



**Masterthesis** 

# Mid-Term Hydro Power Planning for Energy and Ancillary Services



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## **Kurzfassung**

Diese Arbeit befasst sich mit der mittelfristigen Kraftwerkseinsatzplanung von Wasserkraftwerken. Die Einsatzplanung wird für ein typisches schweizerisches Wasserkraftwerk mit einem Speichersee gemacht. Als Wertschöpfungen werden sowohl die Energieproduktion als auch die Vorhaltung von Sekundärregelleistung betrachtet. Dieser Einbezug von Systemdienstleistungen führt zu einem linearen mixed-integer Problem.

Mit Hilfe eines Szenariobaumes flexibler Grösse werden stochastische, d.h. nicht vorhersehbare und veränderliche Parameter, im mittelfristigen Zeithorizont von einem Jahr in die Optimierung eingebunden. Berücksichtigt werden dabei Wasserzuflüsse und Marktpreise und ein Zeitschritt von einem Monat, so dass schlussendlich ein mehrstufiges stochastisches mixed-integer Problem mit 12 Perioden gelöst werden muss.

Das Ziel dieser Arbeit ist letztlich die Bereitstellung von optimierten monatlichen Referenzwasserständen für den Speichersee des Kraftwerkes und die optimale Menge an offerierter Sekundärregelleistung, welche als Inputparameter für eine kurzfristigere Einsatzplanung dienen können. Optimiert wird bezüglich maximalen Profit für den Kraftwerksbetreiber oder minimalem Risiko (CVaR).

**Stichworte:** Stochastische Programmierung, Kraftwerkseinsatzplanung, Speicherkraftwerke, Systemdienstleistungen, Sekundärregelung, Risikomanagement, Mittelfristig.

# **Abstract**

The topic of this thesis is the medium-term operation planning of a hydro power plant. The operation planning is performed for a typical Swiss hydro power plant modeled with one storage reservoir. Added values are energy production as well as provision of secondary control power. The consideration of ancillary services in an operation planning algorithm leads to a mixed-integer problem.

With the help of scenario trees stochastic values, which means uncertain and fluctuating values, in the medium-term horizon of one year can be considered in the optimization. These stochastic values are water inflows and spot market prizes. The time step is one month therefore finally a multistage stochastic mixed-integer problem with 12 periods has to be solved.

The final goal of this thesis is the provision of optimal monthly reference values for the reservoir of the power plant and the optimal amount of offered secondary control power, which could be used as input parameters in a more short-term operation planning. The optimization is made in respect to maximum profit or minimum risk (CVaR).

**Key words:** stochastic program, operation planning, hydro storage plant, ancillary services, secondary control, risk-management, mid-term.

## **Acknowledgement**

6 months of working on a subject may be a long time. However if such a work also should deliver something, which wasn't considered yet in the research, than the time is short and one is depending on researches, who have got some experience in the particular field.

In my case it was Dr. Marek Zima, who always supported me with his knowledge but also has provided a link to the industry. I want to thank him for his various suggestions and hints which improved this work considerably.

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# **Contents**





# **Chapter 1**

# **Introduction**

Power markets were first implemented in Chile in the early 1980s. But the more important key event for the spread of liberalized electricity markets was its introduction in the UK in 1990 by Margaret Thatcher. 3 years later, the first multinational exchange for trading electric power NordPool (Nordic Power Exchange) was founded in Scandinavia, which is now fully developed and one of the largest electrical markets in the world [38]. The EU took its first step towards liberalized energy markets 1993, as also California did.

So one can say that in the most western countries the liberalization process has been taking root.

In liberalized electricity markets the production of power plants can be sold on various markets with different time scales and properties:

- long-term bilateral contracts
- short-term bilateral contracts
- spot markets
- ancillary services markets

This new opportunities expands the possibilities as well as the complexity of operation planning for power producers. This operation planning is usually done in three different time horizons [37], where each optimization delivers bounds for the ones with smaller time scales:

- **Long-term planning** is usually considered to cover a time range longer than one year, up to several years. Long-term developments of influence factors are considered, even such as climate changes might be relevant. The result is power plants' portfolio expansion etc.
- **Medium-term planning** refers to the time period between one week to one year. A seasonal evolution of conditions is considered, such as inflow variations during the year, electricity prices etc.
- **Short-term planning** is done one week before the production till the production itself, however, primarily on the day ahead production planning.

### Chapter 1 Introduction

The topic of this thesis is the medium-term planning of a hydro power plant with storage but without pumping capabilities. With the help of scenario trees stochastic values, which means uncertain and fluctuating values, in the medium-term horizon of one year can be considered in the optimization. These stochastic values are water inflows and spot market prices.

The planning is performed for a typical Swiss hydro power plant with one storage reservoir. Added values are energy production as well as provision of secondary control power. The consideration of ancillary services in an operation planning algorithm is an important part of this work.

The final goal of this thesis is the provision of optimal monthly reference values for the reservoir of the power plant, which can be used as bounds for a more short-term planning. The optimization is made in respect to maximum profit or minimum risk for lower profit  $(CVaR<sup>1</sup>$ -considerations). This is done by stochastic programs.

After this introduction there is a chapter about relevant literature (chapter 2), where the most important works about optimization of power plants are shown and briefly discussed.

In the next chapter 3 stochastic programming is explained on a simple example where also the differences from deterministic optimizations are shown. The model is then extended to a multi-stage stochastic program in chapter 4 with a multi-level scenario tree. Some implementation details are also mentioned.

Chapter 5 further extends the model with the introduction of ancillary services which leads to a mixed-integer problem. The actual legal situation in Switzerland is also shown there. Different risk measures are introduced and compared in chapter 6. In chapter 7 the input parameter of the operation problem are specified and the structure of the optimization program is explained. Chapter 8 finally shows results of the optimization for two cases.

<sup>1</sup>*Conditional Value at Risk:* Expected outcome of a portfolio for the worst situations

# **Chapter 2**

# **Literature**

In this chapter an overview of available literature for power plant optimizations is given. After a brief discussion about the possible structure for this literature it is finally segmented into papers concerning traditional regulated markets and liberalized ones. For each paper there is given a short summary.

## **2.1 Structure of the literature**

Optimization in the power industry has been a subject of research since the late 1950s. The available literature is therefore comprehensive which also has other reasons:

The development of computational power made it possible to solve optimization problems numerically. At the beginning the models were very rough. The intention often was to find a heuristic approach which than could be used on greater structures. But with time, the models became more and more complex e.g. the optimizations inputs changed from deterministic to stochastic variables with distribution functions. This led to many papers.

The second reason for the availability of numerous papers on this subject is the advent of electricity markets in the last roughly 2 decades in most industrialized countries, as already stated in the introduction. So these countries and their power producers have great interest on the possibilities in energy markets and therefore also in research.

Most of the papers take hydro power plants into account because firstly, these plants, especially storage or pump storage plants, are easy to control. Secondly sufficient storage capacity allows great flexibility on the point of time of production, e.g. you are able to produce energy only in times with a high spot market price.

Thermal power plants however normally don't have these flexibility to act with small time constants. So the freedom of an optimization based on only thermal power plants is more restricted as one of an optimizations with flexible hydro power plants.

There are many possibilities to structure the literature on electric power optimizations. The distinction between papers with long-, medium- or short-term time horizons is a possibility. Or one consider the different optimization techniques. A structure of reservoir management problems, hydrothermal coordination problems as well as hydro power production problems is another good segmentation of the literature in problem topics.

The chosen structure here considers the market environment: firstly traditional regulated markets, which covers mostly older papers and secondly liberalized markets.

# **2.2 Traditional regulated markets or minimized cost problems**

Although most of the newer and in this work considered literature deal with electricity markets in liberalized markets, there are also some interesting literature about optimization problems in traditional regulated structures.

To understand the objective of these papers let us revise one of the simple but big rules in economy:

### *Pro fit* = *Revenue*−*Costs*

The economical purpose of a company is to increase the profit. Therefore one can try to increase the revenue or decrease the costs. For an energy producer in traditional regulated markets it was difficult to increase the revenue because they had normally regulated prices. But a good possibility is the reduction of costs by maintaining the revenue. So the aim and the objective-function of many papers was the minimization of generation costs with a stochastic or deterministic but price-inelastic demand.

For hydro power plants the marginal generation costs can often be neglected. Therefore the algorithms mostly try to maximize the hydro production by minimizing the fuel costs of the thermal power plants. Because the big energy producers often has a portfolio with both hydro and thermal power plants (so-called *hydrothermal* portfolio), these were the optimizations they were seeking for.

Another issue is reservoir management. If a hydro power plant consists of multiple reservoirs the scheduling of it is a complex problem. The challenge is that releases upstream contribute to inflows downstream possibly with a time delay. It gets even more complex when considering stochastic water inflows.

Although this thesis deals with problems in the framework of deregulated markets and maximized profits, papers concerning the traditional regulated structures often laid down the foundations of some of the concepts used later in optimizations in deregulated markets. A collection of papers concerning minimized costs are, ordered by their appearance, as follows:

*V. R. Sherkat, R. Campo, K. Moslehi* **and** *E. O. Lo* **(1985)** try to solve a hydrothermal problem by stochastic programming with successive approximations. It is a long-term optimization with multi-reservoirs. The inflows are modeled as log-normal distributed stochastic variables. They show the advantages against deterministic approaches when modeling water inflows. Because of the raw computing power they divide the problem into subproblems (for each reservoir one) and claim the results should not differ a lot from the optimum. [30]

- *S. Soares* **and** *A. A. F. M. Carneiro* **(1991)** demonstrate in this paper the influence of several factors including water head, discount rate, inflow seasonality and system design on an deterministic optimization. They result that these factors have a great effect on a long-term cost-opimization of hydrothermal power plants. They had no possibility to compute in a stochastic way so they used a deterministic approach to model the stochastic factors. [12]
- *N. Nabona* **(1993)** shows in his paper an alternative approach to solve stochastic optimization long-term problems of hydrothermal power plants. By using multi-commodity network flows the water reservoir inflows, given by probability density functions, can adequately be modeled. [39]
- *X. Guan, Peter B. Luh, Houzhong Yan* **and** *Peter Rogan* **(1994)** present in their paper an optimization-based method for scheduling hydrothermal systems. They use Lagrangian relaxation to solve a class of supproblems involving continuous dynamics and constraints (pond level), discontinuous operation regions and discrete operating states (generation and pumping).They show that their algorithm is efficient and near optimal solutions are obtained. [28]
- *Rebecca Rubisch* **(2006)** concentrates herself in this diploma thesis more on the mathematics on a two-stage stochastic optimization model. She shows problems and peculiarities of such an optimization. Afterwards she applies the optimization on a power plant commitment problem. The first non-anticipative decision variable is about unit commitment of power generation blocks and the second is the dispatch variable for a given power consumption. The objective-function aims minimized production costs. [20]

## **2.3 Liberalized markets or maximized profit problems**

As already mentioned, the newer papers write about problems in the framework of liberalized markets. There a power producer not only have the possibility to minimize costs but he can also choose an optimal market strategy to maximize the profit or to reduce risk exposure.

Because of the many possible energy products (e.g. long-term bilateral contracts, spot markets, ancillary services tenders, intraday over the counter trading etc.), optimizations can be very interesting and profitable but can also get very complex. An other problem is the presence of different energy laws in different countries.

An interesting problem, which is implied by electricity markets, is the uncertainty of market prices. Since most energy investments or operations also involve irreversible decisions, a

### Chapter 2 Literature

stochastic programming approach is meaningfull. That's why stochastic optimizations are often used in literature about liberalized markets.

To structure the literature further, a distinction between the different optimization procedures is made. There are first of all papers concerning deterministic algorithms. This is a mature research field with a comprehensive literature. But because this work is focusing more on stochastic optimizations, only two for this work interesting and recently published examples are mentioned.

Afterwards stochastic optimizations of unit commitment and power scheduling are treated. After two books, which give a good overview over the topic of stochastic optimizations, some more papers and also dissertations ordered by their appearances are shown. The focus was on presenting different authors and institutions and also different ideas.

The third category deals with optimal bidding strategies for generating companies, which papers are more financial ones. Because of the normally liquid<sup>1</sup> electricity market with different products (like futures contracts<sup>2</sup> and options<sup>3</sup>), it is also possible not only to maximize the profit but also to reduce the risk for unwanted fluctuation of uncertainties like the spot market prize. This is called risk-management by hedging<sup>4</sup>.

In the fourth category some other interesting concepts are demonstrated.

## **2.3.1 Deterministic optimizations**

- *J. M. Arroyo* **and** *A. J. Conejo* **(2000)** address in this paper the optimal response of a thermal unit to an electricity spot market by maximizing the unit profit from selling both energy and spinning reserve. They proposes a deterministic 0/1 mixed-integer linear programming approach. This allows a rigorous modeling of nonconvex and nondifferentiable operating costs with exponential start-up costs, available spinning reserve taking into account ramp rate restrictions and minimum up and down time constraints. They propose that this approach overcomes the modeling limitations of dynamic programming approaches and is computationally more efficient. Results from realistic case studies are reported. [40]
- *B. K. Pokharel, G. B. Shrestha, T. T. Lie* **and** *S.-E. Fleten* **(2004)** redefine the classical unit commitment objective of minimized costs to a profit maximization. The solution is obtained by using dynamic programming in coordination with selective enumeration. This selection is based on a heuristic on the costs characteristic of an unit. It is needed

 $<sup>1</sup>$ (of a market) having a high volume of activity</sup>

<sup>&</sup>lt;sup>2</sup> an agreement traded on an organized exchange to buy or sell assets, especially commodities or shares, at a fixed price but to be delivered and paid for later

 $3a$  contract giving the holder the right, if they choose to exercise it, to buy or sell an asset at a set strike price <sup>4</sup>protect (one's investment or an investor) against loss by making balancing or compensating contracts or transactions

to select only the most likely combinations of units and it speeds up the optimization. [29]

### **2.3.2 Stochastic optimizations**

- **Peter Kall and Stein W. Wallace (1994)** cover in this book the whole theoretical topic of stochastic optimizations. First they give the basics of stochastic optimizations like linear and nonlinear programming. Then they discuss solution techniques like the Bellman principle, decision trees, dynamic programming and also, at this time, new approaches. Their next topic is recourse problems: decomposition procedures for stochastic programs and their fitting in the framework of branch-and-cut procedures for integer programs. Monte-Carlo based methods in particular stochastic decomposition and quasi-gradient methods are also discussed. For separate probabilistic constraints with a joint normal distribution of the coefficients, they show how the problem can be transformed into a deterministic convex nonlinear program. Afterwards they discuss how to construct efficiently lower and upper bounds for a multivariate distribution function. They conclude the book with a discussion of preprocessing to simplify calculations and networks like PERT-networks. [37]
- *A. Rusczynski* **and** *A. Shairo* **(2003)** bring together leading experts in the most important sub-fields of stochastic programming to present an overview of basic models, methods and applications of stochastic programming. First they give an introduction on two- and multistage stochastic programming models and their optimality and duality. *A. Rusczynski* ends his part with a discussion on decomposition methods. Afterwards *F. V. Louveaux* and *R. Schultz* give their view on algorithms of stochastic integer programming. *A. Prékopa* concentrates in his part on probabilistic programming. *A. Shapiro* writes about Monte Carlo sampling methods. *G. Ch. Pflug* discusses properties and convergences of stochastic optimization and also statistical interference. The stability of stochastic optimizations is treated by *W. Römisch*. Afterwards applications of stochastic optimizations is given. First, *W. B. Powell* and *H. Topaloglu* show examples of stochastic optimization in transportation and logistics. Secondly, *S. W. Wallace* and *S.-E. Fleten* give a worth-reading overview of stochastic programming models in energy. [46]
- *E. Messina* **and** *G. Mitra* **(1996)** focus in this paper on the difficulties which arise in developing multi-period stochastic models. They discuss the development of a modelling and analysis environment which combines multidimensional databases, declarative modelling languages and procedural languages. Their aim is to develop a versatile tool which generates multi-period stochastic models and supports the modeller in browsing of data and solutions across different time stages and among different scenarios. Although their final software solution is after more than ten years of computer development out-of-date they show descriptively how to implement stochastic

#### Chapter 2 Literature

programming for instance the construction of constraint matrices in a computer environment. [47]

- *Markus Leuzinger* **(1998)** develops in this dissertation a linear model for the medium-term scheduling of hydro power plants including the spot market. This model is integrated into a multistage stochastic program to take the stochastic variables water inflows, prices and the load into account. He also explains how to solve this problem by dynamic programming. Dual decomposition is applied and the number of states is reduced. This procedure is showed on a typical Swiss pumped storage power station and the influences of several parameters is studied. [17]
- *Olav B. Fosso, A. Gjelsvik, A. Haugstad, B. Mo* **and** *I. Wangensteen* **(1999)** describe procedures and tools for short-, medium- and longterm scheduling of generation. Besides that they give an overview of the deregulated electricity supply system in Norway. The future spot market price is modeled by *EFIs Multiarea Power Scheduling Model* (EMPS-model). For the description of the stochastic market price they use a autoregressive (AR) model based on the average price the week before. The stochastic water inflows are treated with the same AR-model but with an additional weighting procedure which guarantees an accumulated water amount for a year for each of seven scenarios. To solve the problem they use stochastic dynamic programming. [13]
- *Thor BjGrkvoll, Stein-Erik Fleten, A. Tomasgard* **and** *S. W. Wallace* **(2001)** present a set of generation planning models and also a model for minimizing the risk of the total electricity portfolio of a price taking generator. Uncertainties involve market price, prices of fuel for thermal power plants, inflow to hydro power plants and generating unit availability. By adopting a stochastic programming approach with a scenario tree the random variables are modeled as a discrete-time stochastic process on a probability space. In the case study, the hydro power plant with uncertainty in spot market price and two inflows is implemented in AMPL and solved by CPLEX. The scenario tree consists of 256 scenarios and five stages. [11]
- *F. S. Wen* **and** *A. K. David* **(2002)** try to solve the decision-making problem on how to build optimally coordinated bidding strategies in both energy and spinning reserve markets. The prices are modeled as clearing prices. The bidding coefficient density functions are estimated by their historical values. The resulting stochastic optimization problem is solved by a *refined genetic algorithm*. [35]
- *Robert Fourer* **and** *Leo Lopes* **(2002)** propose a set of routines that manipulate instances of stochastic programming problems in order to make them more suitable for the different solution approaches of generation of explicit and implicit Deterministic Equivalents, Benders decomposition including stage aggregation and Lagrangian relaxation. Additionally they develop an environment where these routines can be accessed and in which the modeler can examine aspects of the problem structure. The goal of the research is to reduce the amount of work, time, and cost involved in experimenting with different solution methods. [33]

*Gorden Spangardt, Michael Lucht* **and** *Edmund Handschin* **(2004)** discuss in this paper

### 2.3 Liberalized markets or maximized profit problems

the theory of stochastic optimization and the implementation of risk aversion in the power industry. Furthermore, short-term unit commitment, middle-term portfolio optimization, long-term investment planning and emissions-trading are reviewed. For all this topics they give a good overview about the different problems and their solution. [19]

- *J. W. Labadie* **(2004)** assess in this state-of-the-art review optimization of multi-reservoir system management. Optimization methods designed to prevail over the high-dimensional, dynamic, nonlinear, and stochastic characteristics of reservoir systems are scrutinized, as well as extensions into multi-objective optimization. Application of heuristic programming methods using evolutionary and genetic algorithms are described, along with application of neural networks and fuzzy rule-based systems for inferring reservoir system operating rules. [44]
- *Daniel De Ladurantaye, Michel Gendreau* **and** *Jean-Yves Potvin* **(2005)** show a deterministic and a stochastic mathematical model for maximizing the profits obtained by selling electricity produced through a cascade of four reservoirs. In the stochastic model 15 different price scenarios are integrated. The inflows are in both models deterministic which is acceptable under their short-time horizon of 24 hours. They also show that the stochastic procedure is superior to the deterministic one with regard to solution quality. [15]
- *Stein-Erik Fleten* **and** *Trine Krogh Kristoffersen* **(2006)** propose within the framework of multi-stage mixed-integer linear stochastic programming a short-term production plan for a price-taking hydropower plant operating under uncertainty. The uncertainties involve water inflow and market price both modeled by the ARMA (autoregressive moving average)-framework. They solved the problem with the software *CPLEX 9.0* and with use of decomposition approaches based on Langrangian relaxation. [21]
- *Shi-Jie Deng, Y. Shen* **and** *H. Sun* **(2006)** present a stochastic programming framework for maximizing the profit of a hydro-electric power producer who simultaneously participates in multiple markets (spot market and ancillary services market) as a price taker. Operational constraints and market uncertainties are also considered. They propose that the model can also be directly adapted for determining the optimal scheduling and bidding strategy for a power producer subject to additional types of market and operational uncertainties. They show a comparison between the deterministic and the stochastic algorithm concerning the profit and the probability density function of it. [22]
- *Martin Densing* **(2007)** concentrates on this dissertation on the mathematics of an optimal operation of a hydro pumped storage plant under uncertainty (spot market prices and water inflows). The model of the plant is formulated as a multi-stage stochastic linear programming problem. Also considered is a risk-adjusted value as a multi-period constraint on risk. The scenarios are modeled in a scenario tree. He uses both a shortterm time scale of one hour for the bidding-decisions and a long-term time scale of one month for modeling the stochastic water inflows. Almost everything is mathe-

### Chapter 2 Literature

matically verified. The general problem is then sufficient reduced to be numerically solvable. In a case study he shows the influence of tree size and the other parameters. He also tries to benchmark (stability and bias) his method by a Monte Carlo sampling. [16]

- *Trine Krogh Kristoffersen* (2007) shows in this dissertation at the beginning the framework of two- and multi-stage linear and mixed-integer stochastic programs. Afterwards he applies this on optimizing bidding strategies, short-term hydro-production planning and managing power reserves. He also explains the generation of scenario trees and the use of ARMA (autoregressive moving average)-models for modeling of short-term uncertainties. [18]
- *Kristian Nolde, Markus Uhr* **and** *Manfred Morari* **(2008)** present a multistage stochastic programming formulation for monthly production planning of a hydrothermal system. Stochasticity from variations in water reservoir inflows and fluctuations in demand of electric energy are considered explicitly. The problem is solved via *Nested Benders Decomposition*. They implement the solution in a model predictive control setup and performance of this control technique is demonstrated in simulations. Tuning parameters, such as prediction horizon and shape of the stochastic programming tree are identified and their effects are analyzed. [31]

## **2.3.3 Bidding strategies**

- *Gustaf Unger* **and** *Hans-Jakob Luethi* **(2002)** show in this work, how to hedge adverse movements in a portfolio by an intelligent dispatch strategy of a hydro storage plant. The plant is modeled as interdependent options. Their optimization (by linear programming based on a maximization of the profit) gives not only the optimal contract portfolio but also the one on production. They also conclude, that the hedging value of hydro storage plant is significant and that a plain discounted cash flow analysis (DCF) will underestimate the true value of the plant. [3]
- *Heike Brand* **and** *Christoph Weber* **(2002)** introduce a procedure for a optimal short-time bidding strategy of a CHP producer. At first, scenarios are generated using Monte Carlo simulations. Then the resulting scenario tree is reduced and a stochastic optimization model is set up. Out of the solution they determine the bidding curve. The procedure is explained on a case study. [27]
- *J. Barquin, E. Ceinteno* **and** *J. Reneses* **(2005)** present a model to represent a mediumterm operation of the electrical market that introduces uncertainties (hydro conditions and demand, generating units' failures and fuel prices). Utilities are considered to be maximizing their expected profits, and biddings are represented by means of a conjectural variation. Market equilibrium conditions are introduced by means of the optimality conditions of a problem, which has a structure that strongly resembles classical optimization of hydrothermal coordination. A tree-based representation

to include stochastic variables and a model based on it are introduced. They propose, that this approach to market representation would provide three main advantages: Robust decisions could be obtained; technical constraints are included in the problem in a natural way, additionally obtaining dual information; and large-size problems representing real systems in detail could be addressed. [32]

- *G. B. Shrestha, B. K. Pokharel, T. T. Lie* **and** *S. -E. Fleten* **(2007)** propose in this work a method of building an optimal bid curve under price uncertainty using price-based unit commitment (PBUC) on a range of different scenarios (2500 random scenarios considering lognormal-distributed prices). They show that the results indicate a simple yet an effective bidding method. [25]
- *A. J. Conejo, R. Garcia-Bertrand* **and** *M. Carrion* **(2008)** describe a stochastic programming model to determine the electricity market strategy of a producer. Both a financial forward market and a day-ahead pool are considered. Hourly pool prices are modeled as stochastic variables. Decisions pertaining to the forward market are made at monthly/quarterly intervals while decisions involving the pool are made throughout the year. Risk on profit variability is modeled through the CVaR methodology. The resulting decision-making problem is formulated and characterized as a large-scale linear rogramming problem. They use scenario-reduction techniques and decomposition to solve the problem. [26]
- **Prakash Thimmapuram et al. (2008)** present details of integrating an agent-based model  $(EMCAS<sup>5</sup>)$  with a hydro-thermal coordination model (VALORAGUA<sup>6</sup>). In addition, EMCAS uses the price forecasts and weekly hydro schedules from VALORAGUA to provide intra-week hydro plant optimization for hourly supply offers. The integrated model is then applied to the Iberian electricity market (111 thermal and 38 hydro power plants). Then, they analyze the impact of supply offers on the market prices and ways to minimize the generation producers' exposure to price risk. [24]

### **2.3.4 Other concepts**

- *Martin Bachhiesl* **and** *Otto Pirker* **(2000)** propose a model for prediction of water inflows. They concentrate on the area of Austria. Out of measuring stations and weather forecasts they perform a prediction of water inflows for each hydro-power producer. They show that their model is suitable for prediction of the next 24 to 36 hours of hourly water inflow or of the daily means for the next four days. [36]
- *Stein W. Wallace* **and** *Kjetil Høyland* **(2001)** discuss in this paper the procedure to discrete continuous functions properly. The aim is to generate a scenario tree. They present

<sup>5</sup>*Electricity Market Complex Adaptive System* provides a framework for simulating deregulated markets with flexible regulatory structure along with bidding strategies for supply offers and demand bids.

<sup>6</sup>*VALORAGUA* provides longer-term operation plans by optimizing hydro and thermal power plant operation for the entire year.

### Chapter 2 Literature

a method based on nonlinear programming that can be used to generate a limited number of discrete outcomes that satisfy specified statistical properties. Users are free to specify any statistical properties they find relevant, and the method can handle inconsistencies in the specifications. They illustrate the method by a single- and a three-period problem. [41]

- *J. F. Bermúdez* **and** *M. Alvarez* **(2007)** develop in this paper a centralized economic dispatch, where ancillary services (primary reserves requirement in each unit (speed governor primary action), plus the AGC reserve, besides the (tertiary) reserves, associated to the outage of any turbine (n-1 criterion)) are included. The objective function is a minimization of the present and future thermal cost in the context of the stochastic dual dynamic programming (SDDP), simplified for academic purposes. They solve the problem by using a linear programming commercial package. Also a comparison between the traditional energy-only dispatch and the proposed model is discussed. [34]
- *J.-H. Lee* **and** *J. W. Labadie* **(2007)** propose a concept of reinforced learning (by *Q-learning*) to find the optimal operation of multi-reservoir systems. This concept makes it possible to reduce the state space in the ongoing optimization and improves thereby the execution speed. The algorithm also acquires knowledge of a underlying stochastic structure e.g. the water inflows in an actual online environment or through simulated experiences using a model. It is therefore not required to have a priori knowledge of the uncertainties. They show on a two-reservoir case study the outperforming of their method against implicit stochastic dynamic programming and sampling stochastic dynamic programming methods. [43]

# **Chapter 3**

# **Stochastic Programming**

In this chapter stochastic programming is introduced and shown on a simple but descriptive and intuitive example of a hydro power scheduling optimization. The stochastic programming procedure is opposed to a heuristic and to a deterministic approach which leads to some conclusions and to the motivation of using stochastic programming in this thesis.

## **3.1 Heuristic optimization**

To illustrate stochastic programming and to understand the differences to deterministic ones let's discuss an example of a simple hydro profit optimization:

Example 3.1 (Heuristic optimization) An energy producer possesses a hydro power plant for which he want to optimize the profit for the next period. The power plant consists of one reservoir and of a facility to produce electricity from the potential energy of the water.

The energy out of one cubic meter of water is depending on the height of the reservoir. That's why the water in the reservoir level as well as the water inflows are noted in units of producible electricity (MWh).

The produced energy can be sold for a fixed price of 10 CHF/MWh, which is known before.



Figure 3.1: Scenario tree of example 3.1



Figure 3.2: Filling of the hydro reservoir in a heuristic approach

The reservoir is initially filled with the maximal energy amount of 1000 MWh. The generators are not limited.

The water inflows are described by a stochastic variable, which distribution is known before: There exists only two possible evolutions: wet (+500 MWh inflow) and dry (+100 MWh inflow) periods, both with the same probability  $p = 0.5$  (figure 3.1). One has to decide irreversible at the beginning of the period how much energy to produce  $(X_1)$  and therefore sell at the price of 10 CHF/MWh.

It gets interesting, if the remaining amount of water at the end of the period can be sold for a much higher price of e.g. 25 CHF/MWh. Therefore one has to decide, how much water to store in order to be able to sell as much as possible at the end  $(X_2)$  should be maximized). But if one stores too much spillover occurs because of the water inflows and the maximum reservoir filling of 1000 MWh.

The most profitable case would be if the period is wet and one would know that in advance. The heuristic strategy would then be to sell in the first step only that amount of water, which will be refilled by the water inflow. So one can sell at the end the whole reservoir at the high price (figure 3.2 (a)). But if one doesn't know the water inflow in advance, the most inconvenient case could occur if you also sell 500 MWh, but it is a dry period (figure 3.2 (b)).  $\Box$ 

## **3.2 Deterministic Optimization**

A reasonable approach to solve a medium-term hydro power planning problem in respect to maximized profit is a deterministic optimization. A standard procedure to solve such a decision problem is its formulation in a *packing linear program* (LP):

$$
\begin{array}{ll}\n\max & c^T X\\ \n\text{s.t.} & AX \leq b\\ \n& A_{eq} X = b_{eq}\\ \n& X \geq 0\n\end{array} \tag{3.1}
$$

 $A, b, A_{eq}, b_{eq}, c$  in (3.1) are given matrices or vectors. The goal is to find a vector *X*, which maximizes  $c^T X$  but satisfying the inequality and equality constraints. This can be done even for LP with a large number of constraints and decisions by a numerical algorithm. For instance [40] and [29] use even for detailed models deterministic optimization approaches on which they show it can be solved in an efficient way.

For a deterministic optimization all parameters has to be known in advance. In real problem however this is hardly the case. Fluctuating parameters, like market prices, has then to be estimated and by that *determined*. Normally the expected value of such stochastic parameters is taken. The solution of such a problem is then called *wait-and-see*-solution.

Example 3.2 (Deterministic optimization) Let's consider the same profit-maximization problem as in example 3.1. But this time deterministic optimization techniques are applied to reduce the risk of a low profit by optimizing the expected profit.

The stochastic water inflows are determined by their expected value:

$$
\mathbb{E}(I) = p_{wet}I_{2, wet} + p_{dry}I_{2, dry} = 300 \text{ MWh}
$$

Afterwards one can formulate and implement the LP. Matlab $^{\circledR}$  was chosen as optimization software with its optimization package linprog. So the implemented formulation is:

$$
\max s_1 X_1 + s_2 X_2 \qquad \text{subject to (I)–(IV)} \tag{3.2}
$$

$$
X_1 \leq R_{start}
$$
\n
$$
-X_1 - spill \leq R_{max} - R_{start} - \mathbb{E}(I)
$$
\n
$$
X_1 + X_2 + spill \leq R_{start} + \mathbb{E}(I)
$$
\n(III)

$$
X_1, X_2, \text{spill} \geq 0 \tag{IV}
$$



The objective function tries to maximize the expected profit with respect to the different prizes. Constraint (I), (III) and (IV) define the bound for the amount of energy which can be sold. Constraint (II) make the linkage between the two periods and also specify together





Figure 3.3: Hydro reservoir in a deterministic optimization

with constraint (IV) the spillover. It seems, that the spill is unbounded. It is not, because of constraint (II) and (III), which together with the objective function will try to minimize the spillover.

The expected profit, for both scenarios considered, of this deterministic solution is shown in table 3.1 at page 19.

Figure 3.3 shows what would happen in a dry and wet scenario if one would apply the deterministic solution. Because the expected inflow is taken for guaranteed, it is clear that as optimal first decision one sell only that amount of water which will be refilled. But with that procedure, the reservoir won't be full in a dry period, which produces high opportunity costs. We will see, that this procedure also isn't the most optimal in respect to every possible scenario.  $\Box$ 

## **3.3 Stochastic Optimization**

To find a motivation for stochastic programming one can look at its differences to deterministic optimizations. The most important difference between stochastic programming and deterministic optimizations is the handling of *non-anticipaty*:

In deterministic optimizations all the information is assumed to be available at the beginning. In stochastic programs however the decisions don't depend on future results which means they act *non-anticipative*. Although the information has to be also assumed at the beginning, this data gets available only then for the optimizations, when it should. This makes the optimization much more realistic than multi-period deterministic problems.

With a *two-stage stochastic program* one can explain the much more complex multi-stage and multi-period stochastic programs. It can be written as follows:

$$
\min \quad sX + \mathbb{E}[\mathcal{Q}(X,\xi)]\n\text{s.t.} \quad \mathbf{A}X \le b, X \ge 0
$$
\n(3.3)

where  $\mathcal{Q}(X,\xi)$  is the optimal value of the second stage problem:

$$
\begin{array}{ll}\n\text{min} & qY \\
\text{s.t.} & \mathbf{T}X + \mathbf{W}Y = h, Y \ge 0\n\end{array} \tag{3.4}
$$

The first stage describes a decision *X*, which in general can be a vector. The second stage *Y* is like a corrective action to the system after the stochastic evolvement has occurred and is therefore depending on the random elements  $\xi$ . It can also be a vector. The second stage problem (3.4) can be seen as a penalty for a violation of the constraint  $\mathbf{T}X + \mathbf{W}Y = h$ , hence it is also called the *recourse stage*. The expectation in (3.3) is taken with respect to the known probability distribution of ξ .The matrices T and W are called the *technology* and *recourse* matrices. If the matrix **W** is not random (fixed), the above two-stage problem is called a problem with *fixed recourse*.

Example 3.3 (Stochastic optimization) Consider again the simple profit-maximization problem from examples 3.1 and 3.2. Now the scenario tree (figure 3.1) shows all possible evolutions of the random elements (water inflows). The tree also shows all possible combinations of random data over time, which specifies the scenarios. So in this case there are two possible scenarios.

The two-stage stochastic program can by written as LP. Like that, it is a very descriptive example of the advantages of stochastic programming without the need of huge constraint matrices. This so-called *Deterministic Equivalent* can be written in two ways: either implicit or explicit. Here chosen is the implicit one [33] because it is more intuitive. The LP will then look like that:

max  $s_1X_1 + p_{wet}(s_{2, wet}X_{2, wet} - q_{2, wet}Y_{2, wet}) + p_{dry}(s_{2, dry}X_{2, dry} - q_{2, dry}Y_{2, wet})$  (3.5)<br>subject to (I)-(VI)



 $X_1, X_2, \text{wet}, X_2, \text{dry}Y_2, \text{wet}, Y_2, \text{dry} \ge 0$  (VIII)

#### Chapter 3 Stochastic Programming



stochastic evolvement second decision

Figure 3.4: Thought experiment

<b>Variables:</b>	$X_1$ : Decision of sold energy at the beginning of the period		
	$X_{2, wet}$ , $X_{2, dry}$ : Decision of sold energy at the end of the period in wet or dry scenario		
	$Y_{2, wet}, Y_{2, dry}$ : Spill over in wet or dry scenario		
	$R_{2, wet}, R_{2, dry}$ : Filling of the reservoir after the stochastic evolvement		
<b>Parameters:</b>	$p_{wet}, p_{dry}$ Possibility of wet or dry period		
	$s_1, s_2,$ <sub>wet</sub> , $s_2,$ <sub>dry</sub> : Prices at the beginning and end of the period		
	$q_{2, wet}, q_{2, dry}$ Artificial prices of spillover		
	$R_{start}$ : Energy amount of the reservoir at the beginning		
	$R_{max}$ : Maximum energy amount of the reservoir		
	$I_{2, wet}, I_{2,dry}$ : Water inflow in wet or dry period		

The objective function in (3.5) makes sure that the expected outcome of both of the sold energies  $X_1$  and  $X_{2, wet}$  or  $X_{2,drv}$  will be maximized. But it also minimizes the spillover *Y*, which guaranties together with constraints (IV), (V) and (VIII) the correct spillover. Constraints (I), (VI), (VII) and (VIII) specify the bounds for the amount of energy which can be sold. Finally, constraints (II) and (III) link the two periods together.

The state variable *R*, which describes the filling of the reservoir, is not necessary. But it makes the implementation more convenient.

The non-anticipaty doesn't have to be expressed explicitly. It is already included in the tree structure and the chosen variables. This is obvious when you do a small thought experiment: if one would distinguish between first variable for wet-scenario and the one for dry-scenario for instance  $X_{1, wet}$  and  $X_{1,drv}$  (Fig. 3.4), then one would need an explicit non-anticipaty constraint like  $X_{1, wet} = X_{1, dry}$ . The linkage between two periods would then be easier but this procedure would also result in an inefficient way of resource management and a much higher number of constraints.<sup>1</sup>

The stochastic optimized reservoir management is showed in Fig. 3.5. There you only sell that amount of water, which will be refilled in a possibly dry period. That's because the

<sup>&</sup>lt;sup>1</sup>for instance for 12 periods in a binary tree: around 150'000 variables instead of 25'000 with a scenario tree



Figure 3.5: Hydro reservoir stochastic optimization

	Most profitable case:   Most catastrophic case:   Deterministic opt.:   Stochastic opt.:		
$30'000$ CHF	20'000 CHF	25'500 CHF	26'000 CHF

Table 3.1: Expected Profit

optimization takes into account the spillover in a wet period, which cannot be sold. The high price at the end of the period makes it more profitable to sell in every case the full reservoir regardless of the sale at the beginning.

Table 3.1 shows an overview of the expected profit for the discussed solution approaches. One can see the superiority of the stochastic optimization if compared with the deterministic one. This can be also verified mathematically for a general two-stage stochastic optimization as showed at page 21 in [46]. But both the deterministic and stochastic optimization get a lower profit than in the most profitable case. So if one would be lucky the policy presented for this case is the best, but there is high risk for getting a much lower profit.  $\Box$ 

## **3.4 Conclusions**

The reason of this chapter is not only to give an intuitive introduction into stochastic programming but also to show some of its benefits when compared with a deterministic optimization. These benefits give the motivation of the greater part of this master thesis and shall here be summarized again.

### Chapter 3 Stochastic Programming

The first benefit is the handling of *non-anticipaty*. This makes the optimization much more realistic but also more complicated and leads to larger problems. The second benefit is the more profitable and robust solution. For a general two-stage stochastic optimization the bigger profit can be verified mathematically. For a more complicated structure e.g. a multistage stochastic problem the bigger profit is in general not verified. There are cases, where a stochastic optimization leads not to a more profitable solution.

The more important specification of the solution however is its reduced risk awareness which was not shown in this chapter but will be presented later. But also the robustness of the solution gets important: because stochastic optimization "looks" at each possible scenario one will never experience an impossible or unintimate situation. This is not the case with a deterministic optimization, where one can be forced to deviate from the optimized strategy. Especially for problems with a lot of technical constraints like for instance the shown spillover, the possibility to implement these in a natural way is also handy.

The third benefit is that the solution provides not only the first but all optimal decisions with there probability.<sup>2</sup> Like that it is a really suitable input for risk-management computations, which will be presented later.

<sup>2</sup> if the stages are independent to each other, so-called *Markov chains*

# **Chapter 4**

# **Multistage Models**

In the previous chapter 3 an introduction into stochastic programming was given. Apart from a deterministic optimization also a two-stage stochastic program and its implementation as a LP was discussed. In this chapter this problem will be expanded to a multistage stochastic program. Afterwards it will be shown, how to automate the construction of huge constraint matrices in Matlab $^{\circledR}$  and also how to handle not only a binary but a n-level scenario tree. The mathematical derivations are motivated by [33] and [37].

## **4.1 Multistage Stochastic Programming**

It would be possible to expand the two-stage stochastic program (3.3) presented in chapter 3 to a multi-period problem with *n*−time periods. As showed in [46], the decisions in a multi-period two-stage stochastic program depend only on the initial values and the random process. In a next presented multistage stochastic program however the recourse action is applied at every time period, which is more convenient.<sup>1</sup> The decisions will then also be depending on the recourse actions. It can therefore be viewed as a relaxation of the multiperiod two-stage problem and hence has a larger optimal value.

In a multistage programming model the variables and constraints are divided into groups corresponding to stages (time periods)  $t = 1, 2, ..., T$ . Also the *non-anticipaty* has to be considered, which means one has to determine what is known at each stage *t* when decisions associated with this period are made. Let  $X_t$  be a decision vector corresponding to stage  $t$ and  $\xi_t$  the observation. This leads to the following sequence of actions:

decision *X*<sup>1</sup> . . . observation  $ξ_2$ 

<sup>1</sup>A multistage program consists of several time periods. So the term *stage* is used here for time period. This is another definition than in section 3.3 where it was used to distinguish between the decision and the recurse variable in a *two-stage stochastic program.*

```
decision X_2.
.
.
...
   .
   .
   .
observation ξT
decision XT
```
 $\xi_t$  can be denoted as the data which become known at stage *t* and hence  $\xi_{[1,t]}$  is the information available up to time *t*. So the principle of *non-anticipaty* implies that the decision vectors  $X_t$  may depend on  $\xi_{[1,t]}$  but not on the results of observations made at later stages.  $\xi_t$  is assembled from components of the constraints which define the problem and which can be random. So one call ξ1,...,ξ*<sup>T</sup>* a *random process*. This process is said to be *Markovian*, if for each  $t = 2, ..., T - 1$  the conditional distribution of  $\xi_{t+1}$  given  $\xi_{[1,t]}$  is the same as the one of  $\xi_{t+1}$  given  $\xi_t$ . If this is the case, the model is simplified considerably.

So consider the random process  $\xi_1, ..., \xi_T$  be Markovian. Then one can also conclude that the random vectors  $\xi_t, t = 2, ..., T$  are mutually independent. Also introduced here shall be the term *scenario*. Scenarios describe the finitely many different values the problem data can finally take. So with each scenario associated is a probability and a corresponding sequence of decisions. A scenario tree is useful to show the possible scenarios with a depiction of possible sequences of data  $\xi_{[1,t]}.$ 

Figure 4.1 shows a binary scenario tree for a three-stage problem with the above given definitions. A scenario is represented in the tree by a path from the root to a unique leaf of the tree. So there are four different scenarios in this example.

Let the set of all scenarios *s* be S. A subset  $A \subseteq S$  defines the set of all scenarios *s* with the same variables up to some stage *t*. This shall be called *bundles*.  $\Lambda_t$  denotes the set of all bundles at each stage *t*. These definitions are important for the handling of the nonanticipaty. For a precise algebraic formulation of the problem one needs a relation that maps a subset  $A_t$  to the one in stage  $t+1$  which are composed of the same scenario. In a scenario tree that means simply the children of a node. So for each subset  $A_t$ , let:

$$
U(\mathcal{A}) = \{ \mathcal{B} \in \Lambda_{t+1} | \mathcal{B} \subseteq \mathcal{A} \}
$$

By using scenarios, one can create large scale linear programs whose solution is the same as that of the stochastic program. These constructs are called *Deterministic Equivalents* and, as already done in section 3.3, the implicit way is used to formulate the general problem 4.1:

### 4.1 Multistage Stochastic Programming



Figure 4.1: Scenario tree for a three-stage problem

$$
\max \quad s_1 X_1 + \sum_{t \in \{2, \dots, T\}} \sum_{\mathcal{A} \in \Lambda_t} p_{\mathcal{A}} s_t X_{t, \mathcal{A}} \qquad \text{subject to (I)–(IV)} \tag{4.1}
$$

$$
\mathbf{A}_1 X_1 = b_1 \tag{I}
$$

$$
\mathbf{A}_{t,A}X_{t,A} = b_{t,A} \quad \forall t \in \{2,...,T\}, \forall A \in \Lambda_t
$$
\n
$$
\mathbf{T}_{t,B}X_{t-1,A} + \mathbf{W}_{t,B}X_{t,B} = h_{t,B} \quad \forall t \in \{2,...,T\}, \forall A \in \Lambda_{t-1}, \forall B \in U(\mathcal{A}) \quad (\text{III})
$$

$$
X_1, X_{t,A} \geq 0 \qquad \forall t \in \{2, ..., T\}, \forall A \in \Lambda_t \tag{IV}
$$



Similar as in the examples in chapter 3 the objective function tries to maximize the outcome of the decisions. In this general formulation one has to sum over all decisions in every stage *t* in every bundle  $A \in \Lambda_t$ . In each of this bundles the decisions are weighted with the unconditional probability  $p_A$  of the bundle A and a price  $s_t$  for stage *t* of the decision  $X_{t,A}$ .

### Chapter 4 Multistage Models



Figure 4.2: First three stages of the scenario tree for example 4.1

The constraints (I) and (II) define the problem and constraint (III) links subsequent bundles. The data ξ*<sup>t</sup>* which are observed for a stage *t* is depending on all parameter within this stage. Some or all of this parameters may be random.

It is interesting to see, that although the non-anticipaty is a key concept of stochastic programming and a major reason for making the computation of a solution difficult, there is no explicit mentioning of it in the problem formulation in (4.1). This is all down by structuring the problem in a scenario tree. So a scenario tree is not just an evident way to visualize the problem but by its introduction it enables also an elegant way to handle time-dependencies and the non-anticipaty.

In (4.1) no explicit distinction between a decision and a recourse variable is made. This can be done as a next step by introducing  $Y_{t,A}$  which specifies the recourse action at a stage *t* in a bundle A. To make the model more intuitive, additional variables  $R_{t,A}$  can be introduced. These *model state* variables shall summarize the relevant history of our decisions<sup>2</sup>. With this two definitions one can formulate the next example:

**Example 4.1 (Multistage stochastic programming)** Let's consider an energy producer who owns a hydro power plant with one reservoir without pumps. He wants to optimize his energy production as a price-taker with known prices but stochastic water inflows. He looks 3 periods (e.g. a quarterly period) into the future and wants the optimal amount of energy to sell in respect to maximized expected profit.

He considers two possible evolutions of water inflows: wet and dry periods. That results in a binary scenario tree with 3 stages (figure 4.2). The specifications of the problem are shown in table 4.1. The LP (4.2) shows for this problem the formulation for a general amount of stages *T*. For this example *T* would be 3.

<sup>2</sup>This introduction is similar to the one on page 18 in section 3.3.



Parameter		Value
Initial filling of the reservoir:	$R_{start}$	1000 MWh
Maximum filling of the reservoir:	$R_{max}$	1000 MWh
Inflow in wet period:	$I_{wet}$	500 MWh
Inflow in dry period:	$I_{\text{dry}}$	100 MWh
First stage price:	S <sub>1</sub>	25 CHF/MWh
Second stage price:	$S_{2}$	10 CHF/MWh
Third stage price:	$S_3$	40 CHF/MWh
Conditional probability for wet or dry period:		0.5

Table 4.1: Specification of example 4.1

$$
\max \quad s_1 X_1 + \sum_{t \in \{2, \dots, T\}} \sum_{\mathcal{A} \in \Lambda_t} p_{\mathcal{A}} \left[ s_t X_{t, \mathcal{A}} - q Y_{t, \mathcal{A}} \right] \qquad \text{subject to (I)–(VI) } \tag{4.2}
$$

$$
X_1 \leq R_{start} \tag{I}
$$

$$
R_{2,\mathcal{A}} = R_{start} - X_1 + I_{2,\mathcal{A}} \qquad \qquad \forall \mathcal{A} \in \Lambda_2 \tag{II}
$$

$$
Y_{t,A} \geq R_{t,A} - R_{max} \qquad \forall t \in \{2,...,T\}, \forall A \in \Lambda_t
$$
  
\n
$$
X_{t,A} \leq R_{t,A} - Y_{t,A} \qquad \forall t \in \{2,...,T\}, \forall A \in \Lambda_t
$$
 (III)

$$
R_{t, \mathcal{B}} = R_{t-1, \mathcal{A}} - Y_{t-1, \mathcal{A}} - X_{t-1, \mathcal{A}} + I_{t, \mathcal{B}} \quad \forall t \in \{2, ..., T\}, \forall \mathcal{A} \in \Lambda_{t-1}, \forall \mathcal{B} \in U(\mathcal{A}) \quad (\mathbf{V})
$$

$$
X_1, X_{t,A}, Y_{t,A} \ge 0 \qquad \qquad \forall t \in \{2, ..., T\}, \forall A \in \Lambda_t \tag{VI}
$$



#### Chapter 4 Multistage Models



Figure 4.3: Overview of results for example 4.1

The objective function in (4.2) is a maximization of present and future income  $s_t X_{t,A}$  and a minimization of the spillover  $Y_{t,A}$ , which together with constraints (III) and (VI) guaranties the correct spillover. So the price for the spillover  $q$  is an artificial one and doesn't have any meaning. Constraints (I) and (IV) specify the bounds for the amount of energy  $X_t$  <sub>A</sub>, which can be sold. Constraints (II) and (V) finally link subsequent stages.

The formulation is close to (4.1) except there is only one random parameter and dedicated state and recourse variables. It is also quite similar to (3.5) from the example 3.3 in the previous chapter. In fact (4.2) is a more general formulation with the possibility of arbitrary many stages *T*.

Figure 4.3 presents an overview of the results of the optimization of this example. Shown are water inflows, spilling and the decision of sold energy for all nodes in the scenario tree. Because of the high prices at the beginning and at the end, one sell no energy in the second stage. This result in an spillover in the third stage if a wet-wet scenario would occur.  $\Box$ 

# **4.2 Construction of Constraint Matrices and Computational Remarks**

To implement a problem like the example 4.1 into Matlab $^{\circledR}$  one has to find a good structure for automating the construction of the constraint matrices. In the used optimization function *linprog* the usage of both an equality and an inequality constraint matrices is provided. So constraints (I), (III) and (IV) from (4.2) can be put together to form the inequality matrix
#### 4.2 Construction of Constraint Matrices and Computational Remarks



Figure 4.4: Overview of sparse constraint matrices for 5 stages

*Aineq* and constraints (II) and (V) form the equality matrix *Aeq*. It has to be noted, that these constraint matrices only depend on the number of stages. That's why the here chosen algorithm build up the matrices from stage to stage.

In the first stage no equality constraint is needed. In the second stage one needs two equality constraints, one for each branch of the scenario tree in the second stage. In general one always needs as many equality constraints per stage as there are branches.The algorithm constructs in each step the constraints for one stage. Because the equality constraints also links subsequent stages a second algorithm within the first one is a stage behind and make sure of the correct linkage. In Fig. 4.4 (a) the special structure of this matrix is salient.

The inequality matrix *Aineq* has some different layout. The first stage needs only one constraint. For the other stages there is always twice as many constraints as there are branches. Because there is no linkage done between different stages the construction of this matrix is easier than the one for the equality matrix because one needs only variables within the same stage. The structure of *Aineq* is shown in Fig. 4.4 (b).

These two matrices have very few non-zero elements. That's why the memory management of sparse-matrices in Matlab $\mathbb{B}$ , which stores only the non-zero elements and its position in a matrix, gets essential. To optimize problems with 12 time periods and with a binary scenario tree more than 10000 constraints are necessary. The optimized storage of the inequality constraint *Aineq* needs in principle around 300 kBytes, whereas the not optimized version needs 800 MBytes. But not only the amount of used RAM-memory is reduced significantly but also the computational effort to construct these matrices and to work with it in the optimization.

Fig. 4.5 shows for different stages the time that is needed for constructing the constraint matrices and for optimize the LP of example 4.1. Both actions constructing and optimizing take approximately equal time. The figure visualize one of biggest problems in stochastic optimization: its time complexity because these problems run in *O*(*exp*).



Figure 4.5: Elapsed time for optimization with different amount of stages



Figure 4.6: 3-level scenario tree for the first three stages

### **4.3 N-level Scenario Tree**

In a scenario tree the term *level* describes the amount of branches and by that the amount of representations of a stochastic variable per stage. Until now only two possible evolutions were treated in the examples. So all scenario trees consisted of 2 levels, a so-called *binary scenario tree*. Two scenarios per time period is normally not sufficient. Although every scenario tree can be represented by a binary one, it is much more intuitive and also spares memory if one implements directly multilevel trees.

While mathematically the equations in  $(4.1)$  are valid for every possible tree, the implementation will change significantly. The effort to implement scenario trees especially with different amount of levels is big. Because also the interpretation of solutions produced by

the help of multilevel scenario trees is difficult and not much comprehensible n-level scenario trees are used in this thesis. These are scenario trees with an arbitrary but for each stage equal amount of levels. Because also of the Markovian behavior of the random process the same stochastic evolvement repeats in every stage. An example of an 3-level scenario tree for the first three stages is given in figure 4.6.

As one can imagine, the problem complexity grows exponentially with the number of scenario tree levels. So usually one tries to use the smallest possible amount of levels in respect to solution quality, to be able to optimize problems for longer periods.

Chapter 4 Multistage Models

# **Chapter 5**

# **Ancillary Services**

A key part in this thesis is the inclusion of ancillary services in a stochastic optimization. This is done in this chapter. A good presentation of the technical fundamentals of ancillary services as well as a comparison of it for different systems for the most important energy markets is given in [49]. In this work the Swiss system is implemented. At first, the legal specifications in Switzerland are described, close to [48]. Afterwards this concept is implemented in the optimization framework.

## **5.1 Legal Specifications of Ancillary Services**

Responsible for ancillary services in Switzerland is its TSO *swissgrid*. This company is responsible for the secure and reliable operation of the Swiss transmission system and for connections to the transmission systems of foreign operators. To this end swissgrid coordinates operation of the systems with neighbouring transmission system operators and supervises the Swiss control area in cooperation with existing transmission system owners as well as with operators of distribution grids and generating units connected to the transmission system.

Ancillary services in the electricity supply area are defined as services essential for the functioning of the system. Such services are delivered by grid system operators to customers in addition to the transmission and distribution of electricity and hence determine the quality of the electricity supply. Ancillary services include:

- Frequency control reserves: Primary, secondary and tertiary control
- Voltage support
- Compensation of active power losses
- Black start and island operation capability
- System coordination
- Operational measurement

swissgrid is required to purchase ancillary services in accordance with a transparent, nondiscriminatory and market-based procedure. The following overview describes this for the individual ancillary services and summarises the intended procurement processes and procedures.

### **5.1.1 Control Reserves**

Electricity or electrical energy cannot be stored in large quantities by conventional means. Therefore exactly the same amount of electricity must be produced as is consumed at any given time. This equilibrium ensures the reliable operation of the electrical grid at a constant frequency. Unforeseen fluctuations between the feeding of electricity into the grid and its withdrawal from the grid must be balanced at short notice by suppliers of what is referred to as *control reserve* increasing or reducing generating unit output at short notice.

Control reserve is required if, in the current capacity balance of a control area, the sum of the actual infeeds and withdrawals deviates from the sum of the expected capacities. This deviation can originate on the network load side for instance as a result of meteorological influences, natural inaccuracy in the load forecast and on the production side for example due to production restrictions or stoppages, additional output from hydroelectric power plants due to heavy precipitation. Each transmission system operator must therefore continually use control reserve to offset balance capacity variations in its control area.

Technically this is achieved within the synchronous electricity grid of the UCTE in Europe by a three-stage control procedure: *primary, secondary* and *tertiary control*.

### **Primary Control**

Primary control restores the balance between power generation and consumption within seconds of the deviation occurring. The frequency is stabilised within the range of permissible thresholds. Activation takes place directly in the power stations by means of turbine governors. The frequency of the grid is monitored and the required primary control energy is directly activated in the event of deviations. All transmission system operators represented in the UCTE must fulfil the requirements in their country in accordance with the UCTE rules. In Switzerland 74 MW of primary control energy (2009) must be kept in reserve at any time. This value is adjusted annually in November in accordance with the UCTE requirements.

Planned procurement of primary control energy through swissgrid:

- Bid process for control energy provision for one month by means of symmetrical capacity blocks of  $\pm 3$  MW linear activation up to a frequency deviation of  $\pm 200$  MHz
- Condition: pre-qualification of the involved power stations for delivering primary control energy
- Monitoring of control energy must be ensured
- Assignment of control power contract to providers on the basis of bid price
- Remuneration of control power according to bid or threshold price
- No remuneration of primary control energy

### **Secondary Control**

Secondary control is used to comply with the intended exchange of energy in a control area with the rest of the UCTE grid while simultaneously supporting the integrity of the 50 Hz frequency. In the case of an imbalance between production and consumption, the central load frequency controller automatically calls up secondary control energy from the connected generating units. As a condition these generating units must be in operation but not generating the maximum or minimum possible nominal capacity, in order to meet the requirements of the central load frequency controller at all times. Secondary control is activated after a few seconds and is typically complete after 15 minutes. If the reason of the control activation is not eliminated after 15 minutes, secondary control gives way to tertiary control. In Switzerland, 350 MW of positive and negative secondary control energy will be permanently held in reserve (2009).

Planned procurement of secondary control energy through swissgrid:

- Bid process for control energy provision for one month by means of symmetrical capacity blocks of min.  $\pm 10$  MW
- Condition: Pre-qualification of the involved generating units for delivering secondary control energy
- Monitoring of energy reserve must be ensured
- Assignment of control power contract to providers on the basis of bid price
- Remuneration of control power according to bid or threshold price (only bids with a price of under 60 CHF per MW and hour are considered)
- Remuneration of secondary control work with compensation indexed to the stock exchange price of the hour (SwissIX  $\pm 20\%$ )
- Employment proportional to the contracted power

### **Tertiary Control**

Tertiary control is used for the relief of the secondary control and thus for restoring a sufficient secondary control volume. Tertiary control is above all necessary in order to adjust major, persistent control deviations, in particular after production outages or unexpectedly long-lasting load changes. Activation is effected by the swissgrid dispatcher by means of



Figure 5.1: Example of a power plant outage in France [48]

special electronically transmitted messages to the providers, who must then intervene in power plant production to ensure the supply of tertiary control power within 15 minutes, irrespective of the schedule matrix. In Switzerland, 650 MW of positive and 560 MW of negative tertiary control energy will be permanently held in reserve (2009). Planned procurement of tertiary control energy through swissgrid:

- Bid process for control energy provision for one month by means of symmetrical capacity blocks of min.  $\pm 10$  MW.
- Condition: Pre-qualification of the involved power stations for delivering tertiary control energy
- Monitoring of the energy control must be ensured
- Assignment of control power contract to providers on the basis of bid price
- Remuneration of control power according to bid price
- (D-1) tenders: Daily invitations to tender for tertiary energy on the previous day with obligatory bids from providers with power provision (i.e. the providers who have been awarded a contract for the weekly and monthly auctions) supplemented by other voluntary bids. The bids can be adjusted intraday up to the bid deadline.
- Tertiary energy is requested («tertiary request») using the daily energy bids based on the increasing bid price.

### **Example of an Power Plant Outage in France**

The figure 5.1 shows an example of a power station outage in France. There the activation of each stage is described. In the entire UCTE region, primary control is activated directly.



Figure 5.2: Example of a generator performance chart

After 30 seconds, secondary control energy is automatically called up in France, and replaced after 15 minutes by tertiary control, in this example provided by power stations in France and Spain.

### **5.1.2 Other Ancillary Services**

The remaining ancillary services are voltage support, compensation of active power losses, black start and island operation capability, system coordination and operational measurement. These services are less important than the control reserves, but for the sake of completeness a description of them is also given.

### **Voltage Support**

The voltage at a node can be affected by the exchange of reactive power. The voltage in a node is raised by the feed-in of reactive power; the intake of reactive power reduces the voltage. swissgrid provides reference voltages for the feed-in nodes of production units in the transmission system. The voltage at the feed-in point can brought to the given reference voltage through the regulated exchange of reactive power.

The intended concept differentiates between the provisioning of reactive power capacity and the exchange of voltage-conformant reactive energy by generating units:

• Each generating unit must provide a mandatory reactive power range, which must be available at any time if the unit is (synchronised) on the grid. The mandatory range corresponds to that reactive power capacity which is available with full active power delivery of the unit (without active power production becoming limited). Figure 5.2 shows this range in the performance chart of a generator. There is no remuneration for mandatory reactive power provisioning.

- For the supramandatory (voluntary) supply of reactive power capacities, contractual regulations are agreed with generating units. A bid process is not planned at present but will be introduced at a later stage.
- In the entire capacity range (mandatory and voluntary) the exchanged reactive energy is remunerated at a flat rate (CHF/MVarh) provided it is used to reach the reference voltage, i.e. the voltage at the feed-in node is moved towards the reference voltage. An appropriate conformity test is performed during operation (monitoring) as well as for billing purposes. In contrast to the fixed voltage requirements applicable today for power stations, in future swissgrid will provide a day-ahead voltage plan containing the hourly reference voltage for feed-in nodes. This plan will be drawn up daily by swissgrid on the basis of forecasts and available reactive power resources, and published on the Internet.

### **Compensation for Active Power Losses**

Any transport of active or reactive energy in the network leads to active power losses. These losses of energy, which occur at all network levels, must be compensated for i.e. energy must be produced or procured in addition to the energy delivered to end consumers, and fed into the appropriate grid. Active power losses vary at different network levels and depend on a range of parameters. For example, the transit of electricity through Switzerland substantially affects losses in the transmission system. Active losses of energy for the technically defined transmission system can be determined by calculating the difference of all measured feedins and withdrawals. Distribution grid operators are responsible for the procurement of active power losses in the distribution grid. In 2007 and 2008 the average active power losses in the Swiss transmission system amount to approximately 100 MW with a range of approximately 60 to 200 MW.

Planned procurement of active power losses in the transmission system by swissgrid:

- Determination of the requirement of active power losses based on empirical values and daily forecast
- Procurement of the forecast active power losses in three stages: yearly, monthly and daily
- Conditions for participation: balance group contract with swissgrid
- Assignment and remuneration in accordance with bid price
- Delivery via normal schedule

### **Black Start and Island Operation Capability**

Generating units capable to perform black starts ensure the restoration of the grid after major incidents. Special operational sequences and procedures are applied to coordinate

#### 5.1 Legal Specifications of Ancillary Services

the restoration of voltage to the grid. This necessitates a defined number of appropriately equipped generating units with the necessary auxiliary installations, which switch themselves on to the grid in the appropriate operational sequence at the grid operator's request and, in doing so, help to restore the grid. A power station is capable of black-start if it can go from idle to operational mode without requiring the injection of grid-connected electricity. A power station is able to operate in isolation (island operation capability) if it can achieve and maintain a certain operating level without requiring activation of the outgoing lines to the synchronous grid.

Planned procurement of black start/island operation capability by swissgrid:

- Selection of providers on the basis of technical and geographical criteria in accordance with swissgrid's grid restoration concept
- Contractual regulation of capacity provision and payment over two years
- Establishment of secure communication channels if necessary
- Annual test of automatic black-start capability

### **System Coordination**

System coordination covers all higher-level services required at the transmission system level in order to coordinate and ensure the reliable, orderly operation of the transmission system in Switzerland as well as guarantee the integration of the Swiss transmission system in European grid operations. In particular, system coordination includes overall monitoring of the grid, grid management and control, the coordination of international energy exchange programmes, congestion management, as well as various other coordination activities within Switzerland and in the international UCTE grid. In terms of technical operations, essential tasks of system co-ordination include calculations to determine grid safety/security also in the case of topology changes, operation of the Swiss load frequency controller and billing/settlement with neighbouring countries, monitoring the provision of ancillary services and coordinating grid restoration following a major incident.

All these tasks are essential for the reliable, secure and stable operation of the grid, serve all grid customers and are performed by swissgrid.

#### **Operational Measurement**

This includes installation, operation and maintenance of the measuring and metering devices and data communications equipment and systems (communication) in the grid, as well as the provision of information (measuring data) to ensure the smooth operation of the grid. This also includes power handover measurements to neighbouring foreign grids. Operational measurements represent an important interface between the different grids. Installation and maintenance of the measuring and metering devices, measuring and metering data acquisition as well as transmission are guaranteed by the respective grid operator.



Figure 5.3: Segmented power range of a generator

## **5.2 Modeling of Ancillary Services**

An inclusion of all the different kinds of ancillary services into an optimization is not very suitable. The ancillary services voltage support, black start and island operation capability are procured contractual for longer periods. System coordination and operational measurement are performed by the TSO swissgrid. These mentioned services are therefore not very useful for an inclusion in the optimization of this thesis because they do also not affect the water management in a hydro power plant.

To integrate control reserves and also the compensation for active power losses however make sense. They are both procured in a similar way: There is first a bid process for energy provision for a next month, whereas the actually delivered energy is payed, expect primary control, by swissgrid during this next month. So in principle these ancillary services are from an implementation oriented point of view not very different. Therefore there is for the next expansion of the model only one ancillary service considered unprovoked, the secondary control reserve.

After a (positive) pre-qualification by swissgrid a power plant is able offer secondary control reserves. Figure 5.3 shows then how the power range of a power plant can be segmented. The efficiency of the turbines is relatively low when operated at small power. So the power plants often introduce a *technical minimum* to avoid inefficient use of the turbines although it would be possible. The technical minimum is therefore the amount of power which a generator has to produce at least if it is not idle. So if one offers e.g. secondary control of 50 MW for a month, one has to run the generators at any time at a power of at least 50 MW plus the technical minimum. This produced energy has to be sold. On the other hand, one cannot produce more than the maximum rating of the generators minus the offered secondary control power.

#### 5.3 Inclusion into the Model



Figure 5.4: Spreading of technical minimum if one offers multiple ancillary services

If one considers only this secondary control one neglects an optimization possibility. Generating companies often try to offer multiple ancillary services like primary *and* secondary control for the same power plant.

This is economically important, because of the monthly commitment of these services. Thus if one offers secondary or primary control reserves for a power plant, one has to produce even in the off-peak hours which is economically not very efficient. Also can the technical minimum be lower e.g. when you offers apart from secondary control also primary, because one has to produce primary control reserves on a smaller timescale and therefore a more inefficient use can be tolerated. This idea is visualized in figure 5.4.

### **5.3 Inclusion into the Model**

As already mentioned, an auction bid for secondary control has to be placed before a period has started. Therefore one has to *decide* how much energy one wants to sell as secondary control *before* any stochastic evolvements has occurred. There is no possibility to change this decision later on. So in a framework similar to  $(4.1)$  or  $(4.2)$  the decision describing the secondary control bid fits in as a decision control vector.

On the other hand one can sell more or less energy within a time period. So one can react on price changes or other stochastic fluctuations. In the mathematical framework this suits well as a recourse variable like the spillover was modeled in previous chapters. If one introduces also a state variable *R*, which describes the storage level after a stochastic water inflow and leaves the possibility for spilling described with the recourse variable *Z*, one gets a scenario tree, which is presented for two stages in figure 5.5. In there, *X* is denoting the control vector and *Y* the averaged produced power within a period.

As derived in the previous section 5.2, the generators has to be operated at any time if not idle at a power *y* of at least the technical minimum *Ymin* plus the secondary control bid and



Figure 5.5: Scenario tree with considered secondary control reserve

at most the installed generator capacity *Ymax* minus the bid:

$$
Y_{min} + X \le y \le Y_{max} - X
$$

This equation is also valid for *averaged* produced power *Y*, if one offers secondary control, which has to be higher than the minimum bid:

$$
Y_{min} + X \le Y \le Y_{max} - X \qquad \text{if } X \ge X_{min} \tag{5.1}
$$

But if one doesn't provide secondary control for a period, than the averaged produced power *Y* is not limited by the technical minimum because one can always produce nothing within the time periods:

$$
0 \le Y \le Y_{max} \qquad \text{if } X = 0 \tag{5.2}
$$

Before the proposition of a model can be made, one has to introduce another parameter: the time duration for one period ∆*t*. This parameter is needed because there are two different kinds of values in the formulas: power and energy. So ∆*t* enables the conversion from averaged power to actually sold hydro energy.

Difficult to simulate is the actual delivery of secondary control energy. In this model it is assumed that over one period one generates the same amount of positive and negative secondary control energy. This is a valid assumption as reality often shows. Neglected is the profit which is generated even for symmetrical delivery because of the higher prices (compared to the spot market price) for delivery of control energy payed by the  $TSO<sup>1</sup>$ . This amount of money could be also integrated in the price for the secondary control power bid.

<sup>&</sup>lt;sup>1</sup>in Switzerland: swissgrid

With these assumptions and equations (5.1) and (5.2) one can create the following deterministic equivalent:

$$
\max \quad c_1 X_1 + \sum_{t \in \{2,\dots,T\}} \sum_{\mathcal{A} \in \Lambda_t} p_{\mathcal{A}} \left[ s_t Y_{t,\mathcal{A}} - q Z_{t,\mathcal{A}} + c_t X_{t,\mathcal{A}} \right] \n\text{subject to } (I) - (X)
$$
\n
$$
(5.3)
$$

$$
R_{2,\mathcal{A}} = R_{start} + I_{2,\mathcal{A}} \qquad \forall \mathcal{A} \in \Lambda_2
$$
  
\n
$$
Z_{t,\mathcal{A}} \geq R_{t,\mathcal{A}} - \Delta t \cdot Y_{t,\mathcal{A}} - R_{max} \qquad \forall t \in \{2,...,T\}, \forall \mathcal{A} \in \Lambda_t
$$
  
\n(II)

$$
R_{t, \mathcal{B}} = R_{t-1, \mathcal{A}} - \Delta t \cdot Y_{t-1, \mathcal{A}} - Z_{t-1, \mathcal{A}} + I_{t, \mathcal{B}} \quad \forall t \in \{3, ..., T\}, \forall \mathcal{A} \in \Lambda_{t-1}, \forall \mathcal{B} \in U(\mathcal{A}) \quad (\text{III})
$$
  
\n
$$
\Delta t \cdot Y_{t, \mathcal{A}} \leq R_{t, \mathcal{A}} \qquad (\text{IV})
$$

$$
Y_{t-1,\mathcal{A}}^{min} + X_{t-1,\mathcal{A}} \le Y_{t,\mathcal{B}} \le Y_{max} - X_{t-1,\mathcal{A}} \qquad \qquad \forall t \in \{2,...,T\}, \forall \mathcal{A} \in \Lambda_{t-1}, \forall \mathcal{B} \in U(\mathcal{A}) \qquad (V)
$$

$$
Z_{t,A} \geq 0
$$
  
\n
$$
X_{t,A} \in \{0, [X_{min}, \infty] \}
$$
  
\n
$$
Y_{t,A} \in \{2, ..., T \}, \forall A \in \Lambda_t
$$
  
\n
$$
Y_{t,A} \in \{1, ..., T \}, \forall A \in \Lambda_t
$$
  
\n
$$
Y_{t,A} \geq Y_{min}
$$
  
\n
$$
Y_{t} \in \{1, ..., T \}, \forall A \in \Lambda_t
$$
  
\n
$$
Y_{t} \in \{1, ..., T \}, \forall A \in \Lambda_t
$$
  
\n
$$
Y_{t} \in \{1, ..., T \}, \forall A \in \Lambda_t
$$
  
\n
$$
Y_{t} \in \{1, ..., T \}, \forall A \in \Lambda_t
$$
  
\n
$$
(VII)
$$
  
\n
$$
Y_{t} \in \{1, ..., T \}, \forall A \in \Lambda_t
$$
  
\n
$$
(VII)
$$

$$
X_{T,A} = 0 \qquad \qquad \forall A \in \Lambda_T \tag{X}
$$



#### Chapter 5 Ancillary Services

Equation (5.3) proposes the framework for a multistage stochastic program with one stochastic parameter (water inflows), three decision variables (sold energy, spill over and sold secondary control) and one state variable (reservoir filling), as presented in figure 5.5. The objective function tries to maximize the profit resulted by selling of secondary control power *X* and average sold power *Y*. The price *s* for *Y* is always averaged over a period. The secondary control outcome for *X c* can be assembled from the outcome for the provision of secondary control output and a proposition of the average actual payments for secondary control delivery.

The unit of the variables and parameters in constraints (I) to (IV) is energy or MWh. So one has to convert there *Y* from power to energy with the parameter  $\Delta t$ , which is the number of hours between two stages.

In the constraint  $(V)$  however the variables and parameters have a unit of power or MW. This constraint also specifies together with constraints (VII), (VIII) and (IX) the unconstrained power which was shown in figure 5.3 and derived in equations (5.1) and (5.2). Constraint (X) finally defines for the very last stage descion *X* to zero.

Because no value for the remaining energy amount in the reservoir at the end is provided, the algorithm will always try to empty the whole reservoir. Therefore if the time horizon would be one year, the reservoir should be also empty at the beginning so that the optimization is consistent.

However it would be no problem to introduce a final targeted storage level or to assign the remaining water at the end a meaningful value corresponding to the sale potential of the next year.

## **5.4 Mixed integer problem**

Constraint (VII) with its consequences (VIII) and (IX) in equation (5.3) transform the LP to a *mixed integer problem*, which is much more complicated to solve than a LP. Luckily in this particular case it is possible to solve the problem in a heuristic concept based on a *branch-and-bound* algorithm and find a near optimal solution, which quality should be "good enough" for practical purposes.

The concept (figure 5.6 and 5.7) first run the optimization without the possibility to offer no secondary control. If the optimization is feasible the algorithm searches the first variable  $X_{t,A}$  which is at his lower bound and fixes this variable to zero when the profit is higher. Then it searches the next variable and so on. The search begins at the first stage and ends at the last so although one is not considering every possibility (which is not solvable with reasonable resources), the solution is close to the optimal one, because later stages influence the former not much.

If the first optimization is not feasible (e.g. one is not able to offer secondary control in some stages because of a low reservoir filling), the algorithm enlarge the minimum bound for the variable *Y* to zero by fixing *X* to zero and runs the optimization again. This time it has to be solvable, otherwise there exists no solution for the problem. The algorithm takes then every *X* and tests if it would be profitable to offer secondary control.

With this heuristic procedure the stochastic mixed integer problem is solved near optimal but with the need of running the optimization several times depending on the problem parameters. This can increase the elapsed time for solving the problem significantly.

The problem was also solved using the optimization package *TOMLAB* <sup>R</sup> . This package would provide a framework for solving mixed integer problems efficiently. Because of the large amount of mixed integer variables the optimization was far too slow to be practical. Also solutions of synthetic problems were in most cases the same as performed with the heuristic approach.



Figure 5.6: Flow chart if the optimization is feasible



Figure 5.7: Flow chart if the optimization is infeasible





Table 5.1: Specifications for examples 5.1 and 5.2

## **5.5 Example of the introduced model**

The problem described by equation 5.3 as well as the concept for solving this mixed integer problem is presented on an example of an hydro operation planning over three time periods. The specifications of the problem are shown in table 5.1. To get a feeling of the quality of the stochastic optimized strategy, it is compared to a deterministic one.

Example 5.1 (Deterministic optimization) The implementation for a deterministic optimization is close to (3.2), but with the inclusion of ancillary services. The parameters for the problem is specified in table 5.1. Because a deterministic optimization only provide the first optimal decision, it is made for every time-step and every scenario with always adjusted input parameters to show how a deterministic strategy would develop over the time in reality.

If the problem is solved in a deterministic approach there are different infeasible situation possible. In reality one would have to react on that and would therefore not follow the optimized strategy.

Figure 5.8 shows the development of the deterministic strategy with green cubes for the optimal offered secondary control power and yellow cubes for the average sold power. The reservoirs at the end visualize the theoretical residual filling of the reservoir if neither positive nor negative secondary control energy is delivered. The red ones shows the reservoirs with below zero values. This happens because the deterministic only optimize for expected water inflows. In reality possibilities to get out of this dilemma would be making use of pumps, if they are available, to increase the stored energy amount when the reservoir is near empty or to use or buy energy from other power plants for instance. It should be clear that these situations will result in higher costs and therefore lower revenue.

Another possibility for a deterministic optimization is not to take expected water inflows into account to get the so-called *wait-and-see*-solution (see also example 3.2) but to optimize the worst case scenario. This concept is often done in reality, because it produces a much more robust solution compared to a wait-and-see-solution. The strategy out of such an optimization is shown in Fig. 5.9. There it is obvious that no infeasible situation can occur. Even in the very worst case the remaining reservoir filling is optimized to zero. But this comes with high opportunity costs if wet inflows occurs.

The expected profit of this scenario is 1.63 MCHF. It will be later used for a comparison with the profit in a stochastic strategy. One can also consider the possibility to react on different water inflows e.g. that one sells more energy on the spot market if one notice a wet period. With this consideration one gets a little higher expected profit of 1.66 MCHF. The difference here is so small because the determination of the secondary control power bid reduce the degree of freedom for the generator significantly. П

Example 5.2 (Stochastic optimization) The problem with specifications from table 5.1 is now optimized with the mixed integer stochastic program (5.3) presented in the previous section. This optimization has to be done only once because the algorithm delivers all optimal decision variables in all stages and simulated scenarios. Therefore one gets e.g. one different recourse vector for each stage and scenario.

The stochastic strategy is shown in figure 5.10. There the different recourse variables for each scenario are well visible. The algorithm tries to sell as much energy as possible. But because of the production limits for the generators it is not always possible to sell the whole reservoir. There are also no infeasible situation possible but it is also not only optimized for the worst case but with respect to every modeled cases.

The expected profit in the stochastic strategy results to 1.89 MCHF which is a bit higher than in the deterministic strategy. As concluded in chapter 3, the possibly higher expected profit is not a general benefit of stochastic strategy when opposed to deterministic ones. This changes if one applies deterministic optimizations for the worst case scenario instead of the expected one for this particular hydro operation problem. In this case one have a reduced risk for a low profit achieved by accepting a lower expected profit.

In stochastic programs however it is well possible to choose how much "risk" is acceptable. This will be shown next.  $\Box$ 



Figure 5.8: Deterministic strategy developed over time



Figure 5.9: Deterministic strategy developed over time with optimized worst scenario



Figure 5.10: Stochastic strategy

# **Chapter 6**

# **Risk calculations**

The term risk was already mentioned numerous times without any explanation or determination of it. This will be done explicitly in this chapter, because the measuring of risks isn't trivial and different concepts can and are used to evaluate and quantify a risk. After a discussion about the procuring of a distribution function three risk measurements are introduced and compared: standard deviation, Value at Risk and Conditional Value at Risk. An example then applies these concepts on the hydro scheduling optimization.

## **6.1 Procuring of a distribution function**

For the owner of a company it is important to have some knowledge of its risk exposure. Risks can come from uncertainty in financial markets, project failures, legal liabilities, credit risk, accidents and natural causes. So the risk management is the identification, assessment, and prioritization of risks (by *risk measurements*) followed by coordinated and economical application of resources to minimize, monitor and control the probability and/or impact of unfortunate events (by *risk treatments*). [42]

In this thesis considered risk measurements are *standard deviation*, the *Value at Risk* and the *Conditional Value at Risk*. At the beginning of the procedure to specify a risk measurement, one has to determine a probability distribution function of the variable of interest, where a normally distributed function can be assumed. This assumption is often justified in a financial context.

In order to find the right parameters of the distribution function, for instances mean value and standard deviation for a normal distribution, one has to have some data about the variable. One possibility is to take historic values of the variable and/or its frequency of occurrence. The second possibility is to simulate the problem e.g. in a Monte-Carlo simulation and extract out of these results the needed data.

After procuring of data the parameters of a distribution function can be determined by a *Maximum Likelihood Method* for instance. Another method is to use the raw data to construct a discrete distribution.

#### Chapter 6 Risk calculations

The determination of the distribution function is the critical point in the risk measurements: If one assumes a wrong distribution all next calculations will have no use and worse because one doesn't know that this base is invalid one has a cattish safety. This is a serious problem and it is still often experienced in the financial sector.

In this work the variable of interest is the profit out of the revenue of sold electricity and secondary control. The data for constructing the distribution function is given by the stochastic program which is one of its already mentioned benefits. There will be no *Maximum Likelihood Method* used to estimate a distribution function but a discrete distribution is formed. The quality and accuracy of this distribution is depending on the input data of the stochastic program and the modeling of the scenarios.

The next task is to find the suitable risk measurement out of the three considered concepts, which then can be applied on the distribution function.

### **6.2 Standard deviation** σ

After formation of a distribution function several risk measurement concepts can be applied. The first one is the *standard deviation*  $\sigma(X)$ . It's definition is:

$$
\sigma(X) = \sqrt{E(X - E(X))^2}
$$

 $E(X)$  is the expected value or mean of the random variable *X*. So  $\sigma(X)$  is a measure of the variability or dispersion of a data set of a random variable *X*. A low standard deviation indicates that the data points tend to be very close to the expected value, while high standard deviation indicates that the data are spread out over a large range of values. In the case where *X* takes random values from a finite data set with each value having the same probability, the unbiased standard deviation is by using a summation notation:

$$
\sigma(X) = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - E(X))^2}
$$

This equation will be used for the calculation of the standard deviation in the stochastic program.

Both negative and positive deviation from the expected value are treated in the same way. This symmetry and its equal approach to measure the risk is opposed to the human intuition of risk measurement where positive events are weighted less than negative ones. For instance if the standard deviation is the only considered risk factor for a profit in a portfolio than it would be possible to lower the risk by introducing new opportunities for better profit.



Figure 6.1: Value at Risk (VaR) and Conditional Value at Risk (CVaR)

## **6.3 Value at Risk (VaR)**

To consider the importance of negative deviations there are often used "down-side" risk measurements. This measurements capture the negative events and by lowering such a measurement one lower at the same time the number or the extent of these negative events in a distribution.

The so-called *Value at Risk (VaR)* is such a (widely used) measure. For a given portfolio, probability distribution, probability level and time horizon, VaR is defined as a threshold value such that the probability that the loss on the portfolio, based on (fair) market prices without trading in the portfolio and over the given time horizon, exceeds this value is the given probability level. In more mathematical words, the VaR is the negative quantile of a distribution:

$$
VaR_{\alpha}(X) = F_X^{-1}(\alpha) \quad \text{where}
$$

$$
F_X(\alpha) = P(X \le \alpha)
$$

A graphical interpretation of this for a normal distribution is showed in figure 6.1: The value of a portfolio will change in the future and the VaR is the maximum negative change (loss) in e.g. 90 % of the best possible scenarios in the next e.g. 365 days. The present value of the portfolio minus the VaR is called *downside*. For a normal distribution the VaR is simply a multiple of the standard deviation.

Although the VaR is used so often it has some major drawbacks. In a debate about the VaR, David Einhorn<sup>1</sup> compared it to "an airbag that works all the time, except when you *have a car accident."* [50], which demonstrate well the problem with using a VaR as a riskmanagement limit. Because the underlying probability distribution of a portfolio is in a real case hardly a normal distribution but a distribution with *fat tails*, it covers these rare events inadequate. Figure 6.2 shows an example of a distribution function, which has got the same 10 %-VaR and also downside as the one in figure 6.1. But it is obvious that the risk in figure

<sup>&</sup>lt;sup>1</sup> Investor and founder of *Greenlight Capital*, which is a hedge fund

#### Chapter 6 Risk calculations



Figure 6.2: "Fat tail" in a distribution function

6.2 is "higher". The fat tail is for instances an event which arise only in one day during two years but will produce so much loss, so that the company isn't able to survive.

One can also defines e.g. a *Profit at Risk* which in fact is a VaR referring to a profit and not to the value of a portfolio but otherwise with the same characterizations. In this thesis such a *Profit at Risk PaR* could be used as a risk measurement. To reduce the risks it doesn't make much sense to minimize a PaR which is obvious when looking at figure 6.1. More suitable is a maximization of the downside. Like that one can guarantee, that in e.g. 99 % the profit will be higher than this maximized downside.

### **6.4 Conditional Value at Risk (CVaR)**

The *Conditional Value at Risk (CVaR)* is also called *expected shortfall* or *Expected Tail Loss (ETL)*. This concept provides an alternative to the VaR for risk-management. It specifies for a given portfolio, probability distribution, probability level and time horizon the expected return on the portfolio in the worst cases within the given probability level (figure 6.1 and 6.2):

$$
CVaR_{\alpha}(X) = E(X|X < \mu) \quad \text{where}
$$
  

$$
P(X < \mu) = \alpha
$$

Fat tails are well covered with this concept which is its advantage when opposed to a VaR. In a known distribution however a CVaR doesn't contain more information than a VaR does but it provides a better insight in the probability function. If there are no fat tails like in a normal distribution, the CVaR does not provide more information than the VaR, which is therefore then normally used.

In this thesis however the CVaR is also easier to introduce in the model than the VaR. So this is the used concept for measuring the risk. Like the PaR one can also define a CPaR or

#### 6.5 Example for a Risk-management



Figure 6.3: Sorted profit in deterministic and stochastic strategy for each scenario

one can keep in mind that the here used CVaR refers not to a value of a portfolio but to the profit of it.

To minimize the risk, which would be therefore a kind of *risk treatment*, a probability level has to be assumed, so that one can determine how *many* worst events has to be maximized. If one takes a close to zero CVaR maximization, than only the very worst scenario will be maximized. This was done in principle also in the deterministic approach in example 5.1. The stochastic program allows now easily the freedom to choose a probability level, so that a range of worst scenarios are maximized and not just the very worst. If one chooses to maximize the 100 %-CVaR for instance, it will be the same as an optimization with maximized expected profit which was done in example 5.1.

### **6.5 Example for a Risk-management**

The profit out of an scenario is with the hydro operation example highly depending on the stochastic variables water inflows and/or spot market prices (simulated correlation coefficient of minimum 0.9). So in this thesis the assumption is made, that the scenarios with the worst profits are these with the worst water inflows or prices. Alternatively one could also perform an optimization in advance to specify the worst scenarios.

Only the profit out of these worst scenarios (for a given probability level) will then be maximized which will be shown in the next example.

Example 6.1 (Risk calculations) The profits for each scenario out of the results of the examples 5.1 and 5.2 are confronted in figure 6.3. As already stated, the expected profit for the stochastic strategy is higher than that out of a deterministic strategy. But the worst case profit is in the deterministic strategy higher because the problem is there optimized explicitly for this case. If the standard deviation would act as a risk measurement, the stochastic strategy would be better because of its higher expected profit. However as the

#### Chapter 6 Risk calculations



Figure 6.4: Sorted profit for a maximization of the three worst scenarios



Figure 6.5: Risk versus expected profit and worst case profit

responsible person of this power plant one would probably choose the strategy with the higher worst profit.

The nine different scenarios which are produced in this example deliver too less data for a well-founded risk-management. If one e.g. would like to maximize the 5 %-CVaR, one would simply maximize the profit out of the very worst scenario.

However for the sake of a visualization, a possibility would be to maximize the 30 %-CVaR, which in this case would mean the maximization of the expected profit for the three worst scenarios. The resulting profits are shown in figure 6.4. It is interesting, that in this particular case, the very worst profit scenario would deliver the same profit as the deterministic strategy, but the overall expected profit is much higher (by 11 %).

Figure 6.5 shows how the expected profit will decrease and the worst case will increase if one increases the CVaR probability level. With this two figures one can decide, how much risk is appropriate. For instance in this case it would make not much sense to increase the CVaR from 0.6 to 0.3, because the worst case profit won't increase, but the expected profit will decrease.

One can conclude out of this example, that a flexible stochastic program could provide a good tool for risk management. On the other hand one can also well introduce the risk, which is appropriate for a company profile, in a hydro operation planning problem.

 $\Box$ 

Chapter 6 Risk calculations

# **Chapter 7**

## **Implementation**

In this chapter the case study is specified with all its parameters and assumptions. A typical Swiss hydro storage power plant is regarded. The data about the power plant as well as its water inflows were provided by an industry partner and are aggregated for secrecy. The parameters to be specified are listed in table 7.1. This specifications are discussed and validated for two different optimizations: the first with water inflows as the stochastic variable and then the second with spot market prices as stochastic variable. At the end the structure of the optimization program is given and discussed.

### **7.1 Time scale**

The time scale as well as the time horizon is the same for both cases. Without scenario reduction techniques, the problem will grow exponentially with the number of periods considered, as already stated in section 4.2 and shown in figure 4.5. The critical parameter with the now available computers is more the memory consumption of the problem than the processor power, because the optimization problem is linear which can be solved very efficiently. So it is important to use only that amount of time complexity which is needed to solve the optimization problem.

In a medium-term operation planning the horizon usually expands from several months to one year. In this work one year is considered with a monthly step, because also the secondary control power is procured monthly. Therfore 12 time periods have to be optimized. With the number of periods fixed, one can only apply scenarios tree reduction techniques or a reduction of stochastic evolvements per period to make the problem less resource depending.

As it will derived in the next section, the reservoir is always empty at the beginning of May. So this is a good starting point for the optimization. This way the remaining water in the reservoir at the end of the optimization doesn't have to have an assigned value. So

Parameter		Case 1	Case 2
Time scale:			
Horizon	T	12 (months)	12 (months)
Step length	$\Delta t$	720 h	720 h
Starting point		$1.$ May	$1.$ May
Power plant model:			
Starting reservoir filling	$R_{start}$	$0$ GWh	$0$ GWh
Max reservoir filling	$R_{max}$	200 GWh	200 GWh
Technical minimum	$Y_{min}$	24 MW	24 MW
Max generator power	$Y_{max}$	120 MW	120 MW
Conversion factor		$2 \text{ kWh/m}^3$	$2$ kWh/ $m3$
Water inflows:	I	stochastic	monthly averaged
constant tree level		$\mathcal{D}_{\mathcal{L}}$	1
Energy prices:	$\mathcal{S}$	yearly scenarios	stochastic
constant tree level			2
Secondary control price:	$\mathcal{C}$		
at maximum		60 CHF/MW	60 CHF/MW

Table 7.1: Parameters which are specified in this chapter

the algorithm can empty the reservoir completely. This would be different if one takes the hydrological year<sup>1</sup> or another starting point.

It should be also noted here, that all values in the optimization are averaged values for one month. That's a consequence of the only one considered timescale of one month. With this procedure daily and weekly fluctuations are flatted out which makes the problem much less complicated. These fluctuations could be addressed in a short-term optimization where the results of the medium-term optimization would act as bounds.

## **7.2 Power plant model parameter**

Similar to the configurations in the previous examples the power plant consists of an upper reservoir and of a facility to produce electricity from the potential energy of the water. The electricity is sold on the spot market. The conversion factor of the amount of stored water into the amount of produced electricity is assumed to be constant, because it fluctuates not much (at maximum  $\pm 3\%$ ), which is shown by the available data. Hence the amount of water is measured as usually in units of producible electricity (MWh).

<sup>&</sup>lt;sup>1</sup>In Switzerland: 1st October - 31st September

#### 7.2 Power plant model parameter



Figure 7.1: Hydro power plant model with one cascade



Figure 7.2: Reservoir filling in different years

The key data of this power plant is visualized in figure 7.1. The data is the same for both cases.

In reality there is a cascade of two generator stations with one small reservoir in between apart from the big storage reservoir. The rating of the second generator station is around two times larger than the first one, which corresponds to the different heights. It is possible for both generator stations to deliver full power, as long as the reservoir has stored enough water. At the maximum one can therefore produce the sum of both generator ratings, which is 120 MW. Analogue, the conversion factor for the storage reservoir is summed up to 2 kWh/*m* 3 , which takes the full height difference into account.

The technical minimum, which is introduced by the power producers (see also section 5.2), is often about 20% of the rated generator power. This means in this case a technical minimum of 24 MW, which is interestingly exactly consistent with the sum of the technical minimums of each generator from the data-set.



Figure 7.3: Water inflows in different years

Figure 7.2 shows the reservoir fillings at the beginning of each month with visualization of the minimum, maximum and average value and the limit for the reservoir before spilling. As one can see the reservoir is normally at his minimum at the beginning of May and at his maximum in autumn when the reservoir is closest to spillage.

Statistically it would be the best to begin the optimization in the month with the minimal spread, which would be the 1st July. In this work however it is assumed that the reservoir is always empty at the beginning of May. This is the point were the optimization starts and ends. There is also one month reserved for maintenance, which implies an empty reservoir for the month April. This maintenance was performed actually also in the data-set. Like that it is possible to perform the optimization only for 11 months, which save memory and computing time.

So the optimizations begin in May with an empty reservoir and will end 11 months later. The remaining water in the reservoir in the last period is given both no value nor costs. It has therefore just opportunity costs and is normally minimized by the optimization. It is also possible to force the algorithm to an empty reservoir at the end.

## **7.3 Water inflows**

Water inflow data are available daily from 2005 to the present. These data are already aggregated and are listed in  $m<sup>3</sup>$ . It is then multiplied with the constant conversion factor, mentioned already in section 7.2.

Figure 7.3 shows the water inflows since 2005 with maximum, average and minimum curve. This figure shows the typical water inflows for a hydro storage power plant in the Alps:


Figure 7.4: Water inflows per month for a 5-level scenario tree



Figure 7.5: Adjustable input-parameters and the output variables

During the months November to April the water is more or less frozen and almost zero water inflow occurs. Afterwards between May to October both snow and glacier smell and provide the whole water inflow in half of a year. This large fluctuations are one of the reasons for building a storage reservoir to be able to produce energy without being depending too much on the water inflows.

Totally there is around 200 GWh of water inflow a year which is interestingly the maximum filling of the reservoir which would therefore be able to store the inflows for one whole year. The installed power of 140 MW performs 85 GWh of electricity a month, if producing around the clock. So one would need about 2.5 months a year to handle the water inflows or about one fifth of the time. However this means not that even without reservoir there would be no spillage because the data shown in figure 7.3 are averaged data.

Out of this data one can specify the stochastic water inflows for the optimizations. This is done in a monthly view because of the high correlation of over 0.9 between the yearly sets of data.

In literature there is often used autoregressive models with or without a drift term (also called moving average) to estimate the water inflow as stated in [13] and [22]. In these models the values depend on their former ones with a weighting factor as well as on a random value.

In this thesis however the scenario tree is made out of Markov-chains, where values does not depend on their history. This simplifies the optimizations but does also not allow the use of autoregressive models. Therefore and because of the small amount of data where estimations of probability functions doesn't make much sense, maximum and minimum values per month defines the stochastic evolution for the optimization in case 1. In other words, one looks what would happen in best and worst cases per month in a binary scenario tree, or with n-discretized steps between maximum and minimum value per month in a n-level tree (figure 7.4).





Figure 7.6: Optimization in case 1 after 4 and 5 periods with different starting filling of the reservoir depending on the tree level (100%-CVaR)

To specify the level of the scenario tree for case 1, a sensitivity analysis is made. In principle it would be better to make as many steps as possible. But one always have the trade-off between solution accuracy and optimization complexity.

For the sensitivity analysis, simulations with varying input parameters are made to define how many discretized steps per period has to be specified in order to get an accurate solution. Not changed are the fixed parameters of the power plant as well as the time steps. The simulations are done for four and five periods to be able to optimize problems with a high number of tree levels. The sensitivity should generally sinks with higher periods so an estimation on the safe side is made.

The sensitivities of the output parameter (see also figure 7.5) sold energy (or reservoir levels) and secondary control power in respect to the tree level are, in the case 1 with stochastic inflows, highly depending on:

- The starting reservoir filling *Rstart*
- The shape of the prices *s*, *c* for the output parameter

First, if the reservoir is filled with a considerable amount of water, then additional water inflows aren't as important as in a case where the reservoir is nearly empty. Figure 7.6 shows this behavior for different time horizons: The deviation of the expected profit, which is a good indicator for the output parameter, is plotted against the raising tree level. There can be also noted, that this behavior will attenuate with higher time horizons.

Secondly, if the prices for secondary control and energy is constant for the whole time horizon, the algorithm just tries to offer the most secondary control which is possible and which will use all of the available water energy. In this case the tree level won't have a great impact on the results, if the generator capacities are not that limited. This can be verified by comparing figure 7.6 with figure 7.7, where raising and falling prizes were simulated.

Out of the shown figures one can determine the tree level. Even in the worst case a tree level

#### 7.3 Water inflows



Figure 7.7: Optimization in case 1 after 4 periods with different starting filling of the reservoir and falling and rising prices depending on the tree level (100%-CVaR)





of two should be enough for an uncertainty of about 1%. For a practical solution for a power plant owner maybe a tree level of three is more trustworthy than just a best/worst case view on the water inflows also if the algorithm won't deliver a more accurate solution. However the complexity prevents the choose of a tree level of three in this thesis.

Figure 7.8 shows the histogram of the cumulated water inflows for each scenario for one year and a 3-level scenario tree. Also plotted are the averaged value for the yearly water inflows out of the data-set. As it should be the monthly water inflows have a high probability to sum up for 12 periods to the averaged value of the yearly water inflows.

For the second optimization with stochastic spot prices, the average values for the monthly water inflows are taken. These values are shown with the green line in figure 7.3 and would be equivalent to the values of the stochastic water inflows in a 1-level tree.





Figure 7.9: Monthly averaged spot market prizes in different years and histogram of it





#### **7.4 Prices**

The energy market prizes for Switzerland *swissix* are available since its start in December 2006 to the present. So there are only around 3 years of data. Out of these data the spot market prizes for single hours are taken. If one plots the monthly average with minimum, average and maximum values one gets to the figure 7.9. In contrast to the water inflow the yearly data here is not correlated (correlation factor of 0.3). Therefore it makes no sense to build monthly prices out of historic data.

Because of this lack of seasonality in this data as well as the high volatility (standard deviation of more than 30%!), there is assumed the same probability distribution for all months in case 2. Similar to the procedure with the modeling of the water inflows, the stochastic innovation are modeled with a minimum and maximum value with n-discretized steps in



Figure 7.11: Optimization in case 2 after 4 periods with different prices for ancillary services and different CVaR depending on the tree level

between. Because of the structure of the scenario tree the yearly averaged price for each scenario will then be gaussian distributed (figure 7.10).

To define the maximum and minimum value the extreme values out of the monthly prizes, 150 CHF/MWh and 45 CHF/MWh, are taken. Because of the high volatility it is assumed that this model is justified. Additionally the prices are discounted with the risk-free interest rate of  $1.5\%$ <sup>2</sup>.

To determine the tree level there is done again a sensitivity analysis similar to the one which has specified the tree level in the case of stochastic inflows. This time this sensitivity doesn't depend on the starting reservoir level but on the price for offered secondary control and the specified CVaR. It does also not depend much on the water inflows. Figure 7.11 shows the behavior of the expected profit depending on the tree level and the secondary control price and CVaR. If the price for secondary control is below 80 CHF/MW per hour (which is normally the case, as noted later), the expected profit fluctuates up to 10% with a tree level of 2 and below 5% with a tree level of 3. If the price is higher, the expected profit does not fluctuate much because then it is always profitable to offer secondary control.

With a low CVaR, which means only the few worst scenarios are optimized, the expected profit fluctuates much more then with an optimization of all scenarios. This behavior will attenuate when more periods and therefore also more scenarios are regarded (figure 7.12). In this thesis a tree level of two is taken because of the limited computer capacity. However

a more sophisticated choose would be a tree-level of at least three.

For case 1 one could take for the energy price its average value of 90 CHF/MWh. However a constant value is not meaningful because in that case it makes no difference for the algorithm *when* the energy is sold, as long as the reservoir is emptied and secondary control can be offered maximally. So there are different price scenarios simulated like raising and falling energy price as well as a seasonal price.

<sup>2</sup>Historical average for Switzerland

#### Chapter 7 Implementation



Figure 7.12: Optimization in case 2 after 5 periods with different CVaR depending on the tree level

There are no data available for the price for offered secondary control, because this data is kept confidentially by swissgrid. However this data wouldn't be of much use because the bidding procedure for procuring control reserves has been started only half a year ago by the time this thesis were finished and therefore this market isn't fully developed yet. Nevertheless there are some guesswork that the auction bids from the generation companies are relatively high and that this is the reason why swissgrid has recently introduced a price cap of 60 CHF/MW per hour for offered secondary control. So no bids higher than this cap are taken into account anymore in the control reserve auction.

So one can assume that the clearing price for secondary control is somewhere near this cap price but not above it.

#### **7.5 Tree topology**

The tree level for both cases are already specified and the reasons are discussed. Used are scenario trees with a constant tree level of two. Another possibility would be to use nonconstant tree levels. One can argue and also show that the number of stochastic evolutions loose importance when they are closer to the end of the time horizon. This knowledge is especially valuable if the optimization is made once a period and new informations will be available every time period which would make the optimization more accurate. Then the tree level could be reduced for the last stages which essentially would mean that some scenarios in a tree with constant level would be disregarded and therefore a scenario reduction technique would be applied.

Otherwise with no new information an optimization with the constant scenario tree will give all the information needed for the whole time horizon and e.g. a monthly optimization with a reduced scenario tree and a rolling horizon would not deliver benefits. The computing and memory effort would then just be segmented, which in some cases however actually can be a benefit.

The vigilant reader might be thinking why the optimization is regarded in two cases with one case with stochastic inflows and one with stochastic prices, when the mathematical framework is able to handle optimizations with several stochastic variables. The reason is the complexity. If one would regard two stochastic variables with each underlying a binary scenario tree, then one would have essentially an optimization with a four-level scenario tree. This is for 12 periods too much data for the available computers in this thesis, so this segmentation into two cases is made.

#### **7.6 Structure of the implemented optimization program**

To describe the structure of the implemented program a flow chart (figure 7.13) is presented. The root file *stochastic.m* defines the sequence of the algorithm. *init.m* initialize the problem with all input parameters as well as the constraint matrices, which are made by the files *constraint\_eq.m* and *constraint\_ineq* respectively. The file *run.m* runs the optimization with the handling of the mixed integer problem, presented on page 44 and 45 in the flow charts 5.6 and 5.7. After the optimization three different output treatments are possible, specified by the root file: *plots.m* which plots the output vectors as well as some additional variables, *risk.m* which calculates and plots different risk measurements (see also chapter 6) and *sensitivity.m* which makes the sensitivity analysis used in this chapter.



Figure 7.13: Flow chart of the implemented optimization

### **Chapter 8**

### **Optimization results**

After the specification of the optimization (chapter 4, 5 and 6) and the input parameters (chapter 7), the results of the algorithm can be studied. There are various simulations possible with the introduced model depending on where the focus is laying. Here exemplarily presented are optimizations with both cases 1, stochastic inflows and 2, stochastic prices (see also chapter 7).

There should be noted that the objective of this chapter is not to deliver one optimal strategy or to show every possible optimization but to present on this two cases what the introduced algorithm is able to perform.

#### **8.1 Case 1: Optimizations with stochastic inflows**

In this case the water inflows are modeled as stochastic variable whereas for the energy price a price scenario is assumed (for the input parameters see also figure 8.1). There are diverse scenarios meaningful. Chosen to present is a seasonal price scenario. Figure 8.2 (a) shows the shape of the energy price with a high price in winter and autumn and a low price in spring and summer.

Figure 8.2 (b) shows the optimal reservoir filling at the end of each month and for each scenario. On this picture the structure of the binary scenario tree is visible. The reservoir capacity is not fully used, because the energy can be sold after four months at high prices.



Figure 8.1: Specification of the input parameters in case 1 (see also chapter 7)



(a) Seasonal price (blue line) on top of spot market (b) Optimal reservoir filling for each scenario (optiprices from EEX mizations starts in 1st May)

Figure 8.2: Price scenario and optimal reservoir filling in case 1



Figure 8.3: Delivered power and offered secondary control for each scenario

To adopt such an optimal strategy, one would have to follow the corresponding branches in the picture depending on the real water inflows and would be led to the appropriate optimal next decisions.

So the optimization delivers all needed data for a yearly strategy. As noted in chapter 7.5, the optimization could be also done monthly if the real water inflows vary too much from the modeled ones. Then it is possible to modulate the input parameters monthly.

Figure 8.3 shows the monthly averaged delivered power and the optimal offered secondary control power. In the first months one offers only in some wet scenarios secondary control of 10 MW which is the minimum bid. But then corresponding to the start of the high prices in the fifth period the algorithm offers in most scenarios the maximal amount of secondary control power which is 48 MW. Therefore one has to produce exactly 72 MW in the gen-



Figure 8.4: Sorted expected profits for each scenario for different CVaR



Figure 8.5: Specification of the input parameters in case 2 (see also chapter 7)

erator on monthly average. In this case it is also the amount which has to be produced in every time step because no additional freedom is given for production (see also chapter 5.2 on page 38).

Similar to the figure 6.4 on page 56 in the chapter about *risk calculations* one can plot the sorted expected profits of all scenarios (figure 8.4 (a)). There the distribution of the profit is visualized and a risk manager would recognize adverse scenarios. This risk of low profit can be prevented by optimizing not regarding every scenario but only the worst ones. This was done in the second plot (b) in figure 8.4 for optimization with an CVaR of 5%. This means in this case the optimization of the expected profit of the roughly 100 worst scenarios. The cost for this benefit of higher profits for the worst scenarios is the lower overall expected profit of roughly 2 MCHF.







#### **8.2 Case 2: Optimizations with stochastic prices**

In the second case the stochastic variable is the energy price. The water inflows are modeled as the monthly averaged values (figure 8.5). In this case the sensitivity of the price of secondary control for the output results is much higher than in the case with stochastic inflows. Therefore this sensitivity can be addressed in this chapter.

Simulated are cases with different constant secondary control prices. If one increases this price up to the price cap of 60 CHF/MW per hour introduced by swissgrid, then it is profitable in more scenarios and also in more periods to offer secondary control. This behavior is shown in figure 8.6 for the prices 30 and 60 CHF/MW.

Also notable is, that this secondary control is mostly offered in the periods at the end of the optimization. That's because for scenarios with low prices up to the second last stage, not much energy was produced and therefore the reservoir is relatively full although it should be empty at the end. So to avoid the uneconomic case of spilling, one has to produce energy even if low prices would occur in the very last period. So in this case it is profitable to offer secondary control even for a low price.

This procedure can be an indicator for the price of secondary control bids, which are profitable for a company. So one could run the optimization for each month with different secondary control price to gather the profitable amount and price for secondary control. It is interesting that even for high revenue out of secondary control it is not always profitable to offer it, although it is additional added value.

There should be noted, that the bidding auction was not simulated so every bid was accepted in the optimization which in reality would not happen.

There was also no modeled penalizing because of the reduced freedom in a short-term optimization if secondary control is offered. For instances the average sport market price should be lower if secondary control is offered, because in that case one has to produce energy even in the periods with low prices. Therefore this optimization is principally only capable of a decision if secondary control is not profitable.

Chapter 8 Optimization results

## **Chapter 9**

### **Conclusion**

After the introduction of the possibility to offer secondary control in a stochastic program, which lead to a mixed-integer problem, it was shown that this algorithm is able to perform a mid-term operation scheduling of a hydro storage power plant. A time horizon of one year was chosen with a monthly time step. To model the stochastic values scenario trees were used. The algorithm can optimize the problem in respect to maximum profit but also to minimum risk, why the concept of the CVaR was introduced.

Two different cases with stochastic water inflows and stochastic spot market prices respectively have presented possible optimization outputs which can support the decision makers also in the area of risk management. But the algorithm is also flexible enough to handle other problems than operation planning of hydro power plants, after adaption of the input parameters and maybe introduction of additional variables. A big advantage of the stochastic program is also that the sensitivity of the results regarding the modeling of the parameters is not as high as in a deterministic optimization. That is because various scenarios can be taken into account simultaneously.

The only disadvantage of this whole methodology is the high demand of computer memory if the scenario tree is made with more than 2 levels. The handling of the mixed-integer problem made it necessary to run the optimization several times which demands also a high computer power to be able to perform the optimization in a realistic amount of time.

Chapter 9 Conclusion

## **Chapter 10**

### **Outlook**

Further work on this subject can be divided in two topics: the enhancement of the simulation model and the elaboration of more sophisticated optimization results.

For the simulation model the introduction of pumps would increase the optimization possibilities enormously and would also map the real hydro storage power plants better. Especially in the financial focus this would have a great meaning.

One could further increase the complexity by taking cascades of reservoir into account or a whole portfolio of power plants. But this probably exceeds the computer possibilities, that's why a coordination of different optimization could be interesting.

The model could be also addressed for other power plants like gas or wind power plants. The input parameter would have to be changed as well as some variables. But the principle methodology should be also possible to be applied on this power plants.

To reduce the complexity of the optimization different scenario reducing techniques could be discussed better and also mathematical improvements are conceivable.

In the case of elaboration of more sophisticated optimization results there would be possible to make a better comparison of an optimized strategy to a traditional one. This would be also interesting if the market for secondary control has settled itself.

The optimization outputs could be further applied on a more short-term optimization where they could act as bounds. Another possibility would be to perform a financial rating of the assets of a hydro power plant as well as to analyze the possibility to use the power plant for hedging purposes.

Chapter 10 Outlook

# **List of Figures**



### List of Figures



## **List of Tables**



List of Tables

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Bibliography

# **Appendix A**

**Miscellanea**

Appendix A Miscellanea

Figure A.1: Time management

### Appendix A Miscellanea

Literature
Master_thesis
2009_master_thesis_hubert_abgottspon.pdf
Matlab_code
a_02_10_09_stochastic_simple Þ.
a_02_11_09_deterministic_simple_ex3.2 Þ.
a_02_23_09_stochastic_simple_ex3.3 Þ.
Þ a_02_24_09_stochastic_ex4.1
a_03_02_09_stochastic_multileveltree ь
b_04_16_09_ancillary_ex5.1_ex5.2 Þ.
b_05_06_09_CVaR_with mixed_version_1_ex6.1 Þ.
b_05_11_09_CVaR_with_mixed_version_2 Þ.
b_05_12_09_CVaR_with_mixed_real_data Þ.
b_05_12_09_CVaR_without_mixed Þ.
b_05_20_09_case_study_TOMLAB_with 20_40_60 Þ.
b_06_03_09_case_study_TOMLAB_real_DATA_stochastic_prizes Þ.
b_06_12_09_case_study_TOMLAB_variable_Tree
c_05_27_09_data_case_study_inflows Þ.
c_05_27_09_data_case_study_prices Þ.
$\Box$ readme.txt
x_06_18_09_case_study_TOMLAB_variable_Tree_version_2
x_06_26_09_case_study_TOMLAB_stochastic_prizes_variable_Tree_version_2
Praesentationen
2009_abschlusspresentation_eth_hubert_abgottspon.pdf
2009_zwischenpresentation_eth_hubert_abgottspon.pdf

Figure A.2: Content of the CD