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Medium-term optimization of pumped hydro storage with stochastic intrastage subproblems

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Abstract—This paper presents a medium-term self-scheduling of pumped hydro storage plants. A decomposition of the problem into inter- and intrastage subproblems, where the intrastage problems themselves are formulated as multi-stage stochastic programs, allows the detailed consideration of short-term flexibility. The method is presented together with three alternative approaches, where the short-term flexibility is considered differently: (1) with aggregated peak and off-peak prices, (2) with price duration curves and (3) with deterministic intrastage subproblems. The approaches are compared and evaluated in a Monte Carlo operation simulation study. The study is performed on a realistic hydro power plant with consideration of revenue from ancillary services.

Index Terms—hydro power, medium-term self-scheduling, stochastic dynamic programming, multihorizon stochastic programming, ancillary services.

I. INTRODUCTION

The goal of a medium-term hydro optimization is to find a seasonal operation strategy. One way to do this is to estimate future revenue, the *profit-to-go*, and to calculate production opportunity costs out of it, the *water values*. In Switzerland hydro power plants typically have storage reservoirs, which are operated seasonally, connected to smaller daily operated reservoirs. The future revenue is therefore influenced by short-term decisions either operationally because of e.g. empty daily operated reservoirs or because of the hourly energy market. It is hardly possible to consider a hourly time resolution for a medium-term optimization for a yearly time horizon, both computationally as well as because of modeling issues. So aggregations and/or simplifications have to be made where it is difficult not to under- or overvalue short-term flexibility. One important aspect here is how and when information about uncertain variables is disclosed in the model.

A. Proposed model: Stochastic intrastage subproblems: A multihorizon stochastic programming approach

The proposed modeling approach is based on two obvious observations: First, that the water management in seasonal operated reservoirs can be considered in a longer time scale than in daily operated reservoirs and secondly, that the filling of the latter one is less important for a revenue estimation. The idea now is, that only for seasonal operated reservoirs water values are calculated. The optimization is done for weekly time stages where for each time stage water values are calculated. This multi-stage stochastic program is decomposed into *interstage* and *intrastage* subproblems (similar ideas were

TABLE I
VARIABLES

Variables	Explanation
$t \in \mathbb{T} = \{1, \dots, T\}$	time stages, $T = 52$ [week]
$x(t)$	state and decision variables
$\xi_t \in \Xi_t$	realized data, possibly random
$\theta_t(x_{t-1})$	profit-to-go (function) [€]
c_t	objective function coefficients
A_t, b_t	equality constraints (water and financial balances)
D_t, d_t	inequality constraints (frequency control reserves)
lb_t, ub_t	lower / upper bounds
$v(t), s(t)$	filling and spill of reservoirs [m^3]
$a(t)$	charges from upstream reservoirs/inflows [m^3]
$u(t), p(t)$	generating / pumping [MW]
$m(t)$	position on energy market [MW]
$f_u(u), f_p(p)$	used/produced energy to water flow (function) [m^3/MW]
$q(t)$	binary: provision of frequency control reserves [0/1]
q^{min}	minimum generation [MW]
q^{max}	maximum frequency control reserves capacity [MW]
c_t^q	remuneration for frequency control reserves [€/MW]
$c_t^{peak}, c_t^{off-peak}$	aggregated peak and off-peak prices [€/MW]
W_t	discretized sum of weekly water discharges [m^3]
pdc	price duration curve (function) [€/MWh/h]
h^u, h^p	amount of generating/pumping hours per week [h]
$\tau \in \{1, \dots, 168h\}$	hourly intrastage time steps [h]
$c_t^{pool}(\tau)$	hourly day-ahead prices [€/MWh]
Q_{ξ_t}	optimal value of intrastage problem [€]
$v^{seas}(t)$	weekly filling of seasonal reservoirs [m^3]
$v^{daily}(\tau)$	hourly filling of daily reservoirs [m^3]
$s \in \mathcal{S}$	scenarios in a scenario tree
$\mathcal{A}_\tau \subseteq \mathcal{S}$	bundle, same past intrastage decisions
Λ_τ	set of all bundles \mathcal{A}_τ in stage τ
$U(\mathcal{A}_\tau)$	set of bundles for “children” of \mathcal{A}_τ

applied in [1]–[3]). The weekly interstage problem, which is the master problem, handles the water management of the seasonal reservoirs. The intrastage subproblems on the other hand take care of hourly water balances in the daily operated reservoirs as well as day-ahead bidding.

In contrast to previous works the intrastage problems are not modeled deterministically but stochastically. From the modeling point of view this makes sense, since in a weekly perspective hourly water inflows and market prices are not known beforehand. This approach is called, as proposed in

another context in [4], *multihorizon* stochastic programming. Additionally the provision of spinning reserves is modeled as a here-and-now decision within the intrastage problem, similar as presented in our previous work [2].

Summarizing, the proposed model can be described as follows:

- 1) interstage problem (master problem):
 - weekly water values depending on the filling of the seasonal reservoirs
 - decisions about water release from seasonal reservoirs without information about water inflows and day-ahead prices
- 2) intrastage problem:
 - hourly time steps
 - decisions about provision of spinning reserves, day-ahead bidding and production operation
 - stochastic water inflows are revealed weekly
 - stochastic market prices are revealed daily

Stochastic dynamic programming is applied for solving the master problem: the problem is decomposed in time and seasonal reservoirs as well as the amount of water discharge from these reservoirs are discretized. The reasons for choosing this solution method are threefold. First, the consideration of a market for spinning reserves leads to non-concave profit-to-go functions in the master problem (see also Fig. 1). Therefore it is difficult to apply an iteration algorithm like stochastic dual dynamic programming. Secondly, only a few basins have to be considered in the master problem which eases one of the curses of dimensionality in stochastic dynamic programming [5]. Finally, since the goal of this work is to evaluate different modeling methods, the actual solution method is primarily not of interest.

The multi-stage stochastic program in the intrastage problem is difficult to decompose. This is because the given weekly discharge makes the hourly stages in the intrastage problem depending on each other. The problem is therefore formulated as the deterministic equivalent and a solver based on the simplex method is used to solve it.

B. Evaluation of the proposed method against alternative approaches

The second goal of this paper is the comparison and evaluation of several methods of aggregations and simplifications for being able to consider short-term flexibility in a medium-term hydro optimization. Apart from the method with stochastic intrastage subproblems, the approaches are:

- 1) neglecting hourly short-term flexibility with peak and off-peak prices (usual approach)
- 2) price duration curves (e.g. [1], [6])
- 3) deterministic intrastage subproblems (e.g. [2], [3])

The methods are formulated in a similar way as before to allow the application of stochastic dynamic programming. An operation simulation evaluates the methods by applying the suggested policies on a number of trial years. The outcome of the evaluation depends on the market structure. Therefore results with- and without the consideration of the revenue out of spinning reserves provision will be shown for a typical hydro power plant in Switzerland.

C. References and Contributions

Notable references for stochastic programming in the energy sector and stochastic dynamic programming in particular are [7]–[11]. The idea of inter- and intrastage problems for hydro power planning were explicitly introduced in [1] and applied in [2], [3], [12], focusing on bidding or operational feasibility respectively.

The usual alternative modeling approaches are aggregation and/or bundling of market products (e.g. peak and off-peak products), as in [13], and to model different lengths of time stages, as in [12], [14]. Another possibility is shown in [15], where Lagrangian relaxation was used to incorporate long-term guidelines into short-term and vice versa.

Instead of applying approximations to the model another idea is to look for approximate solutions to a detailed model. In [16] different techniques are reviewed on how to improve the convergence speed of stochastic dual dynamic programming algorithms and in [17], [18] water head effects were convexified in order to use such methods. This would be also applicable for this context. Especially for hydro plants with many cascaded basins such a solution approach would be better suited (see also [19]).

The contributions of this paper are threefold: First, to the best of our knowledge the application of stochastic intrastage subproblems is presented for the first time. Secondly, several approaches for how to account for short-term flexibility in a medium-term hydro optimization are evaluated and compared. Finally the approaches are extended by considering revenue out of the provision of spinning reserves.

The remainder of the paper is organized as follows: In the next section the model is explained both conceptually and mathematically for all four methods. Afterwards in section III the methods are evaluated and applied in an operation simulation. Some remarks conclude the paper.

II. MODEL

The overall problem is about acquiring a reasonable operating policy for a hydro plant, i.e. the *water values* as production opportunity costs. The focus of this paper is how short-term flexibility could be modeled, where we describe in this section four different methods. First model assumptions and limitations are discussed. Then the basic mathematical model is introduced which is then extended for each of the four methods.

A. Model assumptions and limitations

It is assumed, that the power plant is built up out of two different kinds of reservoirs: the ones operated in seasonal cycles and the others in daily cycles. Further a valid assumption for power plants in the Alps is, that the yearly amount of water inflows remains stable. Therefore a time horizon of one year is considered.

As spinning reserves market the one for the provision of secondary frequency control reserves is considered, since economically it is the most interesting one in Switzerland. The current market rules require the bidding of symmetric power bands. If the bid is accepted, the power band has to be provided

for the tendered period of one week. The actual demand is requested automatically. It is assumed, that this request is symmetric within the tender period so that the energy delivery is balanced out. Considered profit out of this market is the remuneration for holding the capacity whereas payments for energy delivery is neglected.

The turbines have to be continuously running at a certain set point when they provide control reserves (see Fig. 1). To prevent, that the turbines are operated inefficiently, a minimum generation amount is introduced. The provision of secondary control reserves therefore reduces the production flexibility considerably.

As stochastic variables both water inflows and market prices are considered. The time duration for the main steps is one week, since at the moment the considered spinning reserves are procured weekly in Switzerland. However weekly profit-to-go functions make also sense for a medium-term optimization of the chosen power plant.

B. Mathematical model

The hydro scheduling problem naturally leads to a multi-stage stochastic program which can be formulated in a dynamic way. Let θ_t be the expected future profit, the *profit-to-go*, a function of the fillings of the seasonal reservoirs. Then it can be stated:

$$\theta_t(x_{t-1}) = \max_{c_t^T} c_t^T x_t + \mathbb{E}_{\xi_t \in \Xi_t} [\theta_{t+1}] \quad (1)$$

subject to:

$$\begin{aligned} B_t \cdot x_{t-1} + A_t \cdot x_t &= b_t \\ D_t \cdot x_t &\leq d_t \\ lb_t \leq x_t &\leq ub_t \\ x_t &\in \mathbb{R}^n, \{0, 1\} \end{aligned}$$

x_t specifies the state and decision variables at time stage t , i.e. the fillings of the reservoirs as well as production and bidding decisions. $\xi_t := (c_t, A_t, B_t, b_t, D_t, d_t)$ defines the data vector where c_t, b_t are random and not known in advanced, the market prices and water inflows respectively.

$\mathbb{E}[\dots]$ denotes the expected value over sampled random data $\xi_t \in \Xi_t$. It is maximized in order to find the profit-to-go function. Note, that the stochastic data process ξ_1, \dots, ξ_T is Markovian, so the profit-to-go function θ_t depends only on ξ_t and not on the whole past process ξ_1, \dots, ξ_t .

The stochastic program is subject to equality constraints, defined by B_t, A_t, b_t . In more detail, these constraints ensure correct water and financial balance. The water balance is modeled as follows:

$$v_t = v_{t-1} - s_t - f_u(u_t) + f_p(p_t) + a_t - q_t \cdot f_u(q^{min} + q^{max}) \quad (2)$$

To keep notations simple $a(t)$ denotes both water inflows as well as charge from upstream reservoirs. The functions f_u/f_p convert generation or pumping power to the respective water flow.

The market position $m(t)$ is maximized in the objective function multiplied with the market prices. It is defined as

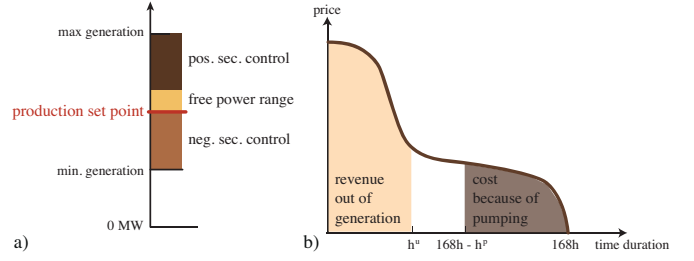


Fig. 1. a) Generation of a turbine with provision of secondary frequency control reserves. Production flexibility is reduced considerably and the substantial minimum of generation results to non-concave profit-to-go functions. b) Schematic example of a price duration curve. Revenue out of generation as well as costs because of pumping are shown. Note that since the overall water discharge is fixed, with more pumping more generation is possible.

another equality constraint:

$$m_t = u_t - p_t + q_t \cdot (q^{min} + q^{max}) \quad (3)$$

The provision of secondary control reserves influences the operation of the turbines. This can be modeled as inequality constraints (D_t, d_t):

$$q_t \cdot (q^{min} + q^{max}) \leq u_t \leq \bar{u} - q_t \cdot q^{max} \quad (4)$$

Note that the provision of spinning reserves is approximated by taking into account either no provision or the maximum quantity for each turbine of the power plant. The remuneration for holding a capacity c_t^q is assumed to be known beforehand. It is estimated as the minimum one can expect to get accepted. The remuneration is also part of the objective function. Finally the lower and upper bounds are:

$$\begin{aligned} 0 \leq v_t \leq \bar{v}, v_t \in \mathbb{R}^v &, \quad 0 \leq s_t \leq \bar{s}, s_t \in \mathbb{R}^v \\ 0 \leq u_t \leq \bar{u}, u_t \in \mathbb{R}^u &, \quad 0 \leq p_t \leq \bar{p}, p_t \in \mathbb{R}^p \\ 0 \leq q_t \leq 1, q_t \in \mathbb{Z}^u &, \quad \underline{m} \leq m_t \leq \bar{m}, m_t \in \mathbb{R} \end{aligned} \quad (5)$$

The position m_t is bounded to some upper and lower values to prevent extreme positions leading to unrealistic high risk exposure.

In the following paragraphs the basic model (1) is adapted for the four mentioned methods.

C. Method 1: Neglecting hourly flexibility with weekly peak and off-peak prices

The first method neglects hourly flexibility. Water inflows and market prices are estimated as expected values over the respective week. Two different prices are assumed. Energy is generated for peak prices c_t^{peak} and pumping for off-peak prices $c_t^{off-peak}$. The daily operated basins are disregarded and turbines and pumps are aggregated accordingly (see also Fig. 3 b)). This result to problem (1), where the state vector x_t consists of one entry for each of the variables v_t, s_t, a_t per aggregated reservoir and one entry for u_t, p_t, q_t for each turbine and pump respectively:

$$\begin{aligned} \theta_t(x_{t-1}) = \max_{W_t} \mathbb{E}_{\xi_t \in \Xi_t} [&c_t^{peak} u_t - c_t^{off-peak} p_t \dots \\ &+ q_t \cdot q^{max} \cdot c_t^q \cdot 168h + \theta_{t+1}(x_t)] \end{aligned} \quad (6)$$

subject to:

constraint (4) and bounds (5) as well as:

$$v_t = v_{t-1} - W_t \quad (7)$$

$$f_u(u_t) - f_p(p_t) + s_t - a_t = W_t \quad (8)$$

Note, that by discretizing the weekly water discharge from the reservoirs W_t it is possible to apply the stochastic dynamic programming scheme. Note also, that random data consists of peak and off-peak prices as well as water inflows a_t .

The advantage with this formulation is the moderate computational burden although stochasticity is considered. For every time stage t and W_t there is only one single constraint of (4), (7) and (8) (for each scenario).

D. Method 2: Price duration curves

A price duration curve (example in Fig. 1 b)) is constructed out of the proportion of hourly prices below a certain price for some time duration. Since the revenue depends on *price x quantity* of sold energy, it can be estimated by integration of the price duration curve.

In [1] such curves are multiplied by quantity-price offers and integrated in respect to prices. Here another approach is followed, where the sum of the water discharge for the next week is discretized. Then for a given water discharge an optimization problem is formulated with the objective to find the time durations of pumping h_t^p and generating h_t^u . The expected short-term profit can then be derived.

It is assumed, that the power plant either generates or pumps fully or not for each hour. Random data involves again prices and water inflows. To estimate random price duration curves the hourly price process itself is sampled and the price duration curve is constructed out of it. The problem can now be formulated as follows:

$$\begin{aligned} \theta_t(x_{t-1}) = \max_{W_t} \mathbb{E}_{\xi_t \in \Xi_t} & \left[u_t \cdot \int_0^{h_t^u} \text{pdc}_t(\tau) d\tau \dots \quad (9) \right. \\ & - \bar{p} \cdot \int_{168h-h_t^p}^{168h} \text{pdc}_t(\tau) d\tau \dots \\ & + q_t \cdot \left((q^{max} + q^{min}) \int_0^{168h} \text{pdc}_t(\tau) d\tau \dots \right. \\ & \left. \left. + q^{max} c_t^q \cdot 168h \right) + \theta_{t+1}(x_t) \right] \end{aligned}$$

subject to:

bounds (5) as well as:

$$v_t = v_{t-1} - W_t$$

$$u_t = \bar{u} - q_t \cdot (q^{max} + q^{min})$$

$$\begin{aligned} h_t^u \sum f_u(\max(u_t)) - h_t^p \sum f_p(\bar{p}) \dots \\ + q_t f_u(q^{max} + q^{min}) \cdot 168h + s_t - a_t = W_t \end{aligned}$$

The problem turns out to be challenging to solve. Therefore the price duration curves are assumed to be piece-wise linear which approximates the problem to a quadratic mixed-integer problem.

From the modeling point of view there are several approximations with this formulation. The most severe is that, similar to the first method, timing is not respected at all. This means it is not considered when and in which order the decisions are taken within a week.¹

The advantage with this formulation is the consideration of a reasonable representation of the opportunities in the hourly day-ahead market.

E. Method 3: Deterministic intrastage subproblems

The idea of intrastage subproblems is already explained in the introduction. For the third method these subproblems are modeled deterministically.

Mathematically the multi-stage stochastic program with intrastage subproblems can be formulated in a similar way to (1):

$$\theta_t(v_{t-1}^{seas}) = \max_{W_t} \mathbb{E}_{\xi_t \in \Xi_t} [Q_{\xi_t, W_t} + \theta_{t+1}(v_t^{seas})] \quad (10)$$

where:

$$\begin{aligned} Q_{\xi_t, W_t}(v_{t-1}^{seas}) = \max_{u_\tau, p_\tau, s_\tau, q_t} & (c_t^{pool})^T m_t(\tau) \dots \quad (11) \\ & + c_t^q \cdot q_t^T q^{max} \cdot 168h \end{aligned}$$

subject to:

1. water balances:

$$v_t^{seas} = v_{t-1}^{seas} - W_t$$

$$v_\tau^{daily} = v_{\tau-1}^{daily} - s_\tau - f_u(u_\tau) + f_p(p_\tau) + a_\tau, \quad \forall \tau$$

$$\sum_\tau [f_u(u_\tau) - f_p(p_\tau) - a_\tau] + s_t = W_t$$

2. financial balance:

$$m_\tau = u_\tau - p_\tau, \quad \forall \tau$$

3. secondary control provision:

$$q_t \cdot (q^{min} + q^{max}) \leq u_\tau \leq \bar{u} - q_t \cdot q^{max}$$

4. bounds similar to (5):

$$lb_t \leq u_\tau, p_\tau, s_\tau, q_t, v_t^{seas}, v_\tau^{daily}, m_\tau \leq ub_t$$

$$u(\tau), p(\tau), s(\tau), v^{daily}(\tau) \in \mathbb{R}^{n \times \tau}, \quad m(\tau) \in \mathbb{R}^\tau$$

$$v_t^{seas} \in \mathbb{R}^n, \quad q_t \in \{0, 1\}^n$$

Q_{ξ_t, W_t} is the optimal value of the deterministic intrastage subproblem. It is a function of the former state, realized random data ξ_t as well as the discretized water release W_t .

The purpose of the problem Q is to estimate the intrastage profit in a realistic way, by hourly deploying W_t most optimally within the week. It is formulated as a two-stage stochastic program. In the first stage the amount of secondary control reserves to bid is decided. This is done for each turbine, which is qualified for this provision. Afterwards ξ_t is disclosed, so the water inflows and prices for the whole week become known. Then actual hourly production decisions take place. As a consequence Q_{ξ_t, W_t} is a deterministic, linear maximization problem with binary variables.

¹It may happen, that e.g. high market prices occur all at the beginning of a week where the reservoirs may be empty and generation not possible. Such cases are not taken care of with the consideration of price duration curves.

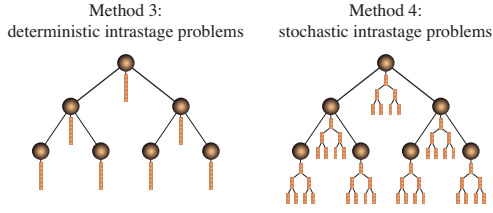


Fig. 2. Decision trees with deterministic and stochastic intrastage subproblems. Whereas random data in the third method is revealed once for each intrastage problem in the fourth method it is revealed daily. Note that interstage decisions do not depend directly on intrastage decisions of previous stages.

Note, that the operation of the power plant is considered in hourly resolution as opposed to the first and second method. Approximations made are first, that the random data are assumed to be known one week in advance. Further the fillings of the daily reservoirs v^{daily} are neglected in the calculation of the profit-to-go functions as well as their water balances are not respected between consecutive weeks. This results to empty fillings of the daily reservoirs at the beginning and end of each week.

F. Method 4: Stochastic intrastage problems

The model from the previous method 3 is now extended by considering stochastic instead of deterministic intrastage subproblems. This is one of the novel contributions of this paper. The idea is depicted in Fig. 2. Whereas in the third method the random data is disclosed at the beginning of a week, the day ahead prices are now revealed daily for one day. So the market prices are known only one day in advance. The water inflows are still revealed weekly out of two reasons: to give a hint about the modeling flexibility and to keep computational burden low.

Note, that the stochastic intrastage problems cannot be formulated in a dynamic way since the sum of the weekly discharge W_t has to be fulfilled for each scenario. It is therefore formulated as the deterministic equivalent.

The mathematical formulation changes in respect to how random data is disclosed if compared to (10) and (11). To keep notation simple the subproblems are described with the help of scenario trees. A scenario is one possible realization path of the random data. Let the set of all scenarios s be \mathcal{S} and consider a *bundle* $\mathcal{A}_\tau \subseteq \mathcal{S}$ a subset of \mathcal{S} with the same intrastage decisions up to some stage τ . Finally let Λ_τ be the set of all bundles in a stage τ and therefore $\mathcal{A}_\tau \in \Lambda_\tau$. Further let the set of bundles $U(\mathcal{A}_\tau)$ be:²

$$U(\mathcal{A}_\tau) = \{\mathcal{B} \in \Lambda_{\tau+1} \mid \mathcal{B} \subseteq \mathcal{A}_\tau\}$$

Now the mathematical formulation of the stochastic intrastage subproblems resembles the deterministic ones. However instead of having one variable per stage τ there is one for each

²As an example consider Fig. 2: For each stochastic intrastage subproblem the cardinality of \mathcal{S} is 4, which means there are 4 different scenarios. Λ_4 is consisted of two bundles: $\Lambda_4 = \Lambda_5 = \Lambda_6 = \{\{1, 2\}, \{3, 4\}\}$. Consider now the bundle $\mathcal{A}_6 = \{1, 2\}$. $U(\mathcal{A}_6)$ then specifies the children of \mathcal{A}_6 , the set of bundles $U(\mathcal{A}_6) = \{\{1\}, \{2\}\}$.

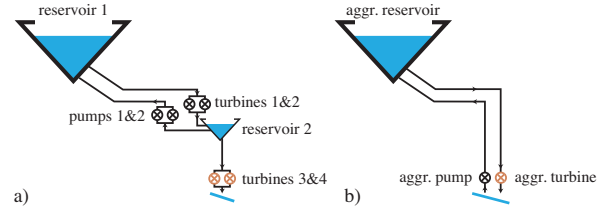


Fig. 3. a) Schematic overview of the considered hydro power plant. Reservoir 1 is the seasonal storage whereas reservoir 2 is the daily one. Turbines 3 and 4 are qualified to deliver secondary frequency control reserves. b) Aggregated hydro power plant for the first and second method. Note, that also water inflows are aggregated and that an infinite lower basin for pumping is assumed. The aggregated turbine is able to provide frequency control reserves.

bundle \mathcal{A}_τ . The problem can be written as follows:

$$\theta_t(v_{t-1}^{seas}) = \max_{W_t} \mathbb{E}_{\xi_\tau \in \Xi_\tau} [Q_{W_t} + \theta_{t+1}(v_t^{seas})] \quad (12)$$

where:

$$Q_{W_t}(v_{t-1}^{seas}) = \max_{u_\tau, p_\tau, s_\tau, q_t} (c_t^{pool})^T \mathbb{E}_{\mathcal{A}_\tau \in \Lambda_\tau} [m_{\mathcal{A}_\tau}] \dots \quad (13)$$

$$+ c_t^q \cdot q_t^T q^{max} \cdot 168h$$

subject to:

1. water balances:

$$v_t^{seas} = v_{t-1}^{seas} - W_t$$

$$v_{\mathcal{B}_\tau}^{daily} = v_{\mathcal{A}_{\tau-1}}^{daily} - s_{\mathcal{B}_\tau} - f_u(u_{\mathcal{B}_\tau}) + f_p(p_{\mathcal{B}_\tau}) + a_{\mathcal{B}_\tau}, \dots$$

$$\forall \mathcal{A}_{\tau-1} \in \Lambda_{\tau-1}, \forall \mathcal{B}_\tau \in U(\mathcal{A}_{\tau-1}), \forall \tau$$

$$\sum_{\tau} [f_u(u_\tau) - f_p(p_\tau) - a_\tau] + s_t = W_t$$

2. financial balance:

$$m_{\mathcal{A}_\tau} = u_{\mathcal{A}_\tau} - p_{\mathcal{A}_\tau}, \forall \mathcal{A}_\tau \in \Lambda_\tau, \forall \tau$$

3. secondary control provision:

$$q_t \cdot (q^{min} + q^{max}) \leq u_{\tau, \mathcal{A}_\tau} \leq \bar{u} - q_t \cdot q^{max}, \dots$$

$$\forall \mathcal{A}_\tau \in \Lambda_\tau, \forall \tau$$

4. bounds similar to (5):

$$lb_t \leq u_{\mathcal{A}_\tau}, p_{\mathcal{A}_\tau}, s_{\mathcal{A}_\tau}, q_t, v_t^{seas}, v_{\mathcal{A}_\tau}^{daily}, m_{\mathcal{A}_\tau} \leq ub_t$$

$$u(\mathcal{A}_\tau), p(\mathcal{A}_\tau), s(\mathcal{A}_\tau), v^{daily}(\mathcal{A}_\tau) \in \mathbb{R}^{n \times \sum_{\tau} |\Lambda_\tau|}$$

$$m(\mathcal{A}_\tau) \in \mathbb{R}^{\sum_{\tau} |\Lambda_\tau|}, v_t^{seas} \in \mathbb{R}^n, q_t \in \{0, 1\}^n$$

The sizes of the variable vectors in the intrastage problem depend on the sum of the number of bundles for each time step $\sum_{\tau} |\Lambda_\tau|$. For example consider two day-ahead price scenarios per day. Each intrastage vector (for each reservoir etc.) then has $24 \cdot (2^1 + 2^2 + \dots + 2^7) = 6096$ entries. Note, that the deterministic formulation would require only $24 \cdot 7 = 168$ entries.

Compared with the method 3 with deterministic intrastage subproblems the method 4 is much more realistic from the modeling point of view. Computationally the same amount of subproblems have to be solved, however the size of these

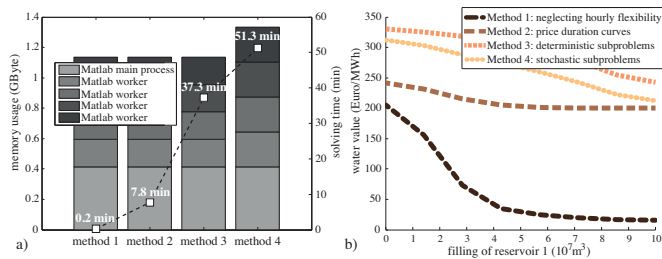


Fig. 4. a) Time duration needed for solving the different optimization methods and memory requirement of the optimizations. b) Water values (gradients of the profit-to-go functions) for the different methods for the first time stage. They depend on reservoir filling and weekly time stages. Note that the first two methods undervalue relatively the short-term revenue whereas the third one overvalues it.

subproblems differ.³

III. EVALUATION

There are no standardized hydro power plant models available for optimization studies as it is the case e.g. for electricity grid analysis. The outcome of the evaluation is however depending on the considered power plant. It seems obvious, that the more complicated structure and the more hourly dynamics are present in the model, the better the proposed method with stochastic intrastage subproblems should perform. Chosen was therefore a typical Swiss hydro power plant (Fig. 3). This plant is not overly complicated but still consists of two different kinds of reservoirs, pumps and turbines. It is also qualified to provide secondary frequency control reserves. Another option would have been to consider several different kinds of power plants, but this was beyond the scope of this paper.

The advantages and disadvantages from the modeling point of view are explained in the previous section. Compared is now the computational burden of the methods, their proposed water values as well as a simulation study where the found water values are applied for several samples of water inflows and market prices.

A. Computational burden

The optimizations were done on a commercial computer with 4 physical processor cores. CPLEX's dual simplex solvers were used for the linear and quadratic programs. For the mixed-integer problem a branch-and-cut algorithm was used. In Fig. 4 the time durations as well as the memory requirements needed for the different methods are depicted. The first method finishes in 15 seconds, which is 30 to 200 times faster than the other methods. This would be a clear advantage in daily use. Method four has higher memory requirements than the other ones. Memory usage of this method (as well as solving time) will further increase exponentially if the amount of intra time stages, stochasticity or power plant complexity (number of state variables) is increased.

³For example for two daily market price scenarios (which results to $2^7=128$ weekly scenarios) one stochastically formulated intrastage subproblem was constructed and solved in 0.41 s whereas the deterministic variant required only 0.12 s.

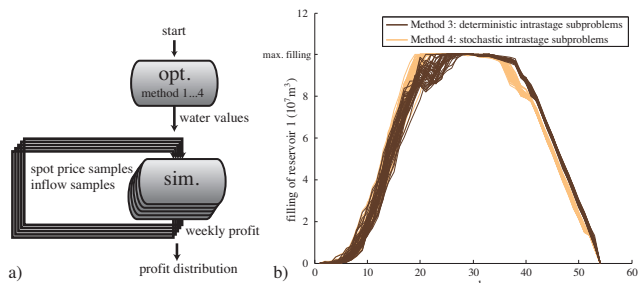


Fig. 5. a) Flowchart of the Monte Carlo operation simulation. b) Filling of seasonal reservoir 1 for each of the samples for water values from method 3 and 4. Note that although the water values are similar the actual operation varies more.

B. Water values comparison

The results of the optimization methods, the water values, are shown and compared in Fig. 4 b) for the first time stage. Notable is, that the more short-term dynamics are considered the higher the water value is. However because the third method assumes perfect weekly knowledge it overvalues the water value. Therefore method 4 should give a more realistic estimation. The water values for methods 3 and 4 are similar not only for the first time stage but also for the others with roughly 80% of them having a difference of less than 10%.

C. Simulation study

A Monte Carlo operation simulation study (Fig. 5 a)) is performed in order to estimate the performance of the different methods. In the simulation part, the power plant operation is mimicked over one year based on the estimated weekly water values. The procedure for each week in the simulation horizon is as follows:

- 1) sampling of correlated water inflows and market prices
- 2) offering of secondary frequency control reserves
- 3) hourly production decision heuristic

Because of lack of sufficient amount of historical data, distributions are estimated out of the available data and water inflows and market prices are sampled out of it. Then it is decided, if secondary frequency control reserves are offered for the next week or not. This is modeled as a mixed-integer problem. After that a heuristic performs hourly production decisions, which simulates what an operator would do in practice: First frequency control reserves obligations are fulfilled and then energy is generated or used for pumping depending on a comparison of the filling depended water values and market prices. This procedure is repeated for every week and for 100 samples in a receding horizon. Outcome of the simulation is a profit distribution.

Fig. 5 b) shows for method 3 and 4 the resulting seasonal reservoir filling for all samples. The reservoirs maximum filling is exploited with both methods (without spilling). One could argue, that the application of water values from method 4 leads to a more conservative strategy i.e. releasing water earlier.

Table II shows the expected profit, the relative standard deviation as well as the mean profit for the 10% worst

TABLE II
COMPARISON OF OPTIMIZATION METHODS WITHOUT / WITH PROVISION OF SECONDARY FREQUENCY CONTROL RESERVES

	Method 1 weekly peak & off-peak prices	Method 2 price duration curves	Method 3 deterministic intrastage	Method 4 stochastic intrastage
expected profit [M€]	34.24 / 34.99	29.13 / 30.44	34.80 / 35.85	33.40 / 39.14
rel. standard deviation	2.19% / 2.53%	1.95% / 2.47%	2.36% / 3.99%	2.57% / 4.10%
CV@R _{10%} [M€]	32.66 / 33.27	27.90 / 29.03	33.27 / 33.47	31.82 / 36.28

scenarios (CV@R_{10%}). The values are shown with and without consideration of provision of control reserves.

Method 1 leads to astonishingly good results. However the performance evaluation was based on market data, where peak and off-peak price periods were clearly present which will or already is not anymore the case.

Method 2 performs worse than expected. An explanation for it could be, that although the price process is considered in a detailed way the power plant itself is simplified considerably. This leads to using non-existing resources more efficiently which may result to less effective policies.

Method 3 outperforms method 4 for optimizations without the consideration of secondary frequency control provision. Interesting is also the increased robustness if compared with method 1: the CV@R_{10%} is considerably higher whereas the relative standard deviation, as an alternative risk measure, would indicate slightly more risk.

Finally the proposed method 4 outperforms the other methods only if secondary control reserves provision is considered. But in this case the increase of both expected profit and CV@R_{10%} is substantially by around 10 %.

IV. CONCLUSIONS

This paper presented four aggregation methods for a medium-term self-scheduling of hydro power plants. The methods were: (1) aggregated peak and off-peak prices, (2) price duration curves, (3) deterministic intrastage subproblems and (4) stochastic intrastage subproblems. Contributions were first the application of stochastic intrastage subproblems to the hydro power planning, second, the comparison and evaluation of the different methods and finally the extension of the methods by considering revenue out of provision of spinning reserves.

The evaluation presented the computational burdens as well as the quality of the proposed optimization outcomes, where a Monte Carlo operation simulation study was performed. The results suggest, that the consideration of stochastic intrastage subproblems makes only sense if the market structure is sufficiently complex. The results also indicate that another reason could be more complex hydro plant structures, whereas the evaluation on such plants is left for future work.

REFERENCES

[1] G. Pritchard, A. Philpott, and P. Neame, "Hydroelectric reservoir optimization in a pool market," *Mathematical Programming*, vol. 103, pp. 445–461, 2005.

[2] H. Abgottspon and G. Andersson, "Approach of integrating ancillary services into a medium-term hydro optimization," in *XII SEPOPE: Symposium of Specialists in Electric Operational and Expansion Planning*, 2012.

[3] N. Löhdorf, D. Wozabal, and S. Minner, "Optimizing trading decisions for hydro storage systems using approximate dual dynamic programming," *Operations Research*, vol. 61, no. 4, pp. 810–823, 2013.

[4] M. Kaut, K. Midthun, A. Werner, A. Tomasgard, L. Hellemo, and M. Fodstad, "Multi-horizon stochastic programming," *Computational Management Science*, pp. 1–15, 2013.

[5] W. B. Powell, *Approximate Dynamic Programming*. John Wiley & Sons, Inc., 2011.

[6] M. Densing, "Dispatch planning using newsvendor dual problems and occupation times: Application to hydropower," *European Journal of Operational Research*, vol. 228, no. 2, pp. 321 – 330, 2013.

[7] S. W. Wallace and S.-E. Fleten, "Stochastic programming models in energy," in *Stochastic Programming*, ser. Handbooks in Operations Research and Management Science, A. Ruszczyński and A. Shapiro, Eds. Elsevier, 2003, vol. 10, pp. 637 – 677.

[8] P. Masse, *Les Reserves et la Regulation de l'Avenir*. Paris: Hermann, 1946.

[9] R. Bellman, *Dynamic Programming*. Princeton, New Jersey: Princeton University Press, 1957.

[10] J. D. C. Little, "The use of storage water in a hydroelectric system," *Journal of the Operations Research Society of America*, vol. 3, no. 2, pp. 187–197, 1955.

[11] J. W. Labadie, "Optimal operation of multireservoir systems: State-of-the-art review," *Journal of Water Resources Planning and Management*, vol. 130, no. 2, pp. 93–111, March 2004.

[12] H. Abgottspon, M. Bucher, and G. Andersson, "Stochastic dynamic programming for unified short- and medium-term planning of hydro power considering market products," in *12th IEEE International Conference on Probabilistic Methods Applied to Power Systems (PMAPS)*, 2012.

[13] A. Conejo, R. Garcia-Bertrand, and M. Carrion, "Forward trading for an electricity producer," in *Third International Conference on Electric Utility Deregulation and Restructuring and Power Technologies*, 2008, pp. 89 –93.

[14] S.-E. Fleten, D. Haugstvedt, J. A. Steinsbo, M. M. Belsnes, and F. Fleischmann, "Bidding hydropower generation: Integrating short- and long-term scheduling," in *17th Power Systems Computation Conference*, 2011.

[15] H. Skjelbred, "Unified modeling of short- and long-term effects in hydropower systems," in *PowerTech, 2011 IEEE Trondheim*, 2011.

[16] V. L. de Matos, A. Philpott, and E. C. Finardi, "Improving the performance of stochastic dual dynamic programming," *Optimization Online*, 2012.

[17] S. Cerisola, J. M. Latorre, and A. Ramos, "Stochastic dual dynamic programming applied to nonconvex hydrothermal models," *European Journal of Operational Research*, vol. 218, no. 3, pp. 687 – 697, 2012.

[18] F. Thome, M. Pereira, S. Granville, and M. Fampa, "Non-convexities representation on hydrothermal operation planning using sddp," working paper, available: <http://www.psr-inc.com.br/portal/psr/publicacoes/>.

[19] H. Abgottspon, G. Andersson, K. Njålsson, and M. A. Bucher, "Risk-averse medium-term hydro optimization considering provision of spinning reserves," accepted for: *2014 International Conference on Probabilistic Methods Applied to Power Systems (PMAPS 2014)*, Durham, UK, 2014.