propose an approach based on SDDP with inter- and intrastage subproblems (as presented in [10] for SDDP), extended by a coherent, time-consistent, nested risk measure (as analyzed in [11]).

To deal with the spinning reserves market, the problem has to be solved using approximations. Proposed is the application of Lagrangian relaxation (as used in [12], [13]) enhanced with concepts from approximate dynamic programming. The idea is to approximate the value function locally with a better quality than the Lagrangian relaxation method would allow. We will show that this leads to more accurate water values.

Finally as a benchmark the problem is also solved with SDP as presented in [2] extended by the risk measure.

Contributions of this paper

The major contribution of this paper is the presentation of two algorithms for solving the medium-term hydro optimization with consideration of three issues: risk-aware operation, short-term production flexibility and provision of spinning reserves. Whereas previous works have shown how to address

TABLE I
VARIABLES

Variables	Explanations
$t \in T = \{1, \dots, T\}$	time stages, $T = 52$ [week]
$\xi_t \in \Xi_t$	realized data, possibly random
$\theta_t(v_{t-1}^{seas})$	risk-aware profit-to-go functions [€]
$\rho(Z), \lambda, \alpha$	risk measure function and tunable parameters
$\tau \in \{1, \dots, 168h\}$	hourly intrastage time steps [h]
Q_{ξ_t}	optimal value of intrastage problem [€]
c_t^{pool}	hourly pool prices [€/MWh]
$v^{seas}(t), v^{daily}(t, \tau)$	filling of seasonal/daily reservoir [m ³]
$s(t, \tau)$	spill [m ³]
$a(t, \tau)$	charges from upstream reservoirs / inflows [m ³]
$u(t, \tau), p(t, \tau)$	generating / pumping [MW]
$m(t, \tau)$	position on energy market [MW]
$f_u(u), f_p(p)$	function of produced/used energy to water flow
$q(t)$	binary: provision of frequency control [0/1]
q^{min}	technical minimum [MW]
q^{max}	maximum amount of frequency control [MW]
q_t^c	remuneration for frequency control [€/MW]
$\bar{v}^{seas}(t)$	trial of seasonal filling [m ³]
$\mu(t)$	slope of a Bender cut [€/m ³]
W_t	discretized water discharge [m ³]

one of these issues at a time, this paper is the first one which can deal with all of it simultaneously, resulting in more realistic results.

Other contributions are: (i) introduction of the concept of locally valid cuts for solving non-convex value functions with SDDP, (ii) the combination of parts of the works [10], [11] and (iii) the application of a risk measure within SDP.

References and structure of the paper

Notable references for stochastic programming in the energy sector and SDP in particular are [14]–[17]. The same for SDDP are [6], [18], [19]. For risk-constrained hydropower scheduling [8], [20]–[24] can be recommended, for approximate dynamic programming the textbook [25]. Finally SDDP for non-convex problems were analyzed in [12], [13], [26].

The remainder of the paper is organized as follows: In section II, the model is explained. Afterwards the proposed solution method is discussed following with an evaluation in section IV. The paper concludes with some remarks.

II. MODEL

The overall problem is about acquiring a reasonable operating policy for a hydro plant, i.e. the *water values* as production opportunity costs. These water values shall take into account both some profit-risk as well as revenue from provision of spinning reserves. The latter can be also seen as some kind of capacity payment whereas the main revenue is achieved in the hourly energy market.

It is assumed that the power plant is built up out of two different kinds of reservoirs: the ones operated in seasonal cycles and the others in daily cycles. Further a valid assumption for power plants in the Alps is that the yearly amount of water

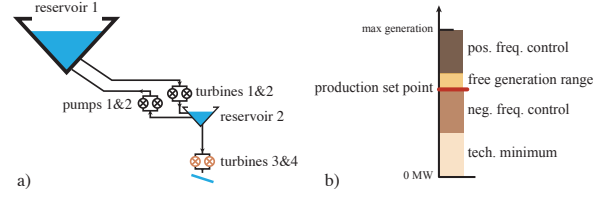


Fig. 1. a) Schematic overview of the considered hydro power plant. Reservoir 1 is the seasonal storage whereas reservoir 2 is the daily one. Turbines 3 and 4 are qualified to provide spinning reserves. b) Generation of a turbine with spinning reserves provision. The presence of a substantial technical minimum results to a mixed-integer problem.

inflows remains stable. Therefore a time horizon of one year is considered.

The outlines of the modeled power plant are shown in Fig. 1 a). As time duration for the main interstage time steps one week was chosen, since the considered spinning reserves are procured weekly in Switzerland at the moment. However weekly profit-to-go functions are also practicable for a medium-term optimization of the chosen power plant. Note that with this approach only for the seasonally operated reservoirs (reservoir 1 in Fig. 1 a)) water values are calculated.

In the next subsections some particularities of the proposed model are explained more in detail:

- A. Provision of spinning reserves
- B. Inter- and intrastage subproblems
- C. Nested single period risk measures

Finally the model is mathematically formulated and discussed.

A. Provision of spinning reserves

In Switzerland the most interesting spinning reserves market is the one for secondary frequency control reserves. The current market rules require the bidding of symmetric power bands. If the bid is accepted the power band has to be provided for the tendered period of one week. The actual demand is requested automatically. It is assumed, that this request is symmetric within the tender period so that the energy delivery is balanced out. Considered profit out of this market is the remuneration for holding the capacity whereas payments for energy delivery is neglected.

Fig. 1 b) shows the segmented power range of a hydro power plant in the Swiss system. The turbines have to be continuously running at a certain set point when they provide control reserves. To prevent, that the turbines are operated inefficiently, a technical minimum is introduced. So the provision of secondary control reserves reduces the production flexibility considerably.

This behavior can be modeled with mixed-integer constraints, where one binary variable per turbine indicates if secondary control reserves is provided or not. This results in non-concave profit-to-go functions.

B. Inter- and intrastage subproblems

The problem is decomposed into inter- and intrastage subproblems (Fig. 2 a)). Pool market decisions, provision of

spinning reserves and hourly operational reservoir decisions are modeled as part of the intrastage subproblem. This two-stage stochastic problem can be formulated as a mixed-integer linear problem (MILP).

Decisions about seasonal reservoir management are modeled in the interstage problem which can be formulated as a multi-stage stochastic program also called Markov decision process. From the modeling point of view this decomposition means, that the seasonal reservoir in the power plant is considered for every interstage time step, i.e. weekly. However, the operation of the daily reservoir is considered hourly in the intrastage subproblems. The intrastage problems estimate the profit which can be achieved on a hourly basis for a given filling state of the seasonal reservoir. With this decomposition the complexity of the problem is reduced significantly without unreasonable valuation of short-term flexibility.

There are several approximations made here. First, the random data is assumed to be known within each intrastage subproblem. This means prices and inflows are known one week in advance which is an optimistic view. Further, the filling of the daily reservoir is neglected in the calculation of the profit-to-go functions as well as the water balance is not respected between consecutive weeks. Practically this means that the daily reservoir is empty at the beginning and end of each week. In [4] we have shown that despite these approximations the quality of the obtained water values are reasonably good.

The water inflows are modeled as stochastic variables. The hourly inflows uncertainty is higher than the weekly one. Therefore two inflow scenarios for each interstage problem is modeled and 100 scenarios, that are correlated to the interstage scenario, for the intrastage subproblems.

C. Nested single period risk measures

In line with the authors of [8], [9] we consider as risk measure a weighted sum of the expected profit and Average-Value-at-Risk $AV@R_\alpha$. The latter is also called *Conditional-Value-at-Risk*, where with *Value* we actually mean profit. The single period coherent risk measure $\rho(Z)$ for a profit Z [21]–[23] is then:

$$\rho(Z) = (1 - \lambda) \mathbb{E}[Z] - \lambda AV@R_\alpha[Z] \quad (1)$$

where $AV@R_\alpha$ is:

$$AV@R_\alpha[Z] = \sup_{\iota} \{ \iota - \alpha^{-1} \mathbb{E}[(Z - \iota)_-] \} \quad (2)$$

In other words an additional penalization term is introduced: the lower α -quantile of the profit distribution. Tunable parameters are both λ as well as α where for this paper $\lambda = 0.5$ and $\alpha = 10\%$ are used.

Consider now the sequence of future profits Z_1, \dots, Z_T which arise in multi-stage programs. Then there is:

$$\rho(Z_1, \dots, Z_T) = \rho(Z_1 + \rho(Z_2 + \dots + \rho(Z_T) \dots))$$

Hence the (time-consistent) risk measure is formulated in a nested and dynamic way, which makes it convenient for dynamic programming. From the modeling point of view

this construction means, that the operator considers the consequences of decisions less the further away in time the consequences influence the profit risk. We believe that this is a reasonable model. In [24] a more detailed discussion about this risk construction is given.

D. Mathematical model

The hydro scheduling problem naturally leads to a multi-stage stochastic program which can be formulated in a dynamic way. Let θ_t be the risk aware future profit, the *profit-to-go function*, for the seasonal reservoir. Then it can be stated:

$$\begin{aligned} \theta_t(v_{t-1}^{seas}) &= \max_{\rho} \rho(Q_{\xi_t}) = \\ &= \max \left[(1 - \lambda) \mathbb{E}_{\xi_t \in \Xi_t} [Q_{\xi_t}] - \lambda AV@R_\alpha [Q_{\xi_t}] \right] \end{aligned} \quad (3)$$

where:

$$Q_{\xi_t}(v_{t-1}^{seas}) = \max_{u_\tau, p_\tau, s_\tau, q_t} (c_t^{pool})^T m_t(\tau) \dots + q_t^c \cdot q_t^T q^{max} \cdot 168h + \theta_{t+1}(v_t^{seas}) \quad (4)$$

subject to:

1. water balances:

$$v_t^{seas} = v_{t-1}^{seas} - \sum_{\tau} [f_u(u_\tau) - f_p(p_\tau) - a_\tau] \dots - s_t - q_t^T f_u(q^{min} + q^{max}) \cdot 168h \quad (5)$$

$$v_\tau^{daily} = v_{\tau-1}^{daily} - s_\tau - f_u(u_\tau) + f_p(p_\tau) + a_\tau \dots - q_t \cdot f_u(q^{min} + q^{max}), \forall \tau \quad (6)$$

2. financial balance:

$$m_\tau = u_\tau - p_\tau + q_t \cdot (q^{min} + q^{max}), \forall \tau \quad (7)$$

3. spinning reserves provision:

$$q_t \cdot (u^{min} + q^{max}) \leq u_\tau \leq \bar{u} - q_t \cdot q^{max} \quad (8)$$

4. bounds:

$$\begin{aligned} lb_t &\leq u_\tau, p_\tau, s_\tau, q_t, v_t^{seas}, v_\tau^{daily}, m_\tau \leq ub_t \\ u(\tau), p(\tau), s(\tau), v^{daily}(\tau) &\in \mathbb{R}^{n \times \tau}, m(\tau) \in \mathbb{R}^T \\ v_t^{seas} &\in \mathbb{R}^n, q_t \in \{0, 1\}^n \end{aligned}$$

$\mathbb{E}[\dots]$ denotes the expected value over sampled random data $\xi_t \in \Xi_t$. Note, that the stochastic data process ξ_1, \dots, ξ_T is Markovian, so the risk-averse profit-to-go function θ_t depends only on ξ_t and not on the whole past process ξ_1, \dots, ξ_t .

Q_{ξ_t} is the optimal value of the deterministic intrastage problem. It is a function of the former state, time and realized random data ξ_t . The purpose of the problem Q_{ξ_t} is to estimate the intrastage profit in a realistic way, by hourly operating the power plant most profitably within the week. It is formulated as a two-stage stochastic program. In the first stage the amount of spinning reserves to bid is decided. This is done for each turbine, which is qualified for this provision, with the help of the binary variables $q(t)$. Afterward ξ_t is revealed at once, so the water inflows and prices for the whole week become known. Then actual hourly production decisions take place. As a consequence Q_{ξ_t} is a deterministic, linear maximization problem with binary variables.

In the intrastage problem the revenue from the energy market is maximized, where the position on the energy market $m_t(\tau)$

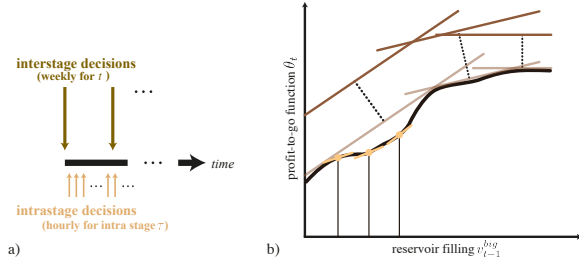


Fig. 2. a) Decomposition of the multi-stage program into interstage and intrastage subproblems. b) Illustration of an approximation of the profit-to-go function. Solid brown lines show cuts generated by Lagrangian relaxation. The cuts are valid however possibly not close to the actual profit-to-go curve. By optimization of the Lagrange multipliers the cuts can be made more accurate (up to the light brown lines). The curve can be approximated further by locally valid cuts. Those cuts are locally accurate however not valid for all reservoir fillings. Their use is primarily for estimating realistic water values, the slope of the profit-to-go function.

is specified in the financial balance constraint (7). Also part of the objective function is the remuneration of the offered spinning reserves, which is based on the predefined value q_t^c . When spinning reserves are provided constraint (8) reduces the free generation capacity.

Constraints (5) and (6) of the intrastage problems ensure correct water balances. To keep notations simple $a(t, \tau)$ denotes both water inflows as well as charge from upstream reservoirs. Finally the variable bounds follow, where n denotes respective amount of turbines, pumps etc.

III. SOLUTION METHODOLOGIES

The problem to solve is a stochastic, dynamic problem with non-convex value functions. Sought are realistic water values that incorporate the above mentioned characteristics. Two approaches are used to solve it:

- A. Risk-averse SDDP with Lagrangian relaxation enhanced with locally valid cuts
- B. Risk-averse SDP without approximation of the profit-to-go functions

The method based on SDDP has the potential to solve larger problems than what is presented here whereas the scalability of the SDP-method is limited. However due to its accuracy SDP will act here as benchmark.

A. Risk-averse SDDP with Lagrangian relaxation enhanced with locally valid cuts

The first method builds up out of several techniques, which are now presented step by step:

1) *Risk-averse SDDP*: In [9] risk-averse SDDP is explained thoroughly. The overall idea is to approximate the profit-to-go function by hyperplanes. For the shown risk measure (1) the additional effort of solving a risk-constrained instead of a risk-neutral SDDP is modest. The critical problem is finding a suitable stopping criterion where we consider the stabilization of the upper bound which produces good results for our not overly complicated problem.

2) *Lagrangian relaxation for non-convex value functions*: For the application of SDDP or similar algorithms to problems with non-convex value functions basically two strategies are promising: (i) convexification of the value function and (ii) the convexification of the problem itself. The latter strategy seems to be of little avail here since one focus of this paper is the detailed consideration of spinning reserves. Therefore the first strategy was chosen.

We use Lagrangian relaxation to convexify the profit-to-go functions in order to get valid Bender cuts [12]: The violation of the coupling constraint, the first water balance (5), is penalized in the objective function by a Lagrange multiplier $\tilde{\mu}_t$, where $(\tilde{\cdot})$ emphasizes that this multiplier is only an estimate and probably not the optimal one:

$$\begin{aligned} \theta_t^{LR}(v_{t-1}^{seas}, \tilde{\mu}_t) = \max \rho(Q_{\xi_t}) - \tilde{\mu}_t \cdot [\dots \\ v_{t-1}^{seas} - v_t^{seas} - \sum_{\tau} [f_u(u_{\tau}) - f_p(p_{\tau}) - a_{\tau}] \dots \\ - s_t - q_t^T f_u(q^{min} + q^{max}) \cdot 168h] \end{aligned} \quad (9)$$

The multiplier $\tilde{\mu}_t$ together with the optimal value of the resulting mixed-integer problem $\theta_t^{LR}(v_{t-1}^{seas}, \tilde{\mu}_t)$ forms a valid Bender cut of the original problem.

3) *Optimization of the Lagrange multipliers*: The cuts generated by the Lagrangian relaxation technique are possibly not close to the actual profit-to-go function (see also Fig. 2 b)). In line with the authors of [13], we optimize the multiplier using a sub-gradient method. Note that the search for a good Lagrangian multiplier is not in order to solve the original problem (3) but to find a more accurate Bender cut. It is therefore interesting that the actual solution of the original problem is known and therefore the updating of the multiplier can be done in an elegant way.

The multiplier optimization has to be repeated for every scenario, for all trials and every time stage. Since the problem is similar for each scenario the optimal Lagrangian multipliers are shared between them.

4) *Locally valid cuts*: As outcome of the optimization water values are sought. With optimized Lagrange multipliers good approximations of the profit-to-go functions can be found however they are of less use for calculating water values. In Fig. 2 b) this is illustrated: the slope of the cuts from Lagrangian relaxation can be quite inaccurate. With the yellow cuts the slope is potentially closer to the real one, however such cuts would be valid only locally.

In order to calculate locally valid cuts two possibilities were evaluated: constructing them by numerical derivation and secondly by solving a locally convexified problem. For the first possibility problem (3) is solved twice for different but close trial fillings. Doing so a locally valid cut can be constructed with the profit $\theta_t(v_{t-1}^{seas})$ and the slope μ_t as numerical derivation:

$$\mu_t = \frac{\theta_t(v_{t-1}^{seas}) - \theta_t(v_{t-1}^{seas} + \Delta)}{\Delta} \quad (10)$$

where Δ denotes the difference of the tried reservoir fillings. The second possibility solves the locally convexified problem,

i.e. problem (3) with fixed binaries, and use the dual variable of the coupling constraint (5) as slope for the locally cut. In both cases the problem is solved twice, two MILPs in the former case and one MILP and one linear problem in the latter case. Both methods produced similar results thus the latter one is used from now on.

The usage of locally valid cuts complicates the calculation of the intrastage subproblem (4) in the backward pass. This is because at time point t the filling v_t^{seas} is not known and therefore the valid cuts for θ_{t+1} can not be determined. To solve this problem we propose a heuristic where the subproblems are solved first with a trial \tilde{v}_t^{seas} and then iterated until the filling converges to a value. In our set-up this procedure results solving a MILP approximately 4 times.

5) *Proposed model:* The Lagrangian relaxation technique and the technique with locally valid cuts complete each other. The proposed methodology is as follows: For the first iterations the Lagrangian relaxation technique is used to quickly find a first approximation of the profit-to-go function. Afterwards the technique with locally valid cuts is employed to refine the approximation locally and to get to accurate water values.

B. SDP with risk measures

Since the considered hydro power plant has only one seasonal reservoir there is the possibility to solve the problem for all discretized realizations of the states instead of only for some trial values. Given sufficient discretizations the result of this optimization is exact and will act as a benchmark.

In order to use SDP for solving the original problem (3) a water discharge W_t is specified, which denotes the amount of water release from the seasonal reservoir for one time stage. The intrastage subproblems task now is to deploy this water amount within the week for maximal profit (see also [2]). The water discharge is discretized and is part of the decision vector in the overall problem.

The problem (3) is therefore implemented with a slight change in the water balance equation (5):

$$v_t^{seas} = v_{t-1}^{seas} - W_t \quad (11)$$

$$W_t = \sum_{\tau} [f_u(u_{\tau}) - f_p(p_{\tau}) - a_{\tau}] + s_t \dots \quad (12)$$

$$+ q_t^T f_u(q^{min} + q^{max}) \cdot 168h$$

As a consequence the subproblems $Q_{\xi_t}(v_{t-1}^{seas}, W_t)$ are solved for each discrete value of the water discharge.

Note that the optimization is done risk-averse, with the same single period risk measure (1) as presented for SDDP. Since this risk measure is already formulated in a dynamic way it can be used here directly.

IV. EVALUATION

In the evaluation first the water values proposed by SDP are compared with the water values from the SDDP approach with and without the extension of locally valid cuts.

Afterwards we apply the water values in a Monte Carlo operation simulation study in order to evaluate the quality of the operation policies as well as if they are indeed risk-averse.

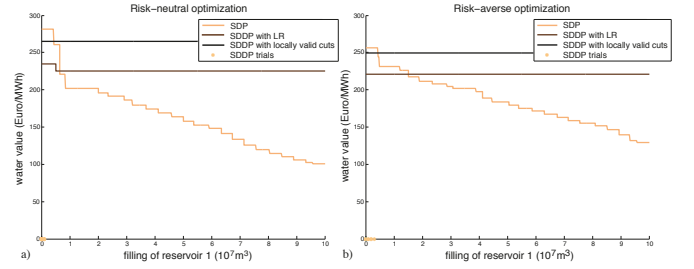


Fig. 3. Comparison of water values proposed by SDP, SDDP with Lagrangian relaxation (LR) and with locally valid cuts. The water values for SDDP are only meaningful where reservoir trials were taken, here for almost empty fillings. The water values are in general lower in risk-averse optimizations that means the reservoir is emptied earlier. The water values from the optimizations with locally valid cuts are closer to the ones from the benchmark method SDP.

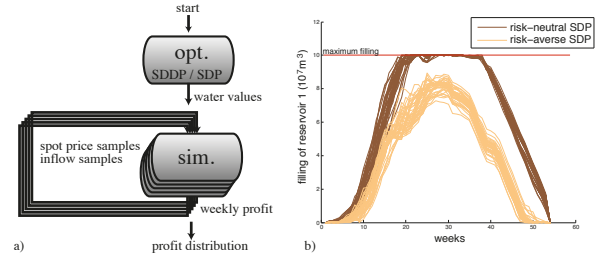


Fig. 4. a) Flowchart of the Monte Carlo operation simulation. First one of the optimization methods is performed. The proposed water values are then used in the simulation part, where the hydro power plant operation is mimicked over one year. This is done for several water inflows and price samples, resulting to a profit distribution for each optimization method. b) Filling of seasonal reservoir 1 for each of the samples for water values from risk-neutral and risk-averse SDP optimizations. Note that with water values from the risk-averse SDP the water release is earlier in time.

The optimizations were performed on a standard computer with 4 physical processor cores. CPLEX dual simplex solvers were used for the linear programs and a branch-and-cut algorithm for the mixed-integer problems.

The number of backward/forward pass iterations for both SDDP methods were limited in order to be comparable: 10 iterations for the Lagrangian relaxation method and 5 iterations with Lagrangian relaxation extended by 5 iterations with locally valid cuts. In both cases the upper bounds would have been also stabilized.

A. Water value comparison

Fig. 3 shows the water values for the benchmark method SDP as well as for the other two SDDP methods. Note, that only values for trial fillings are meaningful for the SDDP methods whereas SDP estimates for all filling levels water values. The values from risk-averse optimizations are lower, what was expected. The values from SDDP with the extension of locally valid cuts are closer to the ones from SDP which could show the benefit of this enhancement.

B. Performance of the optimization methods

The water values are now applied to an operation simulation of the power plant in order to evaluate their quality. In a

TABLE II
COMPARISON OF OPTIMIZATION METHODS OUT OF 100 SCENARIOS

	mean profit [M€]	AV@R _{10%} [M€]
perfect information	53.04	50.90
risk-neutral SDP	44.70	39.14
risk-averse SDP	43.54	41.90
risk-neutral SDDP with LR	43.54	37.31
risk-averse SDDP with LR	42.10	39.42
risk-neutral SDDP with locally cuts	44.32	38.04
risk-averse SDDP with locally cuts	43.49	40.21

Monte Carlo simulation the hydro power plant operation is mimicked over one year for 100 samples of water inflows and market prices (Fig. 4 a)). Fig. 4 b) shows the filling of the seasonal reservoir 1 for all samples for an operation simulation with water values from risk-neutral and risk-averse SDP. As expected with lower water values the reservoir is emptied earlier.

Finally Table II shows the resulting average profits as well as the AV@R_{10%} for all methods as well as for perfect information when the unknown data is known in advance. For simulations with risk-averse water values the AV@R_{10%} increases while mean profit is dropping. The enhancement of SDDP with locally valid cuts results in slightly higher profits compared with SDDP without the enhancement, which is indicating the better quality of the calculated water values. Note, that these results may change for different problem set up as well as scenario constructions.

V. CONCLUSIONS

In this paper two different algorithms for solving the medium-term hydro power scheduling were presented. Considered were risk aware operation, provision of spinning reserves and short-term production flexibility with uncertainty in both inter- and intrastage water inflows. To the best of our knowledge it is the first work which considers all of these issues simultaneously.

Further contributions were the enhancement of the Lagrangian relaxation method with the concept of locally valid cuts for using SDDP for non-convex value functions. Finally we believe it is also the first application of a risk measure within a SDP scheme for the hydro scheduling problem.

It was shown, how locally valid cuts can increase the accuracy of the SDDP method for non-convex value functions, especially for calculating water values. For the considered hydro power plant, SDP produced similar results. Future work has to apply the proposed methods on more complicated power plant structures where SDP would not be applicable.

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