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Hydro power planning: Multi-horizon modeling and its applications

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Preface

This thesis was written during my time as a researcher and assistant at the Power Systems Laboratory of the ETH Zurich between 2010 and 2015.

I would like to thank Prof. Dr. Göran Andersson for providing the opportunity to be a member of the lab and complete my doctoral studies under his guidance. The freedom provided in choice of research topics, along with the support of personal needs, were greatly appreciated.

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I was very fortunate that I was able to collaborate with the Swiss Industry, EGL AG and BKW Energie AG, and would like to thank them for their support and many hours of discussion.

Further, I would like to thank my colleagues, who established an inspiring atmosphere in the lab and broaden my view on many things.

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Hubert Abgottspon
Staldenried, August 2015

Abstract

Hydro power planning problems are well known in academia. However, due to modeling and computational difficulties, not many suggested concepts are applied in practice. Additionally, the recent deregulation of electricity markets initiated different markets, which increased the need for better decision support tools for hydro operators.

Therefore, as a main contribution of this thesis, a novel modeling framework is proposed, the *multi-horizon modeling* approach. This approach allows a very detailed and transparent modeling of many problems in hydro power planning by simultaneously being computationally very efficient. The models are applied in the thesis to pumped storage hydro power plants in a liberalized market environment in order to give decision support for the self-scheduling of them.

In the thesis, first, the manifold challenges in hydro power planning are discussed. Then, state-of-the-art modeling and solution methods to such problems are evaluated, focussing on problems with non-concave value functions and risk averse optimizations. Afterwards, multi-horizon models are analyzed, evaluated, and applied for different medium-term hydro power planning problems:

- consideration of ancillary services,
- risk-averse optimizations,
- long-term evaluations, and
- price-maker bidding in forward and electricity markets.

It is shown how such models outperform traditional methodologies in different ways.

Further, an extension of a solution method, dualized stochastic dual dynamic programming, with *locally valid cutting planes* is proposed. This approach allows to solve problems with non-concave value functions more appropriately. Furthermore, a measure of the severity of non-concavity is introduced in this context, which can lead to reduced computational requirements.

In addition, the bidding into ancillary services markets is discussed and it is presented how delta-hedging can be used to mitigate bidding risk. Finally, short-term planning for hydro power plants is analyzed and decision support tools for the bidding in electricity markets and for strategic bidding in ancillary services markets are given.

With the modeling, solution algorithm, and decision tools presented in this thesis, the planning problems in hydro power can be formulated in a more transparent and meaningful way. Further, the problems can be solved by less computational requirements. Therefore, using such tools, hydro power producers are able to operate their power plants in a more profitable and robust way taking into account multiple markets simultaneously.

Kurzfassung

Die Einsatzplanung von Wasserkraftwerken ist ein altes und bekanntes Problem in der Forschung. Viele der dort entwickelten Konzepte werden jedoch nicht oder nur teilweise in der Praxis verwendet, aufgrund schwieriger Modelle und hohem Rechenaufwand bei der Benutzung dieser. Zusätzlich sind die Anforderungen an Entscheidungshilfen für Wasserkraftsbetreiber gestiegen wegen dem Aufkommen verschiedener Märkte durch die Öffnung des Elektrizitätsmarktes.

Aus diesen Gründen wird in dieser Arbeit eine neue Modellierungsmethode vorgeschlagen, das *multi-horizon* Modellierungskonzept. Es erlaubt ein sehr detailliertes aber transparentes Modellieren von vielen Planungsproblemen im Wasserkraftsbereich, bei gleichzeitig effizienter Nutzung von Rechenkapazitäten. In dieser Arbeit wird das Konzept angewendet auf Speicherwasserkraftwerke im liberalisierten Energiemarkt, um Entscheidungshilfen in der Einsatzplanung zu geben.

In dieser Doktorarbeit werden zuerst die zahlreichen Probleme bei der Einsatzplanung von Wasserkraftwerken diskutiert. Die state-of-the-art Modellierungs- und Lösungsmethoden für solche Probleme werden evaluiert mit speziellem Fokus auf nicht-konkave Nutzenfunktionen und risiko-aversen Optimierungen. Danach werden multi-horizon Modelle analysiert, evaluiert und für verschiedene mittelfristige Einsatzplanungsprobleme angewandt:

- Berücksichtigung von Systemdienstleistungsmärkten im mittelfristigen Einsatz
- Risiko-averse Optimierungen
- Langfristige Bewertungen
- Bidding in Forward- und Elektrizitätsmärkten

Es wird gezeigt, wie solche Modelle den üblichen Methoden überlegen sind.

Des Weiteren wird eine Erweiterung einer Lösungsmethode, von dualized stochastic dual dynamic programming, mit *locally valid cutting planes* vorgeschlagen. Diese Erweiterung erlaubt ein zielführenderes Lösen von Problemen mit nicht-konkaven Nutzenfunktionen. In diesem Zusammenhang wird auch ein Maß für die Stärke der Nicht-Konkavität eingeführt, welche für eine Reduktion des Rechenaufwandes benutzt werden kann.

In dieser Arbeit wird auch das optimale Anbieten von Systemdienstleistungen diskutiert. Es wird präsentiert, wie man das Delta-hedging benutzen kann, um die Risiken aufgrund von Unsicherheiten beim Bieten zu reduzieren. Schlussendlich wird die kurzfristige Einsatzplanung von Wasserkraftwerken analysiert und Entscheidungshilfen werden vorgeschlagen für das Bieten in Energie- und Systemdienstleistungsmärkten.

Die in dieser Arbeit präsentierten Modellierungsmethoden, Lösungsverfahren und Entscheidungshilfen erlauben eine transparente Bearbeitung der Planungsprobleme von Wasserkraftwerken, und dies bei tendenziell geringerem Rechenaufwand. Betreiber von Wasserkraftwerken können somit bei Verwendung dieser Konzepte ihre Kraftwerke profitabler und robuster in den verschiedenen Märkten positionieren.

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List of Acronyms

AV@R	average value at risk
ADP	approximate dynamic programming
DP	dynamic programming
GenCo	generation company
HPP	hydro power plant
KWM	Kraftwerke Mattmark AG
KWO	Kraftwerke Oberhasli AG
LP	linear program
MILP	mixed-integer linear program
MTHP	medium-term hydro power planning
SAA	sample average approximation
SDDP	stochastic dual dynamic programming
SDP	stochastic dynamic programming
SFC	secondary frequency control
STHP	short-term hydro power planning

List of symbols

The notation in this thesis follows mostly the one in mathematical programming. The list of symbols is segmented thematically.

Dynamic programming:

t	$\in \{0, \dots, T\}$, time stage
$Q(t, z)$	Value function
x	$\in X$, (here-and-now) decision vector
z	$\in Z$, state variable vector
$\phi(x, z)$	Contribution function, e.g. immediate revenue return

Scenario trees and sample average approximation:

ξ	$= (\phi, A, B, b), \in \Xi$, data observation
Ξ	$= \{\xi^1, \dots, \xi^N\}$, set of sampled data observation
s	$\in \mathcal{S}$, scenarios
\mathcal{A}_t	$\subseteq \mathcal{S}$, bundle of scenarios with the same decisions up to some stage t
Λ_t	Set of all bundles in a stage t
$U(\mathcal{A}_t)$	Set of bundles, i.e. children of the bundle \mathcal{A}_t

Approximate dynamic programming:

z'	Post decision state
$Q'(t, z')$	Post decision valuing function

Stochastic dual dynamic programming:

i	Algorithm iteration counter
k	$= 1, \dots, M$, sample
\tilde{z}^k	Trial state vectors, $k = 1, \dots, M$
\mathcal{Q}	Collection of supporting hyperplanes
$\tilde{Q}^i(z)$	Approximation of the value function
γ, δ	Intercept and vector of slopes of a cutting plane
π	Dual variables of the complicating constraints
\underline{r}, \bar{r}	Lower/upper bound to the optimal value
ϵ	Maximum allowed optimality gap

Medium-term hydro power planning problem:

$v(t)$	Hydro reservoir fillings	[1000m ³]
$u(t)$	Electricity generation in turbines	[MW]
$p(t)$	Pumped electricity	[MW]
$f_u(u)$	Function: used energy to water flow	[1000m ³]
$f_p(p)$	Function: produced energy to water flow	[1000m ³]
$a(t)$	Charges from upstream reservoirs	[1000m ³]
$o(t)$	Overflow (spillage)	[1000m ³]
$\iota(t)$	Water inflow	[1000m ³]
$c(t)$	Electricity market price	[€/MW]

Dualized stochastic dual dynamic programming:

$\tilde{\mu}$	Estimation of the Lagrange multiplier of the complicating constraint
γ^{LR}, δ^{LR}	Cutting plane for dualized problem
$\gamma^{LR}, \gamma^{opt}$	Optimal values of dualized or original non-concave subproblem

Risk averse optimization:

α	Risk level, quantile of probability distribution of random variable
$\rho(X)$	Risk measure of random variable X
$AV@R_\alpha$	Conditional average value at risk level α
$[X]^{sort}(j)$	Ordering of realizations of a random variable X and returning the j -th entry
$\lceil \cdot \rceil$	Rounding up operator
λ	Cost of risks

Multi-horizon decision trees:

τ	$\in \{1, \dots, \mathcal{T}\}$, intrastage time stage	[h]
$v^{stor}(t)$	Filling of storage reservoirs	[1000m ³]
$v^{bal}(t)$	Filling of balancing reservoirs	[1000m ³]
$W(t)$	Discretized water release from storage reservoirs	[1000m ³]
$m(\tau)$	Position on energy market	[MW]

Provision of ancillary services:

$q(t)$	$\in \mathbb{Z}^n$, Provision of frequency control amount per turbine	[10 MW]
$q^{active}(t)$	$\in \{0, 1\}^n$, Activated frequency control per turbine	
q^{min}	Minimum generation for qualified turbines	[MW]
q^{max}	Maximum providable capacity per turbine	[MW]
$c^q(t)$	Remuneration for the capacity provision for the tender period	[€/MW]

Section 9.3: Bidding of ancillary services:

$i \in \mathbf{I}$	Agents	
$a^i \in \mathbf{A}$	Action a taken by agent i	
$Q^i : \mathbf{A} \rightarrow \mathbb{R}$	Stored function of expected profit for given action	
t	Game round	
$\alpha_t^i \in 0, 1$	Degree of correction	
$r^i(a_t^1, \dots, a_t^n)$	Reward for agent i for actions a_t^1, \dots, a_t^n	
$b \in \mathbf{B}_t$	Bids, constructed out of k quantity-price pairs	
$s_k \in \mathbf{S}_t$	Price, demand charged per quantity	[€/MW/h]
$q_k \in \mathbf{Q}_t$	Offered secondary control power quantity	[MW]
wv_{hpp}	Water value for respective hydro power plant	[€/MWh]

Chapter 1

Introduction

Background and motivation

The optimal operation of hydro power plants (HPPs) for the purpose of delivering electric energy is not a recent problem. It is well treated in academic literature. In practice, however, not many academical concepts are applied. One of the reasons for this is the troublesomeness of such methods in modeling and computational requirements.

Therefore, one of the first goals of this thesis is to provide a meaningful and transparent modeling approach, which can be well applied in practice as well as keeps computational burden as low as possible.

Further, with the advent of liberalized electricity markets, the way HPPs are operated has changed. Instead of providing electricity for least costs in an integrated way, markets were established, where different services can be offered by the generation companies (GenCos). So another goal of this thesis is to give decision support in how to exploit these markets, which requires both better modeling and optimization tools.

In contrast to most of the academic literature, this thesis deals with the self-scheduling of hydro GenCos and not with a hydro dominated *system*. The proposed methods are tested on typical Swiss pumped storage HPPs, however, they are mostly also applicable to other type of power plants. The overall goal of the thesis is to provide decision support for GenCos in order to assist them in operating their HPPs in a more profitable and robust way.

Main contributions

The main contribution of this thesis is the introduction and the evaluation of a novel modeling concept, the multi-horizon modeling approach. This approach is especially well applicable for hydro power planning problems.

The application of multi-horizon models allows both very detailed and computationally efficient formulations of many different planning problems. It outperforms traditional modeling technics considerably. The reasons are manifold, but the most important ones are that both dynamic programming and mathematical solvers are combined in an efficient way, and that physical differences of reservoirs are exploited.

Part of this contribution is also the application of multi-horizon models to different hydro power planning problems, which involves long-term valuation problems, price-makers in forward markets, risk-aware optimizations, as well as the consideration of ancillary services in a medium-term optimization.

Another contribution of this thesis is about bidding problems. Tools to assist hydro GenCos in short-term bidding of electricity and ancillary services markets, and to hedge uncertain cashflows from ancillary services markets in the electricity spot market are presented.

Additionally, this thesis provides a comprehensive review about state-of-the-art stochastic programming methods, their applications and algorithms. Some of these methods are further developed in order to better cope with non-concave value functions with locally valid cutting planes, where also a novel measure of non-concavity is introduced.

Structure of the thesis

The thesis is divided into two parts. The first part introduces the challenges and state-of-the-art methods in hydro power planning. The second part is dedicated to the evolution of the methods and their applications.

Part I: Hydro power planning: challenges and methods:

- **Chapter 2** gives a brief outline of the challenges in hydro power planning without discussing how to tackle them.

-
- **Chapter 3** describes stochastic programming methods. The mathematical notation for this thesis is explained and the basic algorithms are analyzed. The chapter concludes with an application of the methods to a generic HPP.
 - **Chapter 4** extends the stochastic dual dynamic programming algorithm, a well-known solution algorithm especially applicable to hydro power planning problems, in order to cope with some of its shortcomings.
 - **Chapter 5** introduces risk measures and their application in a stochastic multistage setting.

Part II: Model developments and applications:

- **Chapter 6** introduces multi-horizon models. They are analyzed, evaluated, and finally applied in two test cases.
- **Chapter 7** proposes an extension to dualized stochastic dual dynamic programming, locally valid cutting planes, in order to cope better with non-concave value functions. Additionally, a measure for non-concavity is presented.
- **Chapter 8** presents new ideas on how to hedge uncertain cash-flows from the ancillary services market with adapted positions in the electricity spot market.
- **Chapter 9** proposes decision tools to assist GenCos in bidding problems.
- **Chapter 10** concludes the thesis with a summary and outlook.

List of publications

The list of peer-reviewed publications, which lay the basis for this thesis, is as follows:

- H. Abgottspon, M. Bucher, and G. Andersson, “Stochastic dynamic programming for unified short- and medium-term planning of hydro power considering market products,” in *12th IEEE International Conference on Probabilistic Methods Applied to Power Systems (PMAPS), Istanbul, Turkey*, 2012. [1]

- H. Abgottspon and G. Andersson. “Approach of integrating ancillary services into a medium-term hydro optimization.” in *XII SEPOPE: Symposium of Specialists in Electric Operational and Expansion Planning, Rio de Janeiro, Brazil*, 2012. [2]
- H. Abgottspon and G. Andersson. “Stochastic scheduling for a price-maker hydro producer considering forward trading.” in *IEEE PowerTech, Grenoble, France*, 2013. [3]
- H. Abgottspon and G. Andersson. “Strategic bidding of ancillary services for a hydro power producer.” in *10th International Conference on the European Energy Market (EEM), Stockholm, Sweden*, 2013. [4]
- H. Abgottspon and G. Andersson. “Medium-term optimization of pumped hydro storage with stochastic intrastage subproblems.” in *Power Systems Computation Conference (PSCC), Wrocław, Poland*, 2014. [5]
- H. Abgottspon, K. Njálsson, M. A. Bucher, and G. Andersson. “Risk-averse medium-term hydro optimization considering provision of spinning reserves.” in *13th IEEE International Conference on Probabilistic Methods Applied to Power Systems (PMAPS), Durham, UK*, 2014. [6]
- H. Abgottspon and G. Andersson. “Multi-horizon modeling.” submitted to *Energy Procedia*, 2015. [7]

Part I

Hydro power planning: challenges and methods

Chapter 2

Modeling challenges in hydro power planning

In this chapter the challenges in self-scheduling of hydro power plants are discussed. Further, two hydro power plants are introduced which are used to evaluate the proposed concepts in this thesis.

The challenges in hydro power planning problems are well known, but nevertheless are still difficult to approach, because many different complications are simultaneously present.

For the self-scheduling of hydro power plants (HPPs) these challenges are partly mitigated. That is because the whole electricity system with all producers, consumers, and the transmission grid have not to be analyzed altogether, but rather the system can be approximated by assuming exogenous market prices. This allows more detailed modeling of the actual HPPs and their assets, which on the other hand requires the modification of some of the traditional methods used in hydro power planning.

In self-scheduling problems for hydro generation companies (GenCos) the challenges are typically as follows:

- short time scales of markets and water inflows,
- non-linear and non-continuous production functions,
- long time horizons,

- unknown and/or difficult to model data processes,
- multiple markets with different rules, and
- risk-averse behavior of GenCos.

Since electricity demand changes at least hourly the markets for it have a similar time scale. Water inflows into storage reservoirs are due to melting snow and glaciers or precipitation and therefore have short time scales as well. Given short time scales, the consideration of non-linear and non-continuous production functions is required because of varying head and not well-behaved efficiency curves of turbines, pumps, and penstocks.

Storage reservoirs typically are used to balance out dry seasonal periods. Therefore, the periods are coupled and time horizons of such HPPs require to be up to a few years. For such long time horizons, water inflows and market prices are difficult to predict and have to be considered as stochastic variables.

Further, hydro GenCos offer their services in multiple markets, most notably electric energy production and control power for grid frequency stabilization. These markets have typically different rules and time scales and are difficult to formulate in a mathematical way.

Finally, it is often required to find operation strategies which are risk-averse in some sense, which can complicate the problem further.

These challenges are next described in more details giving partly also solutions. Then, additionally, two actual Swiss HPPs are briefly scrutinized, which are used to evaluate the proposed concepts in this thesis.

2.1 Challenges in hydro power self-scheduling

In order to tackle the different time scales in hydro optimization, the problem is usually divided into subproblems. A short-term hydro power planning (STHP) problem tries to find optimal dispatch for the next day(s) and can also include market bidding. As boundaries for this problem a medium-term hydro power planning (MTHP) optimization provides opportunity costs, also called water values, or alternatively target filling values.

The challenges for the two subproblems differ because different simplifications are applicable to them. They are now discussed briefly.

Aggregations

Whereas for STHP optimizations aggregations are not often applied, in MTHP optimization they are of vital importance. A temporal aggregation leads to time steps of typically weeks or even months which allows computational tractability for MTHP optimizations. Given such an aggregation, also a spatial one has to be performed, for instance an aggregation of turbines and reservoirs into representative ones.

The problem of finding a good compromise between the number of time steps, hence computational complexity, as well as accuracy of the model is discussed in more detail in chapter 6.

Production function

The production function relates water flow to energy. It depends on various factors, is non-linear, and non-continuous. It can be approximated by a piecewise linear model as e.g. shown in [8], which is often done for STHP optimizations.

For MTHP optimizations the discussion is a bit more subtle. For instance, if the production function is modeled as dependent on the water head, then some methods are troublesome to use.

In this thesis, non-continuous production functions are assumed which have forbidden operating zones for some turbines and pumps. However, the dependence on the water head is neglected. This is reasonable for HPPs in the Alps with a relative high water head compared with its maximum variation. Additionally, constant efficiencies are assumed, which don't depend on the water flow, which is also reasonable in a medium-term perspective.

Unknown and difficult to model data processes

From a STHP optimization perspective, data processes typically can be modeled well. On the contrary for longer time horizons, some data

processes like water inflows and market prices are very difficult to predict and have to be considered stochastically. However, it is often not clear how to do that. Further on, a comprehensive model will introduce tractability issues.

In practice, GenCos have some models for important data processes. Those processes are used as basis for a number of analyses. Therefore in order to facilitate an introduction of an optimization tool within such a company, the modeling of processes is not considered endogenously as a part of the tool.

Therefore, in this thesis it is assumed that data processes are already sufficiently modeled and are given as input data.

Multiple markets

Hydro GenCos participate in many different markets. The most important ones are bilateral over-the-counter markets, day-ahead and intraday energy markets, forward markets, and ancillary services markets. Except for the one for ancillary services, the markets are based on providing energy for different time periods under varying rules. Bilateral contracts are mostly based on cleared market products and can therefore be substituted by them in a model. Forward markets, consisting of power futures, forwards, and options, are used for hedging purposes whereas the day-ahead and intraday spot markets balance out physical production with the market position.

From the modeling perspective, the day-ahead market is the most important one. This is because it is agreed for most applications that its price describes the price of electricity energy best. Depending on the application, the other markets can also be important, e.g. in STHP planning the intraday market is often reasonable to consider. Whereas in MTHP problems the markets are typically represented only by their prices, in STHP the actual bidding is often important to model, which can be a difficult task.

In chapter 6, 8, and 9 different concepts are shown, which assist GenCos to cope with the challenges in energy markets.

Ancillary services markets are an additional opportunity for hydro GenCos. These services can contribute substantially to the revenue of HPPs, at least in Switzerland. Therefore, it is important to include this opportunity in a MTHP optimization and to consider its obligations in STHP

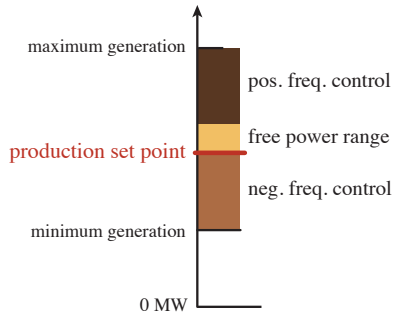


Figure 2.1: Generation of a turbine with provision of SFC reserves. Production flexibility is reduced considerably leading to troublesome problems if considered in MTHP problems.

planning, which both can be troublesome to do.

The most important ancillary service for hydro GenCos is the provision of secondary frequency control. The rules for such markets can differ a lot from system to system and therefore they are described next in more detail for the Swiss system.

Secondary frequency control (SFC) market in Switzerland

In Switzerland, the most valuable ancillary services market is the market for provision of SFC for frequency stabilization (spinning reserve, automatic one-minute control, more details can be found in [9] and for other countries in [10, 11]). It is operated by the Swiss transmission system operator swissgrid. After a pre-qualification, GenCos are invited to bid provision of SFC for a tender period of one week. The power has to be provided for the whole week in symmetrical capacity blocks of at least 5 MW with increments of ± 1 MW. It is allowed to fulfill these requirements from a pool of generating units. A bid itself is defined as a number of combinations of the volume offered and demand charged, so-called quantity-price pairs. The number of combinations are not limited.

The transmission system operator can select at most one of the quantity-price pairs within each bid. This is done by a market clearing optimization, where those pairs are selected that meet control demand with least costs. For Switzerland the control demand is at the moment ± 400 MW.

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The actual energy demand is requested automatically and is almost symmetric and therefore not much energy is delivered or used on average.

For hydro GenCos there are a few requirements which have to be taken into consideration in order to take part in this market. To provide a symmetrical capacity block the turbines have to be continuously running at a certain set point (see figure 2.1). To prevent that the turbines are operated inefficiently, a minimum generation amount is introduced. This is also true if the capacity is provided by pumps or with both turbines and pumps simultaneously (e.g. in a hydraulic short-circuit). The provision of SFC reserves therefore reduces the production flexibility, which justifies its remuneration.

Consider the Kraftwerke Oberhasli AG power plant (more details in the appendix A) as an example, why SFC is important to consider in a MTHP optimization. This plant can offer maximally 280 MW of SFC. For a very conservative average capacity remuneration of 20 CHF/MW per hour this results in a maximum possible revenue of almost 50 Mio CHF, compared with the typical turn-over in the electricity market of the plant of around 140 Mio CHF ¹.

However, the consideration of such a market in a MTHP optimization is difficult. In chapter 6, a solution to this problem is presented.

In a short-term optimization on the other hand, the fulfilling of ancillary services obligations is more important to consider, that is the actual delivery of the unknown energy demand from SFC, as well as the bidding of it. These issues are discussed in chapter 9.

Risk-aware optimization

In practice HPPs are operated risk aversely. The most important risks, apart from operational risk, are currency, cashflow, and volume risks.

Currency risks can be well hedged by financial products. Volume risks refer to the risks that a contract can not be fulfilled due to e.g. empty basins. Cashflow risks are the risks of having during a too long period too low cashflows, which could result in liquidity problems for a company. In hydro power planning, these two risks have to do with price

¹Kraftwerke Oberhasli AG, "KWOKennzahlen2013.pdf", Retrieved from <http://grimselstrom.ch/info/grimselstrom/kennzahlen-und-geschaeftsbericht>, March 2015.

and inflow uncertainties and can be mitigated by adapted operation decisions.

Theoretically, such risks could be managed solely by using power derivatives, because, under some mild assumptions, the production planning can be done independently from hedging [12]. However in practice, many (hydro power) operators do both: (often implicit) risk-averse production planning as well as hedging by financial instruments. This has also to do with the limited availability of suitable financial instruments or because of their high transaction costs. Therefore, it can be relevant to consider some of the risks in an optimization, primarily for MTHP problems, where it is usually quite difficult to define appropriate risk measures as well as to solve the resulting problems. These problems are addressed in chapters 6 and 8.

On the other hand, in STHP, risk-aware optimizations are more straight forward to formulate and mostly have to do with operational risks.

2.2 Evaluation of optimization models and methods

There are no standardized HPP models available for optimization studies, as it is the case e.g. for electricity grid analysis. However, the outcome of an evaluation of a method is depending on the HPP model considerably. It seems obvious that, the more complicated structures are present in a case study, the better a detailed modeling method will perform, and vice versa.

The evaluation in this thesis is therefore based on two HPPs which have different but somewhat typical characteristics for Swiss HPPs. Both plants are real ones located in the Swiss Alps. The first one belongs to Kraftwerke Mattmark AG (KWM), operated by Axpo Trading AG and is from now on called *KWM*. The second one belongs to Kraftwerke Oberhasli AG (KWO) which is operated by BKW Energie AG, from now on called *KWO*.

Practitioners from these companies helped with their modeling, so that their respective characteristics were taken into account realistically. Some of the data are partly confidential and therefore cannot be shown. Nevertheless, the overall ideas and conclusions should be reproducible. In the appendix A a more detailed discussion about these HPPs is given.

The models are also compared regarding their computational requirements. They were solved on a computer with two Intel Xeon E5 processors at 2.2 GHz. 16 individual processor cores were available in total but only 8 of them were used. The computer was usually also occupied by other simulations, therefore, some results have to be taken with caution.

The models are implemented in Matlab and for the applications the mathematical solver IBM ILOG CPLEX 12.4 was used to solve linear, quadratic, and mixed-integer linear programs.

2.3 Summary

In self-scheduling problems of hydro power plants (HPPs) there are many different challenges. A decomposition of the problem into short-term hydro power planning (STHP) and medium-term hydro power planning (MTHP) already allows to address many of them. The cost of this decomposition is to have to define an appropriate linkage between the different optimization parts where water values can be used adequately. In STHP planning, the most difficult issues are to model the production function and to consider multiple markets simultaneously, especially to model market rules in bidding processes. Additionally, the handling of operational risks can sometimes be troublesome.

In MTHP planning the challenges arise typically because of the long time horizon. Thus, appropriate aggregations and simplifications have to be found, both for the HPP model and for markets. It is difficult to model data processes in a meaningful way while maintaining computational tractability. The consideration of multiple markets, like secondary frequency control markets together with an energy spot market, can be troublesome because it complicates the problem in a way that traditional solving methods do not work anymore. Finally, it is also non-trivial to model appropriate risk measures for a risk-aware MTHP problem.

The methods introduced in this thesis try to tackle some of the challenges presented here. In order to evaluate their usefulness from the modeling, computational, and practical point of view, they are evaluated on two different HPPs, which have typical properties for Switzerland.

Chapter 3

Stochastic programming

In this chapter, the mathematical notation and basic algorithms of this thesis are illustrated on generic examples. After the bibliography, multi-stage programs are introduced based on scenario trees. Then, dynamic programs and their stochastic counterparts are described and the algorithms are presented. Further, stochastic dual dynamic programming is explained and its algorithm is discussed. After discussing alternative methods, the chapter concludes with an application example.

3.1 Historical developments and bibliography

Medium-term hydro power planning (MTHP) problems can be formulated in several different ways and can be solved with even more diverse techniques. References [13–15] are some of the few reviews about different algorithms applied to hydro power planning. Some of these techniques became standard in solving of MTHP problems.

Stochastic programs

Research about stochastic programs dates back to the 50's and early 60's of the last century, contributing to a rich literature. It evolved from the mathematical community when trying to introduce uncertainty into

mathematical programs.

Some more recent and also relevant literature about this field are a number of rather technical books like the ones from P. Kall and S.W. Wallace [16], J.R. Birge and F. Louveaux [17], P. Kall and J. Mayer [18], A. Shapiro, D. Dentcheva, and A. Ruszczyński [19], and also the collection edited by A. Ruszczyński and A. Shapiro [20].

From the modeling point of view, the hydro power planning problem fits very well into the concept of multistage programs. This was (partly) realized very early in stochastic programming research, e.g. in 1946 in [21] and 1955 in [22]. A more recent book about the modeling of stochastic programming is the one from A. King and S.W. Wallace [23].

(Stochastic) dynamic programming

Dynamic programming (DP) was also part of research primarily after the second world war, dealing with solving sequential decision processes. Such problems arise in many different applications. Additionally, the basic concepts are quite obvious and feel natural to apply. So it is no surprise that several research communities discovered many findings in parallel and independently.

This is the reason why this research is named in many ways: the mathematical community calls this field (multistage) (stochastic) DP, the computer scientists call it reinforcement learning and in operations research it is known as Markov decision processes.

The term *dynamic programming* itself was proposed by R. Bellman. Bellman described the framework in [24], which made DP popular. Nowadays the field of Markov decision processes is very well treated in the book of M.L. Puterman [25], the same in computer science by D.P. Bertsekas [26]. DP techniques are usually also treated in stochastic programming literature, which was already mentioned in the previous section, as well as of course in literature about approximate DP, like stochastic dual dynamic programming, which will be discussed in the next section.

Hydro power scheduling was used as an application example for stochastic dynamic programming (SDP) from the beginning. Nevertheless, it was actually solved first more than one decade later for a single reservoir problem in [27]. The reason for this was the computational troublesomeness of such problems.

In the 70's and early 80's of the last century, SDP for hydro power problems was an active field in research. The basic algorithms were extended to cope with stochasticity, multi reservoirs, hydro thermal systems, reliability constraints, and improving the model for water inflows.

It is out of the scope here to reference all of these publications. Instead the comprehensive review [28] as well as the introduction of [29] can be used to dig deeper into this early research.

(Stochastic) dual dynamic programming

Stochastic dual dynamic programming (SDDP) is a DP algorithm which, based on multi-stage stochastic Benders' decomposition, approximates some of the problem's elements. Therefore it belongs to the approximate dynamic programming (ADP) algorithms.

Again, research in the field of ADP is almost as old as the one in DP. In line with the increase of computing power, it gained a lot of attention in the 90's of the last century, focusing on implementable algorithms. The book [30] of W.B. Powell as well as [26] of D.P. Bertsekas describe many of these algorithms.

SDDP itself was proposed by J.R. Birge [31] and applied by M. Pereira and L. Pinto [32, 33]. An approach with the same expression was also published in [34] as well as in [35]. However, the former algorithm is more suitable for higher dimensional problems that is why it became popular for applications on multi reservoir scheduling problems.

Only a few years ago, SDDP was analyzed both mathematically as well as from the computational point of view in more detail. In [36–39] techniques to improve the performance of SDDP are shown. A. Shapiro discusses in [40] convergence and statistical properties of SDDP. A review about applications of SDDP in the Nordic countries is given by A. Gjelsvik et al. [41], the same for Brazil in [42], and for New Zealand in [43].

3.2 Multistage stochastic programs

In literature, multistage stochastic programming is usually approached by first analyzing single and then two-stage programs, before going to the multistage ones. For the sake of brevity, only multistage programs are discussed here.

When formulating a multistage program, variables and constraints are divided into groups corresponding to stages. The term *stage* most often relates to time periods, as it is also the case for this thesis, but it can be used differently.

In each stage, a decision has to be taken, after which some previously unknown data is revealed. This is repeated several times, thus the term *multistage* programming. Let x_t be a decision vector corresponding to stage $t = 1, \dots, T$ and ξ_t the observation at this stage. The course of a multistage stochastic programs then looks as follows:

$$x_1, \xi_1, x_2, \dots, \xi_{T-1}, x_T$$

ξ_t are denoted as the data, which become known at stage t and are possibly unknown before. Hence $\xi_{[1,t]} = \xi_1, \dots, \xi_t$ is the information available up to time t . Finally ξ_1, \dots, ξ_T is called a *random* or *stochastic process*.

The decisions $x(t)$ are often rearranged and separated into two types. The first one are decisions at stage t , which have to be done under uncertainty regarding the next data observation ξ_t . They are called *here-and-now* decisions. The second type are decisions, which can react on the realized data so they are aware of it. However, they still belong to stage t and not $t + 1$. Those decisions are called *wait-and-see* or *recourse* decisions. Whereas mathematically this construction would not be needed, it can be very meaningful from the modeling perspective.

In stochastic multistage programs, it is very important to respect the flow of information, i.e. that decisions depend at most only on $\xi_{[1,t]}$, therefore act *non-anticipative*. Practically this can be achieved either by introducing additional equality constraints (to force decisions to be equal for identical history), or by setting up the problem with the help of scenario trees.

In order to formulate stochastic multistage programs correctly, typically a filtration \mathcal{F} of σ -algebras is defined. This construction allows a notation where variables depend only on already disclosed information, i.e. on random variables which are measurable for some filtration. In this thesis this notation is avoided by formulating such problems for sampled random data in scenario trees.

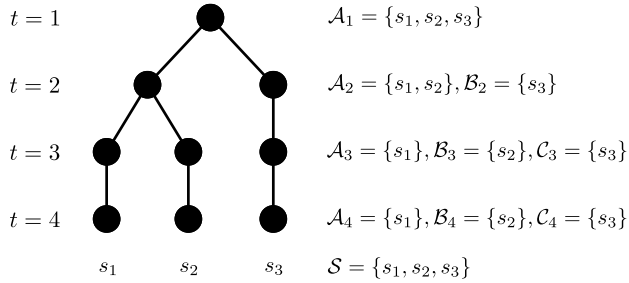


Figure 3.1: Example 3.1: Mathematical notation for scenario trees.

Scenario trees

A scenario tree contains the information about a random data process, i.e. at which stage which information is disclosed. A scenario is one possible realization path of this information for the whole time horizon, i.e. possible sequences of data observations $\xi_{[1,T]}$.

Let the set of all scenarios s be \mathcal{S} and consider bundles $\mathcal{A}_t, \mathcal{B}_t, \mathcal{C}_t, \dots \subseteq \mathcal{S}$ as a subset of \mathcal{S} with the same decisions up to some stage t . Then let Λ_t be the set of all bundles in a stage t and therefore $\mathcal{A}_t \in \Lambda_t$. Further let the set of bundles $U(\mathcal{A}_t)$ be:

$$U(\mathcal{A}_t) = \{\mathcal{B} \in \Lambda_{t+1} \mid \mathcal{B} \subseteq \mathcal{A}_t\}$$

With these definitions scenario trees can mathematically be described in an appropriate and well defined way.

Example 3.1. Scenario tree: Consider figure 3.1: The cardinality of \mathcal{S} is 3, which means there are 3 different scenarios s_1, s_2, s_3 . Λ_2 consists of two bundles: $\Lambda_2 = \{\{s_1, s_2\}, \{s_3\}\}$. Consider now one of these bundles, $\mathcal{A}_2 = \{s_1, s_2\}$. $U(\mathcal{A}_2)$ then denotes the children of \mathcal{A}_2 , the set of bundles $U(\mathcal{A}_2) = \{\{s_1\}, \{s_2\}\}$. ■

Modeling of the (random) data process

The modeling of data processes is a topic by itself. It is out of the scope here to describe and evaluate methods in this respect. Nevertheless,

since this modeling is crucial for the success of stochastic programs a few important references are given.

A data process is usually modeled by using historical observations of it or of other processes which influence it. So one can fit some function or process simulation model (e.g. a fundamental model) to these observations. More information on this can be found in [44, chapter 2,3].

The resulting model is called the probability model, which can be considered also ambiguously. The probability model can typically not be used directly in an optimization model. Therefore a scenario model (e.g. a scenario tree) tries to represent the probability model in a tractable way.

The book by G. Pflug and A. Pichler [45] describes some recent findings about how to construct scenario trees, e.g. by minimizing a nested Wasserstein distance. An alternative is to use reduction techniques for already built but large scenario trees [46, 47], e.g. based on the *expected value of perfect information*, or to expand a simple tree [48, 49], e.g. by the contamination method.

Solving stochastic multistage programs

Stochastic multistage programs can be very difficult to solve. One way to solve them is to formulate their so-called *deterministic equivalent*: The random data process is first sampled to finitely many outcomes per stage. Then a mathematical program is formulated. The objective function maximizes or minimizes all future decisions for all possible outcomes of the random variables, weighted by their conditional probabilities and costs. The resulting problem can then often be solved by commercially available solvers, e.g. for linear problems based on the simplex or interior point solution algorithms.

This approach is very powerful for many stochastic programs. However, the size and complexity of problems formulated as deterministic equivalents grow exponentially with the number of stages. This makes this approach applicable only for a modest number of it.

Depending on the type of stochastic program, other methods can be more efficient. E.g. in [18] different specialized solvers are presented for the class of stochastic linear programs.

Example 3.2. Deterministic equivalent: In this example the scheduling problem of the simple hydro power plant in figure 3.4 is

formulated as its deterministic equivalent. The formulation is made with the help of scenario trees. The notation follows the application example at page 42.

$$\begin{aligned} \max \quad & c_1(u_1 - p_1) + \sum_{t=2}^T \sum_{\mathcal{A}_t \in \Lambda_t} \rho_{\mathcal{A}_t} c_t(u_{t,\mathcal{A}_t} - p_{t,\mathcal{A}_t}) \\ \text{s.t.} \quad & \left\{ \begin{array}{l} v_{t,\mathcal{B}_t}^1 = v_{t-1,\mathcal{A}_{t-1}}^1 - u_{t-1,\mathcal{A}_{t-1}} + p_{t-1,\mathcal{A}_{t-1}} \\ \quad - o_{t-1,\mathcal{A}_{t-1}}^1 + a_{t-1,\mathcal{A}_{t-1}}^1 \quad \forall t \in 2, \dots, T, \forall \mathcal{A}_{t-1} \in \Lambda_{t-1}, \forall \mathcal{B}_t \in U(\mathcal{A}_{t-1}) \\ v_{t,\mathcal{B}_t}^2 = v_{t-1,\mathcal{A}_{t-1}}^2 - o_{t-1,\mathcal{A}_{t-1}}^2 + a_{t-1,\mathcal{A}_{t-1}}^2 \\ \quad \forall t \in 2, \dots, T, \forall \mathcal{A}_{t-1} \in \Lambda_{t-1}, \forall \mathcal{B}_t \in U(\mathcal{A}_{t-1}) \\ \text{lb}_t \leq u, v, o, a \leq \text{ub}_t \\ u, v, o, a \in \mathbb{R} \end{array} \right. \end{aligned} \quad (3.1)$$

Note, how the conditional probabilities of a bundle $\rho_{\mathcal{A}_t}$ enter the objective function. This allows the weighted summation of all energy generation decisions. The only constraint is the water balance constraint (see page 42 for more details). Since the variables (and also an actual implementation) are based on nodes in a scenario tree, there is no need for additional non-anticipatry constraints. ■

3.3 Dynamic programming (DP)

Dynamic programming (DP) problems can be seen as sequential decision problems. If a problem is modeled as a DP, powerful solving tools can be applied. Those solvers typically exploit the characteristic structure of a DP in order to find solutions more time and memory efficiently. One idea is to break down the original problem into simpler subproblems. It is then often the case that solving the simpler subproblems can be performed more efficiently than solving the original complex problem.

Bellman's equation: a path to find optimal decisions

Sequential decision problems inherently have a decomposable structure, since the problem to find one of the decisions can be formulated as a subproblem. In order to find the optimal decisions, there are several different strategies. One way of doing this is expressed in the well

known *Bellman's equation* [24]. In mathematical programming, it is formulated as follows (for a maximization problem as well as for explicit state variables):

$$Q_t(z_{t-1}) = \max_{x_t, z_t} \phi_t(x_t, z_{t-1}) + Q_{t+1}(z_t) \quad (3.2)$$

$$\text{s.t.} : \begin{cases} B_t z_{t-1} + A_t \begin{bmatrix} x_t \\ z_t \end{bmatrix} = b_t \\ lb_t \leq z_{t-1}, z_t, x_t \leq ub_t, z_{t-1}, z_t, x_t \in \mathbb{R}^n \end{cases}$$

where:

- t : stage (time period);
- z_t : state variables;
- x_t : decision variables: one set of decisions per stage and state;
- $\phi_t(x_t, z_{t-1})$: Contribution function, e.g. immediate revenue return for time step t ;
- B_t, A_t, b_t : System matrices specifying system behavior and state transition; and
- $Q_t(z_{t-1})$: Value function (profit-to-go): total expected future income from time step t to terminal time T .

The equation (3.2) expresses that the value of being in a state z_{t-1} is composed of the immediate return of this state plus its future value. In the context of cost minimization problems this value function $Q_t(z_{t-1})$ is called *cost-to-go*, in the context of profit or revenue maximization it is called *profit-to-go*.

A sequential decision problem can be formulated in the form of (3.2), if two conditions are satisfied. First, the objective function has to be separable (e.g. by introduction of state variables). Second, the value functions ϕ are monotonically nondecreasing in time, which is fulfilled in most practical problems. Note that for the hydro power planning problem both conditions are fulfilled.

If the contribution function ϕ returns a monetary value, then the value function is discounted by the risk-free interest rate. In this thesis this is omitted in the formulations.

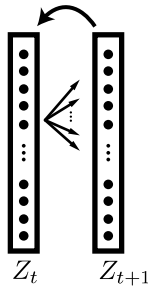


Figure 3.2: Dynamic program as finding the shortest path from nodes (discretized state points) at t to $t + 1$.

If one can formulate the problem in the form of (3.2), optimal decisions can be found recursively, i.e. backwardly in time. This was expressed by R. Bellman in the following way:

“An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.”

This quote introduces the terminology in DP¹:

- *State variables* describe the state of the system for a given time point. A state is the (minimum) information one needs to know at some time in order to be able to make the next decision.
- *Decision variables* are the variables which are under one’s control. They are also called actions or control variables.
- *Policies* are a set of decisions. The typical goal is to find the (optimal) policy which leads to a maximized or minimized objective value.

Bellman’s (somewhat obvious) principle allows the problem to be solved recursively for each subproblem, which can be advantageous over solving the whole problem (e.g. as a large mathematical program). The solution

¹See also [30] for a thorough discussion.

Algorithm 1 Dynamic programming (DP)**Require:** Discretization of time $t = 1, 2, \dots, T$ **Require:** Discretization of states space $Z(t)$ **Require:** Profit-to-go function Q_{T+1} // often zero value

```

1: for  $t = T \rightarrow 1$  do
2:   for all  $z_{t-1} \in Z_{t-1}$  do // for all discretized state points
3:      $x_{t,z_{t-1}} \leftarrow \begin{cases} \arg \max_{x_t} \phi_t(x_t, z_{t-1}) + Q_{t+1}(z_t) \\ \text{s.t. } B_t z_{t-1} + A_t \begin{bmatrix} x_t \\ z_t \end{bmatrix} = b_t \end{cases}$  // store decisions
4:      $Q_{t,z_{t-1}} \leftarrow \begin{cases} \max_{x_t, z_t} \phi_t(x_t, z_{t-1}) + Q_{t+1}(z_t) \\ \text{s.t. } B_t z_{t-1} + A_t \begin{bmatrix} x_t \\ z_t \end{bmatrix} = b_t \end{cases}$  // store profit-to-go
5:   end for
6: end for

```

algorithm is similar to finding the shortest path between nodes (see figure 3.2). For being able to do so, the state of the system has to be discretized to finitely many elements for each also discretized time period.

The shown algorithm 1 is one possible implementation. Going backwards in time from terminal time point T , all discretized state points z_{t-1} are visited. Given a state point a subproblem is formulated in order to find optimal sub-decisions and objective values. This optimization can be solved in many ways. One simple possibility is by discretizing also the decision space and to test all of them.

The optimal decisions $x_{t,z_{t-1}}$ and values $Q_{t,z_{t-1}}$ are stored in a look-up table. Then, for a given starting state z_0 , the optimal policy can be inferred out of this table.

3.4 Stochastic dynamic programming (SDP)

The power of DP is revealed fully only for sequential decision problems under uncertainty. The formulation and the algorithm of stochastic dynamic programming (SDP) is similar to what was presented beforehand. Let $\xi_t = (\phi_t, A_t, B_t, b_t)$ be the data, which is possibly random and gets revealed at time point t (as before) and Ξ_t be the set of all possible

real-valued outcomes of the data so that $\xi_t \in \Xi_t$. Then the recursive optimality equation is as follows:

$$Q_t(z_{t-1}) = \max_{x_t, z_t} \mathbb{E}_{\xi_t \in \Xi_t} [\phi_t(x_t, z_{t-1}) + Q_{t+1}(z_t)] \quad (3.3)$$

$$\text{s.t.:} \begin{cases} B_t z_{t-1} + A_t \begin{bmatrix} x_t \\ z_t \end{bmatrix} = b_t \\ b_t \leq z_{t-1}, z_t, x_t \leq ub_{t, z_{t-1}, z_t, x_t} \in \mathbb{R}^n \end{cases}$$

In contrast to (3.2), the problem is maximized for the expected future profit-to-go. The most optimal policy will therefore maximize the objective on average. If the set Ξ_t has only one element $\forall t$, then the problem coincides with the deterministic case.

Note that the stochastic data process ξ_1, \dots, ξ_T is Markovian. This is the reason why such problems are also called *Markov decision processes*. In other words, the problem has to be stage-wise independent, which is made possible with the introduction of state variables.

The expected value operator $\mathbb{E}[\cdot]$ is linear. Suppose now that the problem would be also linear in the random elements of the data vector ξ . The expectation can then be moved into the contribution and state transition function. The problem would be reduced to the deterministic one (3.2) for the expected data vector $\mathbb{E}[\xi]$. So instead of computing a supposedly difficult expectation over a function one would only need to compute it over a random vector.

However, realistic problems are seldom linear on the random data process and that's why a stochastic viewpoint can be beneficial (see also example 3.3 on page 47).

Sample average approximation (SAA) problem

As for the algorithm for deterministic DP, the time and state variables have to be discretized in order to apply the ordinary SDP algorithm. Similarly, the possible random data outcome space Ξ does not necessarily have to be discretized. However, practically, it is often difficult to estimate the expectation with respect to the random data, if Ξ is not sampled.

Suppose from now on sampled random data, i.e. N_t number of samples $\xi_t^1, \dots, \xi_t^{N_t}$ of the data process ξ_t are generated at stage t , and let Ξ_t be the set of these samples so that $\Xi_t = \{\xi_t^1, \dots, \xi_t^{N_t}\}$. A problem, where

the data process is approximated in such a way, is called *sample average approximation (SAA)* problem.

The generated samples are typically independent within each time stage i.e. they are independent and identically distributed random variables. However, they can be conditional on samples at the previous time stage. Nevertheless, in our case, we consider a stage-wise independent process. Note that the total amount of scenarios in a SAA problem is $N = \prod_{t=1}^T N_t$ and can thus be very large.

Algorithm for solving stochastic dynamic programs

Consider a SAA be given. The stochastic optimality equation (3.3) can now be reformulated, with the expectation operator being simply averaging:

$$Q_t(z_{t-1}) = \max_{x_t} \frac{1}{N_t} \sum_{j=1}^{N_t} \left[\phi_t^j(x_t, z_{t-1}) + Q_{t+1}(z_t^j) \right] \quad (3.4)$$

It can still be computationally expensive to calculate this maximization problem. Therefore the decision space $X(t)$ is also often discretized. The problem can then be solved by the algorithm 2 by going through all combinations of time, state, decision, and sampled random data points and calculate the associated optimal values $Q_{t,z_{t-1},x_t,j}$. In line 7 of the algorithm the expected value over all possible random data realizations define the value for a given decision. By taking the maximum value of these values $Q_{t,z_{t-1}}(x_t), \forall x_t \in X_t$, the optimal value and associated decision, given a state point z_{t-1} , is found.

Note that because of the nested “for loops”, a possibly enormous amount of subproblems in line 5 have to be solved. These subproblems are usually very small, since state, decision, and sampled data are given. Further note that for such an implementation decisions which depend on realized data, the wait-and-see decisions, can not be considered.

Forward simulation

When the decision space is discretized, the application of the found optimal policy may be difficult. Consider for instance a situation, where the system happens to be between states. Whereas a profit-to-go value could be estimated by interpolation, the optimal decision is very difficult to find.

Algorithm 2 Stochastic dynamic programming (SDP)

Require: Discretization of time $t = 1, 2, \dots, T$ **Require:** Discretization of state space $Z(t)$ **Require:** Discretization of decision space $X(t)$ **Require:** SAA // Sampling of data $\xi(t) = (\phi, A, B, b) \in \Xi(t)$ **Require:** Profit-to-go function Q_{T+1} // often zero value

```

1: for  $t = T \rightarrow 1$  do
2:   for all  $z_{t-1} \in Z_{t-1}$  do // for all discretized state points
3:     for all  $x_t \in X_t$  do // for all discretized decision points
4:       for  $j = 1 \rightarrow N_t$  do // for each sampled data element  $\xi_t^j \in \Xi_t$ 
5:          $Q_{t,z_{t-1},x_t,j} = \begin{cases} \max_{x_t,z_t} \phi_t^j(x_t, z_{t-1}) + Q_{t+1}(z_t) \\ \text{s.t. } B_t z_{t-1} + A_t^j \begin{bmatrix} x_t \\ z_t \end{bmatrix} = b_t^j \end{cases}$ 
6:       end for
7:        $Q_{t,z_{t-1},x_t} \leftarrow \frac{1}{N_t} \sum_{j=1}^{N_t} Q_{t,z_{t-1},x_t,j}$  // store profit, given decision
8:     end for
9:      $x_{t,z_{t-1}} \leftarrow \arg \max_{\forall x_t \in X_t} Q_{t,z_{t-1}}(x_t)$  // store decisions
10:     $Q_{t,z_{t-1}} \leftarrow \max_{\forall x_t \in X_t} Q_{t,z_{t-1}}(x_t)$  // store profit-to-go
11:   end for
12: end for

```

This issue can be addressed by performing a *forward simulation* as in algorithm 3. There, for a given scenario of the data process, optimal decisions are computed by going forward in time while a continuous decision space is considered.

Note that the model used in this simulation does not necessarily have to be identical to the one used in the SDP algorithm, but should preferably be more detailed.

Such a forward simulation can be repeated for different scenarios and thus a Monte Carlo simulation study is performed. This procedure results in a distribution of values of the policy. This distribution can be used to show the effectiveness of the policy, as well as for risk analysis.

Link to approximate dynamic programming (ADP)

In the field of approximate dynamic programming (ADP), the objective function in (3.3) is usually formulated differently using, what is called,

Algorithm 3 Forward Simulation

Require: Discretization of time $t = 1, 2, \dots, T$
Require: Discretization of state space $z(t)$
Require: Sampled data process $\xi(t) = (\phi, A, B, b)$ // only one scenario
Require: Profit-to-go function $Q(t, z(t))$ // look-up table or analytical function
Require: Initial state z_0
Require: Initial value of the optimal policy v_0

1: **for** $t = 1 \rightarrow T$ **do**
2: $\{x_t, z_t\} \leftarrow \begin{cases} \arg \max_{x_t, z_t} \phi_t(x_t, z_{t-1}) + Q_{t+1}(z_t) \\ \text{s.t. } B_t z_{t-1} + A_t \begin{bmatrix} x_t \\ z_t \end{bmatrix} = b_t \end{cases}$
3: $v_t = v_{t-1} + \phi_t(x_t, z_{t-1})$
4: **end for**

the *post-decision state* z' . This state describes the state of the system after a decision was taken, but before uncertainty occurs.

Consider now that the conditional probability of the state transition $\mathbb{P}(z_t|z'_t)$ is given. Equation (3.3) can now be formulated equivalently as:

$$Q_t(z_{t-1}) = \max_{x_t, z_t} \mathbb{E}_{\xi_t \in \Xi_t} \left[\phi_t(x_t, z_{t-1}) + \sum_{z_t \in Z_t} \mathbb{P}(z_t|z'_t) Q_{t+1}(z_t) \right] \quad (3.5)$$

Let $Q'_t(z'_t)$ be the function of the value being in the post-decision states. Then we obtain:

$$\begin{aligned} Q'_t(z'_t) &= \sum_{z_t \in Z_t} \mathbb{P}(z_t|z'_t) Q_{t+1}(z_t) \\ &= \sum_{z_t \in Z_t} \mathbb{P}(z_t|z'_t) \max_{x_{t+1}, z_{t+1}} \mathbb{E}_{\xi_{t+1} \in \Xi_{t+1}} \left[\phi_{t+1}(x_{t+1}, z_t) + Q'_{t+1}(z'_{t+1}) \right] \end{aligned} \quad (3.6)$$

Equations (3.5) and (3.6) are mathematically equivalent. The latter can lead to a computational advantage, since the expectation operator associated with the state transition is now outside of the maximization problem.

Note that the state transition can depend on the actual decision taken beforehand. In this case, the decision has to be part of the post-decision state vector. This leads to the problem which was formulated earlier in

(3.4) for discretized decision spaces. The post-decision value function $Q'_t(z'_t)$ would in this case have been calculated in algorithm 2, line 7. However, the way of thinking in ADP can be meaningful for processes where the state transition and its probability are important.

Usability of dynamic programming algorithms

Most practical (decision) problems can be formulated as a DP. This allows a much more efficient implementation compared with a general multistage stochastic program. The main advantage is that the complexity scales linearly with the amount of time steps.

Solving DPs requires the solution of many subproblems. These subproblems are independent of each other (for one time point). So DPs are, what the computer science community calls, *embarrassingly parallel* problems. Such problems are promising candidates to make use of high performance computing e.g. parallelized computation across CPUs and even GPUs.

DP also has its downsides. The most severe one are the so-called *curses of dimensionality*: The complexity (number of subproblems to solve) increases exponentially with the number of discretizations of

- state space Z ,
- decision space X , and
- outcome space of (random) realized data Ξ .

Typically, fine discretizations of all of them are needed for an appropriate model. Mathematically it can be shown (see also [19, section 5.8]): If a problem with continuous distributions is approximated by an SAA, then the number of samples required is large (tend to go to infinity) in order to approximate it reasonably well.

On the other hand, in practice, these theoretical findings are often not that relevant. This is because the original problem itself is the result of many simplifications and often arbitrary modeling decisions. Therefore, it can be more interesting to find an implementable and transparent solution with reasonable effort rather than solving the original problem as exactly as possible.

Some of the curses of dimensionality can be mitigated as discussed earlier. The discretization of the decision space can be avoided if the

subproblems can be formulated as a mathematical program, e.g. as a mathematical program. Then the number of subproblems to solve can be reduced significantly, however, at the cost that these are more complex (see also example 3.3 on page 46).

The same is also true for the outcome space of the data, whereas the discretization of the state space typically is necessary.

3.5 Stochastic dual dynamic programming (SDDP)

As mentioned in the previous section, SDP suffers from the curses of dimensionality. Stochastic dual dynamic programming (SDDP) is a method that tries to overcome one of these curses, i.e. the reduction of the computational complexity for larger state spaces. The basic idea is to find approximations for the valuing function $Q(t, z)$ in such a way that not every discretized state combination is needed to pass. SDDP therefore belongs to the class of ADP algorithms.

In SDDP, these approximations are done by a collection of linear hyperplanes, forming an polyhedral outer approximation of the value function. For a profit-to-go function $Q_t(z_{t-1})$ being concave (or cost-to-go function convex) with respect to the state space $z(t)$, each hyperplane is an upper (lower) bound to $Q_t(z_{t-1})$. The hyperplanes are close to the actual unknown value function only near the states which were visited to construct them (see also figure 3.3). Therefore, it is necessary to find the states which correspond to the optimal policy.

Such states are found in SDDP by performing a forward simulation, similar to the already presented algorithm 3. In this forward simulation, only an approximation of the value function can be used. Therefore, the states found there are called *trial states* since they are not necessarily the ones corresponding to the optimal policy.

As stated earlier, reasonably sized SAA are often too large to be solved exactly. The second advantage of SDDP is that it inherently allows to find feasible policies for possibly much more difficult models with e.g. continuous distributions.

SDDP is an iterative algorithm, where the approximation of the value function is improved after each iteration. Unlike many other algorithms, however, the accuracy can be checked against both an upper and lower

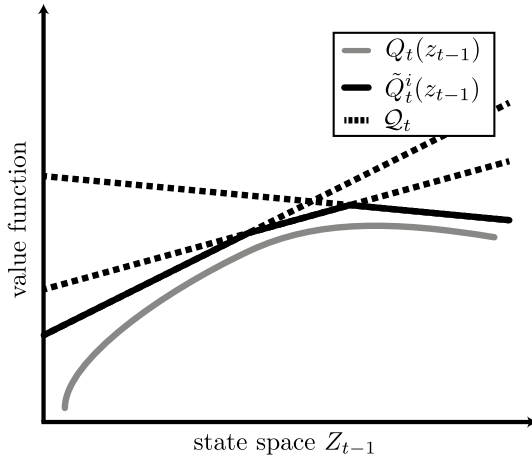


Figure 3.3: Approximation $\tilde{Q}_t^i(z_{t-1})$ of the (unknown) value function $Q_t(z_{t-1})$ by the minimum of a collection of hyperplanes Q_t for a one dimensional state space Z_{t-1}

bound (under some conditions even for the original problem), which could be considered as its third advantage.

The SDDP algorithm is best described if it is separated into two parts, the approximation of the value function, i.e. the backward step, and the searching of the trial states, i.e. the forward step. In order to formulate the algorithm mathematically, many different variables and indices are necessary. The notation used here follows basically the one in [50]. A possible implementation of SDDP is given in algorithm 4.

For the demonstration of the SDDP algorithm the problem is first assumed to be stage-wise independent with continuous distributions given for random variables. The problem is then sampled as a SAA. Further, the recursive optimality equation shall be as follows:

$$\begin{aligned}
 Q_t(z_{t-1}) &= \frac{1}{N_t} \sum_{j=1}^{N_t} \left[\max_{x_t^j, z_t^j} \phi_t^j(x_t^j, z_{t-1}) + Q_{t+1}(z_t^j) \right] & (3.7) \\
 \text{s.t.:} & \begin{cases} B_t^j z_{t-1} + A_t^j \begin{bmatrix} x_t^j \\ z_t^j \end{bmatrix} = b_t^j \\ lb_t \leq z_{t-1}, z_t^j, x_t^j \leq ub_t, z_{t-1}, z_t^j, x_t^j \in \mathbb{R}^n \end{cases}
 \end{aligned}$$

Note that in contrast to (3.4) the expectation operator is now outside of the maximization problem. This means that the decisions x_t^j are wait-and-see decisions. Such a formulation is more streamlined. Here-and-now decisions can be considered in SDDP by its discretization similarly as it was done in algorithm 2 (see also the discussion about multi-cut SDDP in the next chapter 4).

Backward step: the outer approximation

In SDDP, finding approximations for the valuing function is called *backward step* or *outer approximation*.

Assume the i -th iteration of the SDDP algorithm. Suppose, that *trial states* $\tilde{z}_t^k, t = 0, \dots, T-1, k = 1, \dots, M$ are given, for each time stage M different trials.

The profit-to-go function $Q_t(z_{t-1})$ is approximated by the minimum of a collection of affine hyperplanes, also called cutting, supporting planes or Benders' cuts. Let \mathcal{Q}_t be this collection and $\tilde{Q}_t^i(z_{t-1})$ the approximation of the value function $Q_t(z_{t-1})$. In each iteration i of the algorithm, M cutting planes with intercepts γ_t^k and vectors of slopes δ_t^k are added to the collection \mathcal{Q}_t , which refines the approximation of the value function $\tilde{Q}_t^i(z_{t-1})$. So there is:

$$\tilde{Q}_t^i(z_{t-1}) = \min \mathcal{Q}_t = \min_{k=1, \dots, i \cdot M} \gamma_t^k + \delta_t^k z_{t-1} \quad (3.8)$$

In order to construct the intercept γ_t^k of a cutting plane the recursive optimality equation (3.7) is solved, however, only for a given trial state \tilde{z}_{t-1}^k :

$$\begin{aligned} \gamma_t^k &= \frac{1}{N_t} \sum_{j=1}^{N_t} \left[\max_{x_t^j, z_t^j} \phi_t^j(x_t^j, \tilde{z}_{t-1}^k) + \tilde{Q}_{t+1}^j(z_t^j) \right] \\ \text{s.t.:} &\begin{cases} B_t^j \tilde{z}_{t-1}^k + A_t^j \begin{bmatrix} x_t^j \\ z_t^j \end{bmatrix} = b_t^j \\ lb_t \leq z_t^j, x_t^j \leq ub_t, z_t^j, x_t^j \in \mathbb{R}^n \end{cases} \end{aligned} \quad (3.9)$$

Because the approximation of the value function $\tilde{Q}_{t+1}^i(z_t)$ is given as a minimum of a set of affine functions, one can insert (3.8) directly into

(3.9):

$$\begin{aligned} \gamma_t^k &= \frac{1}{N_t} \sum_{j=1}^{N_t} \left[\max_{x_t^j, z_t^j} \phi_t^j(x_t^j, \tilde{z}_{t-1}^k) + \theta \right] \\ \text{s.t.:} &\begin{cases} \theta \leq \gamma_{t+1}^l + \delta_{t+1}^l z_t^j, \forall l=1, \dots, (i-1) \cdot M \\ B_t^j z_{t-1}^k + A_t^j \begin{bmatrix} x_t^j \\ z_t^j \end{bmatrix} = b_t^j & [\pi_{t,k}^j] \\ b_t \leq z_t^j, x_t^j \leq ub_t, z_t^j, x_t^j \in \mathbb{R}^n \end{cases} \end{aligned} \quad (3.10)$$

Consider now $\pi_{t,k}^j$ as the dual variables of the complicating constraints in (3.10), i.e. the ones which specify the state transitions. Qualitatively, these variables are measures of the sensitivities of the profit-to-go value for varying states. The vector of slopes δ_t^k of a cutting plane can be found as the subgradient of \tilde{Q}_t^j at the point \tilde{z}_{t-1}^k . It is calculated as follows:

$$\delta_t^k = -\frac{1}{N_t} \sum_{j=1}^{N_t} (B_t^j)^T \pi_{t,k}^j \quad (3.11)$$

Thus, in the backward step, for each trial state a supporting hyperplane is added to the collection \mathcal{Q}_t . Therefore, the subproblems (3.10) grow in size after each iteration of the algorithm.

Note that by the dualization of the state transition constraints one applies a nested Benders' decomposition (also called L-shaped method). That is why the supporting hyperplanes are also called Benders' cuts. Since SDDP solves a multi-stage problem, the method is also known as a multistage nested Benders' decomposition algorithm.

Upper bound

Since the approximated value function $\tilde{Q}^i(t, z)$ is the minimum of a collection of cutting planes of the value function $Q(t, z)$, it is also an upper bound of it. At time point $t = 1$ the approximated value function at the initial state $\tilde{Q}_1^i(z_0)$ is therefore an upper bound \bar{r} to the optimal value of the SAA:

$$\bar{r} = \min \mathcal{Q}_1 = \tilde{Q}_1^i(z_0) \quad (3.12)$$

This bound is deterministic, i.e. it is not based on sampling. Since the optimal value of the original problem with continuous distributions of the random variables is lower or equal to the one from the SAA, \bar{r} is also an upper bound (on average) to the original problem.

Forward step: the inner approximation

The forward step is sometimes also called forward simulation or *inner approximation*. It serves primarily two purposes: to find trial states as well as to calculate a lower bound.

Consider approximations $\mathcal{Q}_t, t = 1, \dots, T$ as well as the initial state $\tilde{z}_0^k = z_0, k = 1, \dots, M$ be given. In the forward step, first one realization of the data process, a scenario $\xi^k = \xi_1^k, \dots, \xi_T^k$, is sampled or chosen. After that, the trial states for the backward step $\{\tilde{z}_1^k, \dots, \tilde{z}_{T-1}^k\}$ can be found by going forward as follows:

$$\begin{aligned} \{\tilde{x}_t^k, \tilde{z}_t^k\} &= \arg \max_{\tilde{x}_t^k, \tilde{z}_t^k} \phi_t^k(\tilde{x}_t^k, \tilde{z}_{t-1}^k) + \theta & (3.13) \\ \text{s.t.:} & \begin{cases} \theta \leq \gamma_{t+1}^l + \delta_{t+1}^l \tilde{z}_t^k, \forall l=1, \dots, (i-1) \cdot M \\ B_t^k \tilde{z}_{t-1}^k + A_t^k \begin{bmatrix} \tilde{x}_t^k \\ \tilde{z}_t^k \end{bmatrix} = b_t^k \\ lb_t \leq \tilde{z}_t^k, \tilde{x}_t^k \leq ub_t, \tilde{z}_t^k, \tilde{x}_t^k \in \mathbb{R}^n \end{cases} \end{aligned}$$

The forward step can be repeated by taking another scenario, resulting in an additional set $\{\tilde{x}^k, \tilde{z}^k\}$, because the policy depends on the realizations of the random data.

A policy $\{\tilde{x}_1^k, \dots, \tilde{x}_T^k\}$ is non-anticipative, feasible, and implementable in respect with the chosen scenario. If the sampling is restricted to scenarios from the SAA, then the policy becomes feasible and implementable for the SAA (similar to policies found by SDP). However, if the samples are drawn from distributions from the original problem then the policy is feasible and implementable for it and not only for the SAA.

Lower bound

Given the policies for equally probable scenarios, a lower bound for the optimal value can be found with the following:

$$\underline{r} = \frac{1}{M} \sum_{k=1}^M \sum_{t=1}^T \phi_t^k(\tilde{x}_t^k, \tilde{z}_{t-1}^k) \quad (3.14)$$

Note that \underline{r} is a statistical bound to the SAA or original problem, depending on which distributions the scenarios were constructed from. The bound is stochastic and is a function of the considered scenarios.

Therefore, it can vary from one iteration to the next.

Additionally, with the central limit theorem one can show that, provided that the number of scenarios M is large enough, the returns $\sum_{t=1}^T \phi_t^k(\tilde{x}_t^k, \tilde{z}_{t-1}^k)$ are approximately normally distributed. Consequently one can construct a $(1 - \alpha)$ -confidence lower bound:

$$r_{\alpha, M} = \underline{r} + cdf^{-1}(1 - \alpha) \frac{var(\sum_{t=1}^T \phi_t^k(\tilde{x}_t^k, \tilde{z}_{t-1}^k))}{\sqrt{M}} \quad (3.15)$$

where $(1 - \alpha)$ is the confidence level, cdf the cumulated distribution function of the standard normal distribution, and var the variance of the returns.

Usability of the SDDP algorithm

SDDP is an algorithm, which can be applied to problems where SDP is computationally intractable. Typically for smaller problems (one or two state variables) SDP outperforms SDDP computationally, whereas for problems with a few more states, SDDP can be more efficient. Nevertheless, SDDP is not an algorithm for dealing with problems with high-dimensional states (say more than 20 states). For such problems, other algorithms are better suited.

In SDDP, good performance requires parameter tuning. There is a trade-off between solving time per iteration and conversion rate.

Consider algorithm 4. For a fast solving time, the number of samples M should be as small as possible. On the other hand, the more trial states, the higher the chance for an improvement of the approximation of the value function. Therefore, it can be a good idea that for earlier time stages less trial states are used. Then, in later time stages, more trial states are considered because of the higher uncertainty about the location of the state points with respect to the optimal policy.

Similar to SDP, parallel processing can be used in SDDP. The calculation of cutting planes per trial state and/or the calculation of it for each sampled realization can be performed in parallel.

In each iteration of the algorithm, new cutting planes are added to both the forward as well as the backward step subproblems, leading to higher computational complexity per iteration. Therefore, the quality of each cut is often measured in some way in order to be able to consider only the best cuts.

Algorithm 4 Stochastic dual dynamic programming

Require: Discretization of time $t = 1, 2, \dots, T$ and SAA

Require: Profit-to-go function Q_{T+1} // often zero value

Require: Initial state z_0

Require: $\epsilon > 0$ // maximum allowed optimality gap

Require: $Q_t, t = 1, \dots, T$ // initial outer approximation

1: Initialize: $\underline{r} = -\infty, \bar{r} = \infty, i \leftarrow 1$ // lower/upper bound, iteration counter

2: **while** $\bar{r} - \underline{r} \geq \epsilon$ **do**

Sampling:

3: Choose or sample M scenarios ξ^1, \dots, ξ^M

Forward step:

4: **for** $k = 1 \rightarrow M$ **do**

5: **for** $t = 1 \rightarrow T$ **do**

6:
$$\{\bar{x}_t^k, \bar{z}_t^k\} \leftarrow \begin{cases} \arg \max_{\bar{x}_t^k, \bar{z}_t^k} \phi_t^k(\bar{x}_t^k, \bar{z}_{t-1}^k) + \theta \\ \text{s.t.} \begin{cases} \theta \leq \gamma_{t+1}^l + \delta_{t+1}^l \bar{z}_t^k, \forall l=1, \dots, (i-1) \cdot M \\ B_t^k \bar{z}_{t-1}^k + A_t^k \begin{bmatrix} \bar{x}_t^k \\ \bar{z}_t^k \end{bmatrix} = b_t^k \end{cases} \end{cases}$$

7: **end for**

8:
$$\underline{r}^k = \sum_{t=1}^T \phi_t^k(\bar{x}_t^k, \bar{z}_{t-1}^k)$$
 // store return per scenario

9: **end for**

10:
$$\underline{r}_{\alpha, M} \leftarrow \frac{1}{M} \sum_{k=1}^M \underline{r}^k + \text{cdf}^{-1}(1 - \alpha) \frac{\text{var}(\underline{r}^k)}{\sqrt{M}}$$
 // update lower bound

Backward step:

11: **for** $t = T \rightarrow 1$ **do**

12: **for** $k = 1 \rightarrow M$ **do** // for each trial state \bar{z}_{t-1}^k

13: **for** $j = 1 \rightarrow N_t$ **do** // for each sampled data element $\xi_t^j \in \Xi_t$

14:
$$\gamma_{t,k,j} = \begin{cases} \max_{x_t^j, z_t^j} \phi_t^j(x_t^j, \bar{z}_{t-1}^k) + \theta \\ \text{s.t.} \begin{cases} \theta \leq \gamma_{t+1}^l + \delta_{t+1}^l z_t^j, \forall l=1, \dots, (i-1) \cdot M \\ B_t^j z_{t-1}^k + A_t^j \begin{bmatrix} x_t^j \\ z_t^j \end{bmatrix} = b_t^j \quad [\pi_{t,k}^j] \end{cases} \end{cases}$$

15: **end for**

16:
$$\gamma_t^k = \frac{1}{N_t} \sum_{j=1}^{N_t} \gamma_{t,k,j}$$

17:
$$\delta_t^k = -\frac{1}{N_t} \sum_{j=1}^{N_t} (B_t^j)^T \pi_{t,k}^j$$

18:
$$\mathcal{Q}_t \leftarrow \mathcal{Q}_t \cup \{\gamma_t^k, \delta_t^k\}$$
 // update collection of cuts

19: **end for**

20: **end for**

21:
$$\bar{r} \leftarrow \min \mathcal{Q}_1 = \tilde{Q}_1^i(z_0)$$
 // update upper bound

22:
$$i \leftarrow i + 1$$
 // Update iteration counter

23: **end while**

The cut selection can be based on its age: the older a cut is, the less important it is. Another possibility is to choose cuts based on their dominance: if a cut at the state it was constructed is dominated by another cut, it will not be considered (so-called level one dominance).

Typically, the convergence rate of the algorithm decreases with the number of iterations because the approximation of the value functions are not improved any more. The algorithm is then “stuck”. This can be prevented by increasing the number of trial states, where sometimes even random trial states are meaningful.

In the presented algorithm 4, the stop criterion is when the gap between upper and lower bound gets smaller a predefined threshold. In practice, however, the algorithm is often stopped after a fixed number of iterations, or when the upper bound is not improving anymore.

3.6 Alternative methods

Apart from stochastic programming, hydro power planning problems can also be approached with alternative methods. These methods are typically less suited for medium-term hydro power planning (MTHP) problems. Therefore, only some references about them are given.

Deterministic methods

Although, from a theoretical point of view, stochastic methods clearly outperform deterministic ones, in practice this may not be the case. The reason for that is mainly that the computational complexity of deterministic models is much lower than for their stochastic counterparts. Therefore, given a certain amount of computing power, deterministic models can be more detailed with respect to the representation of the system. Typically, one or more of the following complications of a hydro power model are modeled in a more detailed way:

- head variations effects,
- turbine efficiencies,
- network constraints, or

- delayed water flows.

Additionally, it may be very difficult or costly in practice to estimate stochastic processes of random variables. Such models also tend to be more complicated than deterministic ones, which reduces the transparency of the model.

Most stochastic methods can also be formulated deterministically. This is also the case for SDP and SDDP. It would go too far here to introduce and compare all of these methods. In academia, there is a general consensus that models for hydro power planning should be formulated stochastically if it is possible, since the importance of modeling stochasticity is much higher than that of a more detailed representation of a system.²

A counterexample is for instance the Brazilian hydrothermal optimization problem, which is so complex such that the performance of deterministic, but less aggregated models, seems to be comparable with stochastic ones [52–54].

Model predictive control

Model predictive control is a method originally developed for process control. The general idea is to perform an optimization with a receding or moving horizon. The found optimal control strategy, the policy, is applied only for the first time stage and the optimization is repeated. The optimization is typically a deterministic one, but it can also be stochastic.

Such an optimization strategy with receding horizon also mimics the way how operators think in MTHP problems: every day they repeat some optimization and apply the first optimal decision. This procedure only makes sense, if there are updated informations available, otherwise the optimization result would not change from one day to the other.

A possible procedure for a MTHP optimization is that a (deterministic) optimization is performed stage-wise with receding horizon and updated states. So the operation of the plant would be simulated throughout the time horizon.

Note that such a model predictive control based simulation is exactly the same as the forward step in SDDP, where for (updated) states the

²E.g. for a typical Swiss system setup this was analyzed in [51].

optimal policy is sought. In model predictive control, it is not specified how the optimal policy is found. If the optimization is based on a SDP-scheme, then the same policy as in SDDP would be found if the value function is concave. However, note that such a model predictive control based simulation would require the calculation of a SDP algorithm n -times, with n as the number of time stages.

In order to avoid this, offline model predictive control is typically applied, where the policy is calculated for all possible states only once. This scheme is then equivalent to SDP.

For MTHP optimizations, the procedure of solving daily stochastic optimizations, which are triggered manually, seems to be more convenient. Nevertheless, there are a few publications [55, 56], which apply model predictive control approaches to a MTHP optimization.

For short-term optimizations, model predictive control can be of more interests, since faster dynamics are present. For instance, the fulfillment of ancillary services obligations is a good application for such methods.

Decomposition and aggregation

In medium-term hydrothermal problems, the difficulty is usually the number of different power plants which has to be considered. These plants are often only weakly connected to each other, for instance through a common electricity market. These coupling constraints can be relaxed which results in solving individual small subproblems which are connected only through a master problem. Examples for works with such ideas are [57–59].

In [60], another heuristic is proposed, where an interconnected hydro system is extended by discretizing the energy interchange between subsystems. This allows a decomposition of the system.

Whereas academically such approaches can be very interesting, they would suffer in practice from the issue that the methods are based on heuristics, which have to be tuned. Additionally, there is usually not much insight on how good the found policies are.

Temporal or spatial aggregations can be very reasonable for specific problems. This is also the present approach to apply SDP and SDDP to the Brazilian system [42, 61] and Scandinavian system [41].

In order to consider hourly electricity market products in a MTHP problem, the products can be bundled (e.g. in peak and off-peak products), as in [62]. Another idea is to model different time stages lengths: finer

ones in the near term and coarser ones going forward as in [1, 63]. In chapter 6, decomposition and aggregation technics are further discussed and also applied to a MTHP problem.

Monte Carlo regression analyses

The MTHP problem can also be seen as an asset valuation problem. Real option theory can be applied, where a (pumped) hydro power plant can be modeled as a series of path-dependent American options. The general idea in Monte Carlo regression analyses is that the unknown stopping time for the option in a scenario is found by some regression analysis on the stopping times of the other scenarios. The regression is often based on least squares, leading to some variant of the *Longstaff and Schwartz* algorithm [64].

Such algorithms are formulated recursively, but also with a simulation part. Therefore, they are similar to SDDP. The advantage is their scalability in number of scenarios and their flexibility in how uncertainties are modeled. That's why they are well suited for asset valuation problems. On the other hand, these algorithms are computationally troublesome for more complex systems.

Additionally, regression simulation methods partially lack physical interpretations. This is because a *basis function* has to be found, which models the relationship between stopping times throughout scenarios. Typically, this is an affine or quadratic function in order to work well with the regression method. However, whereas the motivation of correlated stopping times can be considered questionable in hydro power plant applications, its modeling by a simple function is even more dubious.

There is more research necessary to compare such methods with more traditional ones like SDDP. In literature, there are a number of works valuing gas storages with Monte Carlo regression methods (e.g. [65]). For hydro power storages there are fewer works, which typically model the plants in a generic way in order to value their performance in different markets or to analyze expansion plans, as e.g. in [66].

Approximate dynamic programming schemes

As previously stated, SDDP belongs to the approximate dynamic programming (ADP) algorithms. ADP is “based on an algorithmic strategy that steps forward through time” [30]. The forward simulation only makes sense if there is some estimation of the value function and/or a good policy. In ADP, there are many different approaches to find them, e.g. by performing a backward optimization as in SDDP. Many of these approaches are based on heuristics and can work well for a specific problem.

From the perspective of a MTHP problem, the mentioned approaches in literature are very similar to the ideas in SDDP. For instance in [67], value functions are constructed by hyperplanes per cluster of states. A cluster of states contains states which are strongly coupled. The value function of the overall system is then found as the linear sum of the value functions of the clusters. Obviously, such a method can work well, when states can be clustered reasonably.

In a similar way in [44, chapter 5], a SDP scheme for decoupled basins is presented. The overall system’s value function is again the linear sum of the decoupled ones. Nevertheless a comparison with SDDP did not show any advantage.

In [68, 69], the convex hull of the future cost-to-go function was used to approximate it instead of Benders’ cuts as in SDDP. However, this approach does not introduce substantial advantages. It not only approximates the value function from the “wrong” side, but its benefit to use less constraints is negligible for problems which are solvable by such methods.

3.7 Application example: Medium-term hydro power planning problem

In this thesis, if not stated otherwise, the medium-term hydro power planning (MTHP) problem is related to finding the optimal medium-term scheduling for a single power plant. In a medium-term perspective, a time horizon of some months up to a few years is considered. The plant belongs to a generation company (GenCo), which sells its production on a deregulated electricity market.

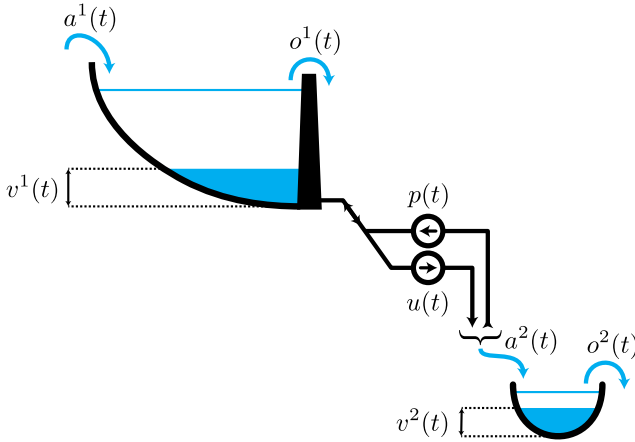


Figure 3.4: Notation for medium-term hydro power planning (MTHP) problems in this thesis.

Notation

When formulating the MTHP problem as a multistage program, variables and constraints are divided into groups corresponding to stages, the time periods. An optimization should find the optimal operating decisions. The notation is as follows (see also figure 3.4):

- State variables $z(t)$:
reservoir fillings: $v(t)$.
- Decision variables $x(t)$:
water release decisions: generation $u(t)$ and pumping $p(t)$,
charges from upstream reservoirs and inflows $\iota(t)$: $a(t)$, and
overflow (spillage): $o(t)$.
- Constraints:
coupling constraints: water balance:
$$v_{t+1} = v_t + a_t - o_t - f_u(u_t) + f_p(p_t).$$
- Objective:

maximized revenue:

$$\max_{u(t), p(t)} c(t)(u(t) - p(t)).$$

The functions $f_u(u)$ and $f_p(p)$ relate used or produced energy in the turbines and pumps to respective water flows. These functions depend on head differences and efficiencies of the machines and can easily be deduced from the energy conservation equations.

Variables for reservoir fillings are not needed in general, however, they make the formulation much more streamlined. The production decisions $u(t), p(t)$ (one for each time period, thus *multistage*) should lead to maximum revenue. For limited water inflows, the question of the hydro scheduling problem is therefore if water should be released now or stored for future use.

Note that in this formulation the time duration of a time stage is not accounted for explicitly. In this thesis it is assumed that the functions $f_u(u), f_p(p)$ as well as the electricity price $c(t)$ are adapted accordingly.

In a medium-term perspective, the sequential decisions have to be made under uncertainty. Stochastic variables taken into account are typically water inflows as well as prices for some markets. DP schemes can now be applied, since the objective function can be separated with the introduction of state variables and since the objective value, the revenue, can only increase in time.

Operation policy: water values

The MTHP optimization should provide operational decision support. Well suited for that are marginal values of basin fillings, i.e. the water values. They are the opportunity costs of the stored water for a basin and depend on time and current filling of all the basins. Typically, water values can be used directly as a policy in short-term optimizations or simulations.

The water values can be calculated out of the value function $Q_t(v_{t-1})$. In SDP-based algorithms, the value function is known only at discrete points. An interpolation between them is used to estimate the full function. The water values then follow from the gradient of it. If the subproblems in the algorithm are formulated as linear programs (LPs), then the dual variables of the water balance constraints give further usable information about the value function.

In SDDP-based algorithms, the cutting planes already include the information about the gradients of the value function. The water values therefore are found by selecting the active cutting plane for a specific filling. The active cut from the set of cutting planes is the one with the least value for the given filling.

Here-and-now and wait-and-see decisions

Decisions about overflow $o(t)$ as well as charges from upstream reservoirs $a(t)$ are usually modeled as wait-and-see decisions. On the other hand, generating and pumping decisions can be modeled as both here-and-now or wait-and-see decisions. The next example 3.3 analyzes the associated differences.

Example 3.3. MTHP problem modeling: First, the problem is modeled with only wait-and-see decisions. The recursive optimality equation for the MTHP problem for the plant in figure 3.4 is then as follows:

$$\begin{aligned}
 Q_t(v_{t-1}^1, v_{t-1}^2) &= \frac{1}{N_t} \sum_{j=1}^{N_t} \left[\theta_t^j(v_{t-1}^1, v_{t-1}^2) \right] & (3.16) \\
 \theta_t^j(v_{t-1}^1, v_{t-1}^2) &= \max_{v_t^j, u_t^j, p_t^j, a_t^j, o_t^j} c_t^j(u_t^j - p_t^j) + Q_{t+1}(v_t^{1,j}, v_t^{2,j}) \\
 \text{s.t.:} & \begin{cases} v_t^{1,j} = v_{t-1}^1 - f_u(u_t^j) + f_p(p_t^j) - o_t^{1,j} + a_t^{1,j} \\ v_t^{2,j} = v_{t-1}^2 - o_t^{2,j} + a_t^{2,j} \\ a_t^{1,j} = \iota_t^{1,j} \\ a_t^{2,j} = \iota_t^{1,j} + f_u(u_t^j) - f_p(p_t^j) \\ \text{lb}_t \leq v, u, p, a, o \leq \text{ub}_t \end{cases}
 \end{aligned}$$

Note the indexes j for all decisions, which means they are adapted to current realizations of the uncertainties. Although all decisions are wait-and-see decisions, they have to be taken under uncertainty of future evolution of the random data. The future is anticipated only by the value function that is the expected future average revenue.

Note also that per realization j there is one independent maximization problem to solve.

Next, the generating decisions $u(t)$ and pumping decisions $p(t)$ shall be modeled as here-and-now decisions. The recursive optimality equation

is now:

$$\begin{aligned}
 Q_t(v_{t-1}^1, v_{t-1}^2) &= \max_{u_t, p_t} \frac{1}{N_t} \sum_{j=1}^{N_t} \left[\theta_t^j(v_{t-1}^1, v_{t-1}^2, u_t, p_t) \right] \quad (3.17) \\
 \theta_t^j(v_{t-1}^1, v_{t-1}^2, u_t, p_t) &= \max_{v_t^j, a_t^j, o_t^j} c_t^j(u_t - p_t) + Q_{t+1}(v_t^{1,j}, v_t^{2,j}) \\
 \text{s.t.} &: \begin{cases} v_t^{1,j} = v_{t-1}^1 - f_u(u_t) + f_p(p_t) - o_t^{1,j} + a_t^{1,j} \\ v_t^{2,j} = v_{t-1}^2 - o_t^{2,j} + a_t^{2,j} \\ a_t^{1,j} = \iota_t^{1,j} \\ a_t^{2,j} = \iota_t^{1,j} + f_u(u_t) - f_p(p_t) \\ \text{lb}_t \leq v, u, p, a, o \leq \text{ub}_t \end{cases}
 \end{aligned}$$

Note that the maximization operator is now outside of the summation which couples the previous independent subproblems. From this formulation alone, it is not clear how to solve the problem. This issue will be treated in the continuation of this example.

Since in the first formulation all decisions are adapted to the random data realizations, the problem is overly optimistic. In general, one can say that the objective function value of such formulated models are higher compared with models with here-and-now decisions (see also table 3.2). ■

Computational complexity

Stochastic problems are computationally troublesome due to the exponential growth of their scenario tree. Therefore, it is clear that a decomposition is meaningful, e.g. the presented dynamic programming algorithms.

Similarly, the here-and-now decisions can be discretized (e.g. as in algorithm 2) in order to maintain computational tractability. However, such a decomposition may be not always meaningful. In recent years, solvers for mathematical programs have become very powerful. The speed up of the LP algorithms from the year 1990 to 2010 is considered to be more than 3000 times. Together with the advances in processor speed of more than 1500 times, this results in a speed up for solving LPs of about 4.5 millions [70].

Typically, the main problem experienced nowadays when solving LPs,

are the limited amount of memory available. On the other hand, as long as the modeled system matrices remain manageable, it is very difficult to find more efficient solution methods, making a further decomposition of the problem obsolete.

Example 3.3. continued: The formulation (3.17) shall now be implemented. In algorithm 2 the decision space was discretized in order to be able to solve the problem. In this case, this would mean introducing discrete generation and pumping steps. For instance, the steps could be threesome: fully pumping, doing nothing, or full generation. Although this seems a crude model, in practice this can be a reasonable assumption for a MTHP optimization. It can be even shown mathematically that, for simple models, such *bang-bang* solutions are optimal [71].

Another possibility is to solve the coupled problem directly as one maximization problem, formulated as the deterministic equivalent for one time stage. This problem can still be linear depending on how the value function $Q_t(v_{t-1})$ is modeled.

Table 3.1 shows the computational complexity of both implementations for one time step and varying number of scenarios N_t . When the here-and-now decisions are discretized, then the complexity is linearly dependent on the amount of scenarios. This is obvious, since the subproblems remain the same, but only a larger number of them have to be solved. Note that, for comparison reasons, the computations were performed on a single processor core and not in parallel.

The second implementation formulates the coupled problem as a single LP. As solver, the function *linprog* from the optimization toolbox in Matlab was used.

This implementation is clearly superior to the first one, yielding to results more than 60 times faster. Further, although the complexity of this implementation grows more than linearly with number of scenarios, one would need a massive amount of them in order to undermine it. ■

Deterministic optimization of expected random data

As stated in section 3.4, the expectation operator can be moved into the contribution and state transition function if the problem is linear in the random elements. This would make solving the problem much easier,

Table 3.1: Example 3.3: Computational complexity for implementations of (3.17).

Scenarios N_t :	10	100	1000	10000	100000
Decomposed [s]:	1.21	3.08	33.66	320.57	3198.48
Single LP [s]:	0.02	0.05	0.33	40.75	out of memory
Ratio:	60	62	102	8	-

since the problem could be considered deterministically for expected random data.

A closer look at the MTHP problem, as stated e.g. in (3.17), reveals that the problem is indeed linear in the prices $c(t)$ and water inflows $\iota(t)$. However, if the constraints of θ become binding because of an actual realization of the random data, then the overall problem is not linear anymore, which is illustrated in the next example.

Example 3.3. continued: The MTHP problem is now also formulated as a deterministic DP for expected random data:

$$Q_t(v_{t-1}^1, v_{t-1}^2) = \theta_t(v_{t-1}^1, v_{t-1}^2) \tag{3.18}$$

$$\theta_t(v_{t-1}^1, v_{t-1}^2) = \max_{v_t, u_t, p_t, a_t, o_t} c_t(u_t - p_t) + Q_{t+1}(v_t^1, v_t^2)$$

$$\text{s.t.:} \begin{cases} v_t^1 = v_{t-1}^1 - f_u(u_t) + f_p(p_t) - o_t^1 + a_t^1 \\ v_t^2 = v_{t-1}^2 - o_t^2 + a_t^2 \\ a_t^1 = \iota_t^1 \\ a_t^2 = \iota_t^2 + f_u(u_t) - f_p(p_t^j) \\ \text{lb}_t \leq v, u, p, a, o \leq \text{ub}_t \end{cases}$$

$$\text{where } c_t = \frac{1}{N_t} \sum_{j=1}^{N_t} c_t^j, \quad \iota_t^1 = \frac{1}{N_t} \sum_{j=1}^{N_t} \iota_t^j$$

In this formulation, the prices c_t^j and water inflows ι_t^j are considered in the optimization only by their expected value. Similarly to the implementation of (3.17) as a single LP, the optimization here consists also of a single LP per time stage. However, it is a deterministic one and therefore possibly much smaller.

Consider now the implementation of the formulations (3.16), (3.17) and (3.18). Table 3.2 shows some results for one time stage. In order to

make this example both simple and interesting, only two water inflow scenarios are considered. Additionally, there is no pumping allowed. The basins are large and empty at the beginning, the time duration of the time step is one hour, efficiencies are $= 1$, 1000 m^3 of water flow relates to 1 MWh, and there is no value given to stored water and therefore $Q_{t+1} = 0$.

In the wait-and-see formulation (3.16), the generation decision $p(t)$ is adapted to the water inflows. Therefore, the production will make use of all of them. In the here-and-now formulation (3.17), the turbine does not run at all, since in one scenario there is no water available. The deterministic formulation (3.18) finally makes use of the expected water inflow. But it does not realize that in one scenario the turbine capacity would not be enough to process all of the inflow.

Also shown are the water values for the two basins. They can be obtained from the dual variables of the respective water balance constraints. For all formulations, the water value is zero for the basin 2, since $Q_{t+1} = 0$ and since no pumping is allowed. Therefore, the filling of this basin does not influence the objective function at all. For the basin 1, the formulations get to different values. The values can be explained by noticing the change in the objective value, if the filling of this reservoir would increase by one unit.

Although this example shows only the results for one time step, some fundamental characteristics become apparent. The stochastic wait-and-see formulation is different to the deterministic case only if some variable bounds of it become binding. In this case, it was the turbine capacity and therefore, the overall problem is not linear in the realization of the water inflows.

The need for formulating a problem stochastically can be analyzed by the *expected value of perfect information (EVPI)* measure. The EVPI is the price one would be willing to pay for perfect information about the random data. In the shown example it would be $2500 \text{ €} - 0 \text{ €} = 2500 \text{ €}$, the difference of the objective values of the wait-and-see and the here-and-now formulation (only valid for such a two-stage stochastic program). If the stochastic formulation would result in the same objective value, the EVPI would be zero. This would indicate that considering random data is not needed, since it does not have an impact on the problem.

The benefit of using the stochastic formulation can be expressed by the *value of the stochastic solution (VSS)* measure. The VSS can be cal-

Table 3.2: Example 3.3: Comparison of some results for implementations of (3.16) (wait-and-see), (3.17) (here-and-now) and (3.18) (deterministic) formulations.

Input data for time stage t :			
Inflow c_t^j :	= $\{0, 200\}$	[1000 m ³]	
Price $c_t^j, \forall j$:	= 50	[€/MWh]	
Turbine $p(t)$:	$\in [0, 100]$	[MW]	
Pump $u(t)$:	= 0	[MW]	
Results:	wait-and-see	here-and-now	deterministic
Obj. value: [€]	2500	0	5000
Turbine p_t^j : [MW]	0/100	0	100
Water values: [€/1000m ³]	25/0	50/0	0/0

culated, in this case, as the difference of the here-and-now objective function and the revenue resulting from applying the optimal policy from the deterministic formulation. Applying the deterministic solution in this simple example would lead to an infeasible solution, and therefore the VSS is infinite.

■

Chapter 4

Extended stochastic dual dynamic programming

Stochastic dual dynamic programming is an algorithm, which can be very powerful, but at the costs of requiring concavity of the value function with respect to the state space and stage-wise independency. These two conditions are sometimes difficult to meet. Therefore extensions to the basic stochastic dual dynamic programming algorithm were proposed which can handle them in an appropriate way. This chapter introduces these methods as well as describes some exotic variants.

4.1 Bibliography

Stochastic dual dynamic programming (SDDP) was applied for several different problems by adapting or extending the algorithm. Many authors approximated the problem itself in order to be able to apply SDDP, e.g. in order to consider transmission network constraints [72], price-maker agents [73], or non-linear water head effects [74].

Another approach is to use a relatively simple model in the backward step, but a detailed one in the forward step. Although convergence is not guaranteed then, in practice, for a small solution gap, one can argue that a good policy for the detailed model is found. E.g. N.Löhndorf et al. were able to solve with this approach a hydro scheduling problem where the hydro power plant was a price setter in the intraday market

[75].

An extension of SDDP was proposed by A.Gjelsvik et al. in [76]. They applied SDDP for hydrothermal scheduling, considering spot price uncertainty, where the price process was a discrete Markov model. With this construction, the cost-to-go function is non-convex with respect to the price state. By making the price process discrete, they were able to apply a variant of SDDP where the cuts are not shared for different price points, thus combining stochastic dynamic programming (SDP) with SDDP.

Non-convex value functions cannot be approximated by a set of linear constraints which is needed in the traditional SDDP scheme. The usual approach to cope with this in SDDP is to approximate the problem. E.g. in [77], integer variables from an investment problem were relaxed to continuous ones or similarly in [73], for the consideration of a price-maker hydro plant.

Another way is to approximate the value function itself instead of the problem. S.Cerisola et al. considered nonlinear water head effects by applying Lagrangian relaxation to the coupling constraints in order to find valid Benders' cuts [74]. F.Thome et al. further enhanced this approach in the working paper [78] by optimizing the Lagrange multipliers, leading to closer cutting planes and thus faster convergence of the algorithm.

4.2 SDDP for stage-wise dependent sample average approximations

SDDP requires a stage-wise independent problem. However, a stage-wise independent sample average approximation (SAA) seems at first to be a very limited construct. For instance, consider an hourly model for electricity prices and water inflows. Both prices and inflows are strongly autocorrelated. Therefore, some kind of autoregressive modeling is required, which would prevent constructing a stage-wise independent SAA and consequently also the application of SDDP.

However, with a simple trick, such a stage-wise dependent SAA can be reformulated stage-wise independently by augmenting the state space. The role of the additional state variables is to provide the relevant history. As long as the formulation stays concave with respect to the newly introduced state variables, SDDP can be applied.

A downside of this method is the increase in computational complexity due to the increased state space. Even a simple autoregressive model quickly requires a couple of additional states, which reduces the conversion rate of SDDP considerably. In such cases, the application of SDP can be more efficient, since a level one dependency can be employed there inherently.

Another approach is to combine SDP with SDDP, which allows also autoregressive considerations for data processes where the value function would be non-concave in their respect. The method is also known as multi-cut SDDP and follows directly out of the formulation (3.6) for the post-decision state.

Consider a random process, which depends on an underlying finite Markov chain. Let the state transition of the Markov chain process be sampled. In the backward step of SDDP, the cuts are now not shared for different sampled states of the Markov chain. Thus, an individual collection of hyperplanes per state point is constructed. Algorithm 4 would then have to be slightly adjusted so that the subproblem in line 14 would be solved also for each sampled state point of the Markov chain process.

Note that by not sharing the cuts, the value function does not have to be concave for the respective process. The Markov chain can be quite general and can therefore model complicated systems.

Note further that a multi-cut scheme can be applied for problems where a single-cut would be also possible. The main disadvantage of multi-cut SDDP is the combinatorially increased number of subproblems, which have to be solved per number of sampled Markov chain discrete points. However, in comparison with a single-cut formulation, the conversion rate is higher.

Example 4.1. Extended SDDP: Consider the bidding problem for a hydro power plant with stochastic market prices and water inflows. A bid consists of a price and an associated quantity of energy. Therefore, the value function is non-concave with respect to realized market prices. In order to apply SDDP, the multi-cut scheme with augmented state space is applied.

For the price process, a discrete and finite Markov Chain is constructed. A possibility could be to model a temperature process as a Markov Chain. Then both stochastic water inflows and market prices could realize depending on a temperature state.

Nevertheless, assume that the price process itself is modeled as a simple

Markov chain. The price is discretized to a sparse representation of only 2 price states, e.g. to a low and high price point. The state transition function would then specify the four transitions low-low, low-high, high-low, and high-high with appropriate probabilities.

In contrast, the water inflows process shall be modeled as a autoregressive process of order 1. Therefore, the state space is augmented by the realized inflow at $t - 1$. Note that with such a construction, the inflow process remains continuous.

Regarding the computational complexity, the modeling of the prices and inflows have a different impact. For the price states low/high, the set of cuts can not be shared. Therefore, the backward step in SDDP has to be performed basically twice. Additionally, instead of calculating only one subproblem per trial state and sampled random data, two of them have to be considered now. So, in total, the complexity grows $2 \cdot 2 = 4$ times. However, note that with sufficient parallelization, the complexity would stay the same. Note also that the conversion rate of the algorithm would not be influenced.

In contrast, the additional state for the water inflow process does not change the formulation of the algorithm 4. Because the value function is concave in respect to a water inflow state, cuts can be shared. The increase in complexity of the subproblems, which are solved in the forward and backward step, is negligible. But most probably, the conversion rate of the algorithm will be lower. It is now more difficult to find the right trial states and, therefore, a lower quality of the cutting planes has to be accepted.

To make this clearer, consider an hourly time step. The hourly uncertainty in the water inflows is typically very high. Therefore the forward step will have problems to estimate the state correctly. ■

4.3 SDDP for non-concave value functions

Depending on the type of problem, there are different approaches to handle meaningfully non-concave value functions. In table 4.1, a brief overview of some of them is given. Those methods are now explained in more detail.

Table 4.1: Dealing with non-concave value functions

Method	Primarily use
Multi-cut SDDP:	If non-concave only for some states.
Concavification:	Source of non-concavity is not that important.
Reformulation:	Non-concavity can be reformulated reasonably.
Ordinary SDDP:	Value function is only slightly non-concave.
Dualized SDDP:	Value function is moderately non-concave.
Ordinary SDP:	Value function is not well-behaved.

Multi-cut SDDP

Multi-cut SDDP was already explained in the previous section. This method can handle states in which the value function is non-concave. It is important to note here that the value function is non-concave only for *some* states and not for all. Otherwise multi-cut SDDP would be nothing else than an ordinary SDP algorithm.

Concavification or reformulation of the problem and ordinary SDDP

Another way of dealing with non-concave value functions is its concavification either by approximation or reformulation. Of course, such strategies make only sense if the source of the non-concavity is not that important and can be approximated or reformulated reasonably.

In practice, value functions are often only slightly non-concave. In such cases, ordinary SDDP can still provide acceptable policies.

Dualized SDDP

Cutting planes are theoretically no longer upper bounds in SDDP if the problem is non-concave for some states. A way around this issue can be found with the help of Lagrangian relaxation. The idea is that the state transition function, which is the complicating constraint, is relaxed. Therefore, the violation of these constraints is penalized with a Lagrangian multiplier in the objective function. One can then show that the optimal value of the resulting problem as well as the multiplier can be used to construct a valid upper bound even for non-concave functions.

Consider again problem (3.7) and assume a non-concave value function. Let an estimate of the Lagrange multiplier $\tilde{\mu}_{t,k}^j$ for time stage t , trial state k , and sample j for the complicating constraint be given. The constraints $\{B, A, b\}$ are split up into complicating and non complicating constraints $\{B_c, A_c, b_c\}$ and $\{B_{nonc}, A_{nonc}, b_{nonc}\}$. Then (3.10) is reformulated, where the complicating constraints are dualized:

$$\begin{aligned} \tilde{\gamma}_{t,k}^{LR} = \frac{1}{N_t} \sum_{j=1}^{N_t} \left[\max_{x_t^j, z_t^j} \phi_t^j(x_t^j, z_{t-1}^k) + \theta - \right. \\ \left. \tilde{\mu}_{t,k}^j \cdot \left(b_{t,c}^j - B_{t,c}^j z_{t-1}^k - A_{t,c}^j \begin{bmatrix} x_t^j \\ z_t^j \end{bmatrix} \right) \right] \\ \text{s.t.:} \begin{cases} \theta \leq \gamma_{t+1,l} + \delta_{t+1,l} z_t^j, \forall l=1, \dots, (i-1) \cdot M \\ B_{t,nonc}^j z_{t-1}^k + A_{t,nonc}^j \begin{bmatrix} x_t^j \\ z_t^j \end{bmatrix} = b_{t,nonc}^j \\ lb_t \leq z_t^j, x_t^j \leq ub_t, z_t^j, x_t^j \in \mathbb{R}^n \end{cases} \end{aligned} \quad (4.1)$$

The vector of slopes $\tilde{\delta}_{t,k}^{LR}$ for the construction of a cutting plane can be found by slightly adapting (3.11):

$$\tilde{\delta}_{t,k}^{LR} = -\frac{1}{N_t} \sum_{j=1}^{N_t} (B_{t,c}^j)^T \tilde{\mu}_{t,k}^j \quad (4.2)$$

$\{\tilde{\gamma}_{t,k}^{LR}, \tilde{\delta}_{t,k}^{LR}\}$ forms a valid cutting plane. The cut is possibly not “close” to the actual value function (see also Fig. 4.1), if the estimate of the Lagrange multiplier $\tilde{\mu}_{t,k}^j$ was not good. But qualitatively good cuts are important for the convergence of SDDP.

The multipliers can be optimized in order to get more restrictive cutting planes. The procedure for such an optimization can be as follows:

1. Solve the non-concave subproblem to get the optimal value $\gamma_{t,k}^{j,opt}$.
2. Find the initial guess of a Lagrange multiplier by solving the local convexified problem.
3. Solve the Lagrangian relaxed problem (4.1):
 - (a) Solve it for the initial multiplier guess.

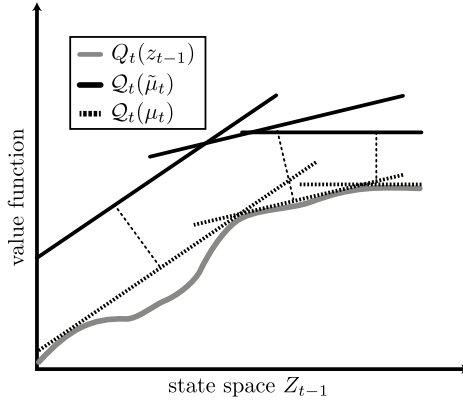


Figure 4.1: The collection of cutting planes for guessed multipliers $Q_t(\tilde{\mu}_t)$ are possibly not “close” to the actual value function $Q_t(z_{t-1})$. The optimization of the Lagrange multipliers leads to closer bounds up to the collection $Q_t(\mu_t)$.

- (b) If the resulting $\tilde{\gamma}_{t,k}^{j,LR}$ is “close” to $\gamma_{t,k}^{j,opt}$ or no progress was done, then a good multiplier $\tilde{\mu}_{t,k}^j$ is found.
 - (c) Otherwise, solve Lagrangian relaxed problem again with an updated multiplier.
4. Repeat the procedure for all samples $j = 1, \dots, N_t$ and construct the cutting plane $\{\tilde{\gamma}_{t,k}^{LR}, \tilde{\delta}_{t,k}^{LR}\}$.

The updating of the multipliers can be done by using a sub-gradient method, where the step size depends on how far the optimal value $\tilde{\gamma}^{LR}$ of the Lagrangian relaxed problem is from the one of the original problem γ^{opt} , normalized by the violation of the complicating constraint:

$$\tilde{\mu}_{t,k}^{j,new} = \tilde{\mu}_{t,k}^{j,old} - \frac{\tilde{\gamma}_{t,k}^{j,LR} - \gamma_{t,k}^{j,opt}}{\phi_{t,k}^j} \quad (4.3)$$

$$\phi_{t,k}^j = \left(b_{t,c}^j - B_{t,c}^j \tilde{z}_{t-1}^k - A_{t,c}^j \begin{bmatrix} x_t^j \\ z_t^j \end{bmatrix} \right)$$

Note that the search for a good Lagrange multiplier is not in order to solve the subproblem (3.10) or to decompose it, but to find a more

restrictive cutting plane. In this respect, it is interesting to note that the actual solution of the subproblem is known and therefore the updating of the multiplier can be done in an elegant way.

Note also that if the value function is concave, then the initial guess of the Lagrange multiplier corresponds to the dual variable of the complicating constraint π and the same cutting plane as in ordinary SDDP is found.

The main disadvantage of this method is the increase in computational complexity. When the value function is only moderately non-concave, then two additional subproblems have to be solved, which triples the needed computational effort. However, already found optimal multipliers can often be shared for different trial states and data samples, resulting in only one additional subproblem to be solved.

For not well-behaved value functions many iterations can be necessary in order to find the optimal multipliers and the sharing of those will not be helpful. Additionally, even for optimal multipliers, the cutting planes can be of poor quality, which further lowers the convergence rate of the SDDP algorithm.

As a conclusion, one can note that although this method could handle non-concave value functions in SDDP, in practice, this only makes sense for moderately non-concave value functions.

Example 4.2. Dualized SDDP: In order to illustrate the dualized SDDP method, consider a problem like (3.7), where some decision variables $w(t)$ are integer ones. Such mixed-integer linear programs (MILPs) can lead to non-concavities since the profit-to-go function may become uncontinuous.

In order to apply the SDDP algorithm 4, line 14 has to be adapted as shown in algorithm 5.

Note that instead of having a single subproblem, one additional linear program (LP) with fixed integers has to be solved and then at least one MILP for calculating the optimal profit of the relaxed problem.

In a practical implementation, previous Lagrange multiplier can be reused which makes the calculation of the LP with fixed integers obsolete. Also, previous solutions of the MILPs are used to warm-start their computation. Therefore, the effort to solve a single MILP is very close to solving a LP. As a result, the needed computational effort approximately at least doubles. ■

Algorithm 5 Example 4.2: Dualized SDDP for a MILP

Require: Algorithm 4, line 14 is replaced with the following:

Solving of MILP to get its optimal value and integer decisions:

$$1: \gamma_{t,k}^{j,opt} = \begin{cases} \max_{x_t^j, w_t^j, z_t^j} \phi_t^j(x_t^j, w_t^j, z_{t-1}^k) + \theta \\ \text{s.t.} \begin{cases} \theta \leq \gamma_{t+1}^l + \delta_{t+1}^l z_t^j, \forall l=1, \dots, (i-1) \cdot M \\ B_t^j z_{t-1}^k + A_t^j \begin{bmatrix} x_t^j \\ w_t^j \\ z_t^j \end{bmatrix} = b_t^j \\ lb_t \leq z_t^j, x_t^j, w_t^j \leq ub_t, z_t^j, x_t^j \in \mathbb{R}^n, w_t^j \in \mathbb{N}_0 \end{cases} \end{cases} \quad // \text{ Original MILP}$$

$$2: w_t^j = \begin{cases} \arg \max_{x_t^j, w_t^j, z_t^j} \phi_t^j(x_t^j, w_t^j, z_{t-1}^k) + \theta \\ \text{s.t.} \begin{cases} \theta \leq \gamma_{t+1}^l + \delta_{t+1}^l z_t^j, \forall l=1, \dots, (i-1) \cdot M \\ B_t^j z_{t-1}^k + A_t^j \begin{bmatrix} x_t^j \\ w_t^j \\ z_t^j \end{bmatrix} = b_t^j \\ lb_t \leq z_t^j, x_t^j, w_t^j \leq ub_t, z_t^j, x_t^j \in \mathbb{R}^n, w_t^j \in \mathbb{N}_0 \end{cases} \end{cases} \quad // \text{ Original MILP}$$

Find initial guess of a Lagrange multiplier:

$$3: \gamma_{t,k}^{j,LP} = \begin{cases} \max_{x_t^j, z_t^j} \phi_t^j(x_t^j, w_t^j, z_{t-1}^k) + \theta \\ \text{s.t.} \begin{cases} \theta \leq \gamma_{t+1}^l + \delta_{t+1}^l z_t^j, \forall l=1, \dots, (i-1) \cdot M \\ B_t^j z_{t-1}^k + A_t^j \begin{bmatrix} x_t^j \\ z_t^j \end{bmatrix} = b_t^j(w_t^j) \quad [\pi_{t,k}^j] \\ lb_t \leq z_t^j, x_t^j \leq ub_t, z_t^j, x_t^j \in \mathbb{R}^n \end{cases} \end{cases} \quad // \text{ LP with fixed } w_t^j$$

$$4: \tilde{\mu}_{t,k}^j \leftarrow \pi_{t,k}^j \quad // \text{ update Lagrange multiplier}$$

Optimize Lagrange multiplier:

5: **while** $\tilde{\gamma}_{t,k}^{j,LR} - \gamma_{t,k}^{j,opt} \geq \epsilon$ **do**

$$6: \tilde{\gamma}_{t,k}^{j,LR} = \begin{cases} \max_{x_t^j, w_t^j, z_t^j} \phi_t^j(x_t^j, w_t^j, z_{t-1}^k) + \theta - \tilde{\mu}_{t,k}^j \cdot \left(b_{t,c}^j - B_{t,c}^j z_{t-1}^k - A_{t,c}^j \begin{bmatrix} x_t^j \\ w_t^j \\ z_t^j \end{bmatrix} \right) \\ \text{s.t.} \begin{cases} \theta \leq \gamma_{t+1}^l + \delta_{t+1}^l z_t^j, \forall l=1, \dots, (i-1) \cdot M \\ B_{t,nonc}^j z_{t-1}^k + A_{t,nonc}^j \begin{bmatrix} x_t^j \\ w_t^j \\ z_t^j \end{bmatrix} = b_{t,nonc}^j \\ lb_t \leq z_t^j, x_t^j, w_t^j \leq ub_t, z_t^j, x_t^j \in \mathbb{R}^n, w_t^j \in \mathbb{N}_0 \end{cases} \end{cases}$$

$$7: \phi_{t,k}^j = \left(b_{t,c}^j - B_{t,c}^j z_{t-1}^k - A_{t,c}^j \begin{bmatrix} x_t^j \\ w_t^j \\ z_t^j \end{bmatrix} \right)$$

$$8: \tilde{\mu}_{t,k}^j \leftarrow \tilde{\mu}_{t,k}^j - \frac{\tilde{\gamma}_{t,k}^{j,LR} - \gamma_{t,k}^{j,opt}}{\phi_{t,k}^j}$$

9: **end while**

Construct cutting plane:

$$10: \gamma_{t,k,j} \leftarrow \tilde{\gamma}_{t,k}^{j,LR} \quad // \text{ store intercept}$$

$$11: \delta_{t,k,j} \leftarrow \tilde{\mu}_{t,k}^j \quad // \text{ store vector of slopes}$$

4.4 Summary

Stage-wise dependent problems can be well reformulated in order to allow stochastic dual dynamic programming (SDDP) to solve them by either augmenting the state space or by using multi-cut SDDP.

For non-concave value functions, the discussion is more subtle. Depending on the source and severity of non-concavities some approaches may be more suitable than others. However, it is not really clear a priori, how to measure the extent of non-concavity. This issue is therefore further discussed in chapter 7.

The different methods and their main application are summarized in table 4.1. Note that, for more than moderately non-concave value functions with many state variables, there is no effective method.

Chapter 5

Risk averse optimization

This chapter introduces risk averse optimization. First, coherency of risk measures is discussed. Then, the average value at risk is described. In the multi-stage setting, dynamic risk measures and time-consistency are briefly scrutinized. Finally, the risk measure is built into stochastic dynamic programming and stochastic dual dynamic programming algorithms.

5.1 Bibliography

In literature, many approaches for risk averse hydro power planning strategies are proposed. A relatively recent review about them is given in [79], and for hedging of price risk in energy trading in [80].

Roughly speaking, uncertainties may appear either in the constraints and/or in the objective function. An example of the former case is uncertainty in water inflows. Research about how to mitigate such inflow risks, and with that related also reservoir level risks, began with applications of stochastic dynamic programming (SDP) in the 70' of the last century. In [81], a penalty-based procedure was used to ensure certain water levels. This was extended in [82] by using Lagrange multiplier methods and in [83] these kind of algorithms were investigated in detail. Finally, in [84], it was tried to overcome some issues for such algorithms by modeling them as a set of nested problems, where the

targeted risk was attained by an iterative approach.

The works were further extended by considering chance constraints. If the distribution of the random variables (like water inflows) are assumed to be known analytically, then such problems can be formulated efficiently. E.g. recent examples [85, 86] follow such an approach which is applicable to stochastic dual dynamic programming (SDDP).

With the advent of liberalized electricity markets, the scope of some researchers shifted from achieving a certain reliability of hydro power production to mitigate profit risk. However, this was achieved by similar methods, i.e. by setting target values for some variables. E.g. in [87], an integrated approach was followed, where revenue below a certain target was penalized by taking into account future contracts.

More recently, risk measures in the objective function of multistage stochastic programs that depend on decisions and the underlying random variables achieved great attention in research.

It begun with the discussion of coherent risk measures by P. Artzner et al. in 1999 [88]. F. Riedel contributed by analyzing dynamic risk measures in [89] and A. Eichhorn by investigating general polyhedral risk measures in [90, 91]. Time-consistency of risk measures were analyzed in many publications, e.g. for dynamic risk measures in [92] and applied for hydro power by M. Densing in [93]. A. Ruszczyński discussed such risk measures further in [94] as well as G. Pflug et al. in the textbook [45, chapter 5]. A. Shapiro contributed to this field by various publications and by analyzing how coherent risk measures, especially the average value at risk, could be co-optimized within a SDDP based algorithm [19, 40, 50, 95, 96]. A. Philpott et al. extended such approaches to general coherent risk measures in [97, 98]. Independently, these approaches were also investigated by the Electric Energy Research Center (CEPEL) in Brazil in [99, 100]. Finally T. Homem-de-Mello et al. tried to summarize the findings of risk aversion in multistage stochastic programming in [101].

5.2 Risk aversion in power plant operations

Risk and its mitigation can be modeled as additional constraints in a mathematical program. One possibility is to maintain the risk constraints for all possible scenarios. This leads mostly to quite unrealistic and costly solutions or even to infeasible problems.

One can make the model less restrictive by using chance constrained programming, where the risk constraints have to be maintained only for a high probability. When the distributions of the uncertainties are known and not complicated (like normal distributions), this can be a convenient approach. Otherwise such models can be very difficult to handle numerically.

The next step is to maintain the risk constraints only for some representative scenarios, which is also called robust programming. Whereas such methods can be quite transparent, the choice of the scenarios is on the other hand difficult to justify. Additionally, no guarantees for the quality of such robust policies can be given.

More recently, the so-called *scenario approach* [102] provides, for convex problems, the number of randomly sampled scenarios which has to be considered, in order to fulfill probabilistic constraints. This approach depends only on the amount of decision variables and/or how uncertainty enters the problem. Therefore, the number of scenarios to consider can be quite large. Additionally, although the constraints are fulfilled, the found policy is not the most optimal one as well as there is no insight on its relative quality.

All of the so far mentioned risk measures consider possible shortfalls equally, no matter how large they can be. Penalty schemes, in contrast, are different in this respect. The idea is that the *violation* of the risk constraints is penalized in the objective function. Consider the risk constraints being described by $A^{risk}x \geq b^{risk}$. Then, its violation $\phi(b^{risk} - A^{risk}x)$ is added to the objective function.

It is difficult to motivate the parameters of the penalization ϕ and the lower limit b^{risk} . Additionally, the parameters are in this way not adapted to the current realizations of a random process. Consider for instance a model which would penalize too high usage of water per time stage. When the parameters are not adapted then they are chosen a priori. For the model, it could make sense to adapt the upper limit of water usage depending on if the respective time stage has relatively high or low water inflows.

In order to make the limit b^{risk} adaptive or *conditional* on the observed history of the random process, the value at risk can be taken as the limit. This construction leads to optimizing the conditional *conditional value at risk* or, in order to avoid that awkward terminology in this context, the conditional average value at risk.

Following this qualitative discussion of risk measures a more mathe-

matical one is shown below. First, some properties of risk measures are discussed. Then, the conditional average value at risk is introduced for the multistage case.

5.2.1 Coherency of risk measures

As shown in the previous section of this chapter, there are many possible different risk constructions. *Coherency* for risk measures now guarantees meaningful risk functionals. Let the variables X, Y be random, where again a higher realization of them is considered to be better. A risk measure $\rho(X)$ is called to be coherent [88], if it fulfills the following conditions:

1. Subadditivity::

$$\rho(X + Y) \leq \rho(X) + \rho(Y)$$

2. Positive homogeneity:

$$\rho(\lambda X) = \lambda \rho(X), \lambda \geq 0$$

3. Monotonicity:

$$\rho(X) \leq \rho(Y), X \geq Y$$

4. Translation equivariance:

$$\rho(X + a) = \rho(X) - a, a \in \mathbb{R}$$

The four conditions can be motivated qualitatively. Subadditivity ensures that a combination of two random variables does not increase the risk value (diversification). Monotonicity ensures that if all realizations of a random variable is higher or lower than the other ones, then the same is true for their risk measures. And, finally, translation equivariance and positive homogeneity ensure that a summation or multiplication of all realizations of the random variable can be taken out of the functional.

Note that if lower realizations of the random variable are considered to be better, then the monotonicity and translation equivariance have to be adapted accordingly.

As discussed in [103], many situations can arise where the risk of a position might increase in a nonlinear way with the size of the position, e.g. due to additional liquidity risk. For such cases, the properties of coherent risk measures are proposed to be relaxed, namely the subadditivity

and positive homogeneity conditions, to the property of convexity:

5. Convexity:

$$\rho(tX + (1-t)Y) \leq t\rho(X) + (1-t)\rho(Y), \quad \forall t \in [0, 1]$$

Such risk measures are then called convex risk measures.

It can be shown that a coherent or convex risk measure is meaningful from the mathematical point of view. Mostly this is also the case from the modeling point of view.

5.2.2 Average value at risk and mean-risk models

In general, the average value at risk (AV@R) is defined for a random variable X and a risk level α as follows:

$$\text{AV@R}_\alpha[X] = \mathbb{E}[X|X \leq \text{V@R}_\gamma] = \frac{1}{\alpha} \int_0^\alpha \text{V@R}_\gamma(X) d\gamma$$

where V@R_γ is the value at risk. The AV@R is the expected value of the probability distribution function of the random variable lower than its α quantile. Therefore, it is also called *expected tail loss*. Note also that this notation applies only to the risk of a random variable, where higher realizations of it are considered to be better. Examples for such random variables are portfolio values or cashflows.

It can be shown that the AV@R is a coherent risk measure. In an optimization problem, it can be calculated as follows [104]:

$$\text{AV@R}_\alpha[X] = \max_{\kappa \in \mathbb{R}} \kappa - \alpha^{-1} \mathbb{E}[(X - \kappa)_-]$$

If there are identically and independently distributed random samples X^1, \dots, X^N of realizations of the random variable X available, then this optimization problem can be written as:

$$\text{AV@R}_\alpha[X] = \max_{\kappa \in \mathbb{R}} \kappa - \frac{1}{\alpha N} \sum_{j=1}^N (X^j - \kappa)_- \quad (5.1)$$

In practice, this problem can also be solved by first ordering the set of samples to $X^{1,\text{sort}} \leq X^{2,\text{sort}} \leq \dots \leq X^{N,\text{sort}}$. Consider now the operator $[X]^{\text{sort}}(j)$, which orders the set of the realization of the random

variable X and returns its j -th entry. Then for $\lceil \cdot \rceil$ being a rounding up operator there is:

$$\text{AV@R}_\alpha[X] = \frac{1}{\lceil \alpha N \rceil} \sum_{j=1}^{\lceil \alpha N \rceil} [X]^{\text{sort}}(j) \quad (5.2)$$

A co-optimization of the risk of random costs or revenues and the expected value of them seems to be the most meaningful approach for risk averse dynamic programs. Such constructs are for general risk measures known as *mean-risk* models. If the AV@R is taken as risk functional and with $\lambda \in [0, 1]$ as *costs of risk*, a weighting factor, there is:

$$\rho(X) = (1 - \lambda) \mathbb{E}[X] + \lambda \text{AV@R}_\alpha[X] \quad (5.3)$$

For a two-stage stochastic program the formulation of the objective function then changes from optimizing the expected value of some random variable (e.g. revenue) to its risk adjusted one (5.3).

5.2.3 Dynamic risk measures and time-consistency

For multi-stage problems, *dynamic risk measures* [89] are convenient for the consideration of risks. A dynamic risk measure ϱ is above all a measure, which adapts its value if new information gets available. It is well known that such a behavior can be achieved by using a recursive setting of single period risk measures.

Consider now as X_1, \dots, X_T the sequence of future realizations of a random variable X which arises in multi-stage programs. Further, let $\rho(X_t)$ be a single period risk measure, e.g. as defined in (5.3). Then, the nested formulation of the risk measure ϱ for this sequence is as follows:

$$\varrho(X_1, \dots, X_T) = \rho(X_1 + \rho(X_2 + \dots + \rho(X_T) \dots)) \quad (5.4)$$

Since this formulation coincides with the one for SDPs (3.3), it can be integrated there seamlessly.

Whereas mathematically the complete characterization of ϱ is difficult (it is a risk measure of risk measures)¹, it makes sense from the modeling

¹Recently, some authors argued that multi period risk measures should fulfill information monotonicity, i.e. with more information the risk can not increase [45]. Risk measures based on the concatenation of single period risk measures as in (5.4) fulfill this property only for the single period risk functional being based on the expectation or the max-risk functional [105]. In our view, however, this property is not required for a reasonable setting.

and economical point of view as follows.

Firstly, with such a construction, risks are valued less the further they are away, which is intuitive thinking. Then, one can also try to interpret the objective function economically. At time stage $t = T - 1$, the objective function for a revenue maximization problem represents the risk adjusted value at terminal wealth. In financial terms, it is the *certainty equivalent* of it, the amount of money which is viewed as being comparable to holding the asset. Further, at time point $t = T - 2, \dots, 0$, the objective function describes again the certainty equivalent of holding the asset for *one period*. [106]

For multi period risk measures, *consistency in time* is another important aspect. Time consistency assures that decisions are not contradictory in time. It can be referred as:

“For a multi stage optimization the optimality of the decisions in a stage should not depend on scenarios which are already known that they can not happen in the future.” [19]

An alternative formulation is that “*if an investment opportunity is preferred to another at a future time in all possible events at that time, then it is preferred as of today, too.*” [93].

An example for a time inconsistent risk measure would be the minimization of the AV@R of some quantity at the end of the planning horizon. That is because the policy at some time stage $t > 1$ would depend on realizations which are known that they can not happen any more. Thus, the constructed policies would be awkward and it can be shown that they are in general suboptimal.

Now, if a risk measure ϱ can be formulated in a nested way of coherent risk measures ρ as in (5.4), then it follows mathematically that ϱ is also coherent as well as time consistent [92]. Therefore, such a construct can be a good choice for a risk measure in dynamic programmings (DPs).

5.3 Application to stochastic multi-stage programs

For a stochastic multi-stage program, which can be formulated as a DP, a mean risk model shall be introduced. As single period risk measure,

the coherent AV@R is chosen. In the multi-stage setting the measure is considered adaptively on the realized uncertainty as well as in a nested way. Therefore, a mean risk model with a dynamic and conditional AV@R will be formulated.

The formulation of the objective function of the stochastic optimization (3.4) will change to:

$$\begin{aligned}
 Q_t(z_{t-1}) &= \max_{x_t} (1 - \lambda) \frac{1}{N_t} \sum_{j=1}^{N_t} \left[\phi_t^j(x_t, z_{t-1}) + Q_{t+1}(z_t^j) \right] + \\
 &\quad \lambda \frac{1}{\lceil \alpha N_t \rceil} \sum_{j=1}^{\lceil \alpha N_t \rceil} [\phi_t(x_t, z_{t-1}) + Q_{t+1}(z_t)]^{\text{sort}}(j) \quad (5.5)
 \end{aligned}$$

Note that the calculation of the AV@R requires a certain number of samples N_t in order to be meaningful. Otherwise, with the modification (5.5) of the objective function, the algorithm 2 can be used directly to solve the risk averse multi stage stochastic program.

The problem (5.5) may be difficult or quite expensive to solve. But it was shown in [19, Proposition 6.37] that if a risk measure is monotone (which is the case when it is coherent), then the risk operator can be written outside of the maximization. So, instead of computing one large maximization problem, a number of subproblems can be solved, for each random data realization one. The risk measure can then be computed out of them.

In our context, this is only possible, if all of the decisions $x(t)$ are of the wait-and-see type. Equation (5.5) can then be formulated as follows:

$$\begin{aligned}
 \theta_t^j &= \max_{x_t} \phi_t^j(x_t, z_{t-1}) + Q_{t+1}(z_t^j), \quad \forall j = 1, \dots, N_t \\
 Q_t(z_{t-1}) &= (1 - \lambda) \frac{1}{N_t} \sum_{j=1}^{N_t} [\theta_t^j] + \lambda \frac{1}{\lceil \alpha N_t \rceil} \sum_{j=1}^{\lceil \alpha N_t \rceil} [\theta_t]^{\text{sort}}(j) \quad (5.6)
 \end{aligned}$$

Example 5.1. Risk averse SDDP: Similar to the SDP algorithm, with modest effort, the SDDP algorithm 4 can be modified in order to consider risk aversion.

Consider the application of a dynamic mean risk model with the conditional AV@R as risk measure. The lines 14-17 in the backward step of algorithm 4 have to be adapted as shown in algorithm 6.

Note that the forward step does not need to be adapted, since the risk averse policy, the value function approximations \mathcal{Q} , can be used in the same way as the risk neutral one. Similarly, the expected value of the found policy can still be evaluated as in (3.15). However, there is no easy way to calculate the nested risk averse objective value.² Hence, it is no longer possible to compute a lower bound to the algorithm and therefore another stopping criterions for the algorithm has to be applied. A possibility is to stop the algorithm if the upper bound stabilizes itself or simply after a fixed number of iterations.

Note also that the computational complexity is not increased in comparison to the risk-neutral formulation because the convergence rate stays the same. But as in the case for risk averse SDP, an appropriate amount of samples N_t has to be considered in order to be able to compute the risk measure. ■

Algorithm 6 Example 5.1: Risk averse SDDP

Require: Algorithm 4

Line 16 in algorithm 4 is replaced by:

- 1: $\Gamma_t^{k,sort} = \text{sort}(\gamma_{t,k,j})$ // sort in increasing order
- 2: $\gamma_t^k = (1 - \lambda) \frac{1}{N_t} \sum_{j=1}^{N_t} \gamma_{t,k,j} + \lambda \frac{1}{\lceil \frac{1}{\alpha} N_t \rceil} \sum_{j=1}^{\lceil \frac{1}{\alpha} N_t \rceil} \Gamma_t^{k,sort}(j)$ // compute risk-averse intercept

Line 17 in algorithm 4 is replaced by:

- 3: $\delta_{t,k,j} = -(B_t^j)^T \pi_{t,k}^j, \forall j = 1, \dots, N_t$
 - 4: $\Delta_t^{k,sort} = \text{sort}(\delta_{t,k,j})$ // sort in increasing order
 - 5: $\delta_t^k = (1 - \lambda) \frac{1}{N_t} \sum_{j=1}^{N_t} \delta_{t,k,j} + \lambda \frac{1}{\lceil \frac{1}{\alpha} N_t \rceil} \sum_{j=1}^{\lceil \frac{1}{\alpha} N_t \rceil} \Delta_t^{k,sort}(j)$ // compute risk-averse intercept
-

5.4 Summary

Risk measures can be constructed in many different ways. Coherency and time-consistency are two mathematical properties, which try to describe the meaningfulness of them.

²Recently, there was shown a way around this issue, where in [98] an upper bound to a risk constrained minimization problem was constructed by a separate backward optimization, where the convex hull was used as cost function approximation. But this upper bound is only valid for the sample average approximation (SAA) and requires a high computational effort for higher state dimensions. Since SDDP is typically applied for such problems with higher state dimensions this method gets almost obsolete.

Dynamic mean-risk models, with the conditional average value at risk as risk measure, are good candidates for risk-averse optimization models of stochastic multi-stage programs. It was shown how to implement such models in stochastic dynamic programming (SDP) and stochastic dual dynamic programming (SDDP) type algorithms.

Part II

Model developments and applications

Chapter 6

Multi-horizon modeling

This chapter deals with the analysis of how to model the medium-term hydro power planning problem. The goal is to mimic the operation of the plant as realistically as possible for reasonable computational complexity. A flexible modeling framework, the multi-horizon modeling approach, is proposed and discussed from the modeling and computational point of view.

The model is extended through the consideration of risk measures, where two variants are shown. Then, the revenue from providing ancillary services is included into the model.

The multi-horizon modeling method is evaluated against traditional approaches as well as some alternative modeling methods are compared. Finally, two examples show how such models can be applied for medium-term hydro power planning problems.

This chapter is based on the publications [1–3, 5, 7].

6.1 Bibliography

In this bibliography, first, references about the foundation of multi-horizon models are given. Then, references about alternative models in order to account for ancillary services markets and price makers in medium-term hydro power planning (MTHP) problems are described.

Multi-horizon models

In [107], *inter-* and *intra-stage* problems were termed explicitly and used for a MTHP optimization. The intra-stage problem in this dynamic programming (DP) problem consists of finding a supply function, which is offered for every trading period within an interstage. The electricity market is considered with stochastic price duration curves. This problem is solved for a range of different water release policies. Then, the interstage problem selects the most optimal one.

Similarly, in [71, 93], price duration curves were used as a model for the short-term bidding. The model is formulated as a mixed-integer linear program (MILP) and risk measures are incorporated. By a principal component analysis, it is possible to motivate the number of price levels.

In [75], the idea of inter- and intra-stage problems were applied in order to incorporate day-ahead and intra-day bidding in a MTHP optimization. A stochastic dual dynamic programming (SDDP) type algorithm was used to solve the model. The day-ahead price process is modeled as a Markov Chain depending on the evolution of underlying “environmental states” like temperature and gas prices. The intra-day price process then linearly depends on the respective day-ahead price and the bid volume, thus, a price response is modeled.

As in [108], the bidding is modeled as specifying a supply curve with fixed price points. This can be formulated as a MILP. Nevertheless, in the backward step of SDDP, the problem is solved without a price response. This simplifies the problem considerably, since all trades are now moved into the intra-day market. Therefore, the backward step considers basically only the intra-day market. In their applications, however, the solution gap was reasonably small.

The term of *multi-horizon decision trees* was also proposed by M. Kaut et al. in [109] for a gas field investment problem. They named the inter- and intra-stage decisions *strategic* and *operational* decisions respectively. Some of these authors extended the work by considering risk measures in [44, chapter 8]. An average value at risk (AV@R) is introduced there on a multi-dimensional scenario tree. The risk measure is based only on the operational decisions which is reasonable in their application. However, they also briefly discussed other possibilities.

MTHP optimization with consideration of ancillary services

Ancillary services markets were mostly considered in short-term bidding problems. In [110], a decision support tool for market players is presented, by a deterministic multi-agent based simulator taking into account ancillary services. In [111, 112], deterministic MILPs are presented to solve the bidding problem of a hydro power producer with consideration of an ancillary services market. A number of works [113–115] use stochastic mixed-integer programming to account for uncertainties in the electricity market and the ancillary services market for a more short-term risk constrained scheduling optimization. In [116], the bidding and scheduling problem is optimized for an 11-unit system by a stochastic mixed-integer optimization, solved by an algorithm based on Lagrangian relaxation and stochastic dynamic programming (SDP).

MTHP planning for a price-maker in forward and day-ahead markets

In [62], an hourly day ahead market and a monthly financial future market are considered. Stochasticity in the pool prices is introduced via scenarios. Large scale linear programming, which has the advantage of easily incorporating risk measures, is used. Because of the curse of dimensionality, computational tractability is a problem. So in their example, they consider only two periods with five price scenarios in the recourse stage and three different forward contracts.

In [117], a static hedging strategy is constructed. For a given physical position, the hedge of it by the available forward contracts is found subject to risk constraints. However, the spot market is omitted in their formulation.

In [63], the focus is on constructing bid curves with a large scale mixed integer linear program. A finer model is used on near term and then a coarser one going forward. Whereas the finer model models e.g. non-convex efficiency curves and have hourly time resolution, the coarser one is very simplified with price-segments. Again, the problem is the computational complexity because no decomposition was considered.

The mentioned approaches have not dealt with limited liquidity in their models. The most interesting ones that do are as follows:

In [118], game theory is used to study a duopolistic case where by applying SDP they compute the water values of these two utilities. However, such an approach would not be suitable to consider hourly bidding for a yearly time horizon.

In [119, 120], an optimal scheduling of a price-maker pumped hydro storage producer was performed. By using residual demand curves the influence on the pool market prices was modeled. This mixed-integer problem results in short-term bidding strategies as well as mid-term reservoir management. Apart from issues in modeling the competition as a demand curve, forward contracts were considered only as predefined and fixed. The authors in [73] used a similar approach, but they considered stochastic water inflows, but no forward contracts. They applied SDDP in order to find a long-term operation strategy.

6.2 Proposed model

Hydro power plants (HPPs) in the Alps and in other regions with similar topology, typically, have two different kinds of reservoirs. The first kind is storage reservoirs. They store water inflows in order to be able to produce during dry seasons. Therefore, they are operated seasonally and a daily or even weekly view on them is sufficient.

The second type of basins is balancing reservoirs. In comparison with the storage reservoirs, they are very small. Their role is to balance out water flows, i.e. to provide a necessary amount of water for generation or pumping. Therefore, they can be depleted within some hours.

Whereas the filling of storage reservoirs define the medium-term operation strategy of the HPP, the balancing reservoirs are very important to consider in the daily operation of it. In order to consider both types of reservoirs in a MTHP optimization, a short time step is often modeled. Alternatively, the balancing reservoirs can also be neglected by some aggregation of the HPP in order to reduce the computational complexity.

The proposed modeling approach lies in between these two concepts. It allows both a realistic consideration of short-term operation as well as has limited computational complexity. The model is explained next, both conceptually and mathematically. Then, it is extended by considering risk measures and an ancillary services market.

6.2.1 Multi-horizon modeling approach

The idea of the proposed modeling approach is that storage and balancing reservoirs are considered both with their inherent dynamics. Further, a decomposition of the problem into inter- and intrastage subproblems makes an efficient implementation of the model possible.

Decomposition into inter- and intrastage subproblems

The model is such that the operating policies are calculated for daily or weekly time stages only for the storage reservoirs. This multi-stage stochastic program is decomposed into *interstage* and *intrastage* subproblems. The daily/weekly interstage problem, which is the master problem, handles the water management of the storage reservoirs. The intrastage subproblems on the other hand manage the hourly operation of the plant. In this subproblem, the balancing reservoirs and their water balances and the hourly electricity market are considered.

Summarizing, the proposed model can be described as follows:

1. Interstage problem (master problem):
 - is formulated dynamically with daily/weekly time stages.
 - state variables are the storage reservoirs.
 - here-and-now decision variables are the daily/weekly sum of water release from storage reservoirs.
 - output are operation policies (water values) and profit-to-go function.
2. Intrastage problem:
 - has hourly time steps.
 - is formulated as a multistage stochastic program.
 - state variables are the balancing reservoirs.
 - decision variables are the day-ahead market bidding and production operation.
 - has possibly stochastic water inflows and market prices.

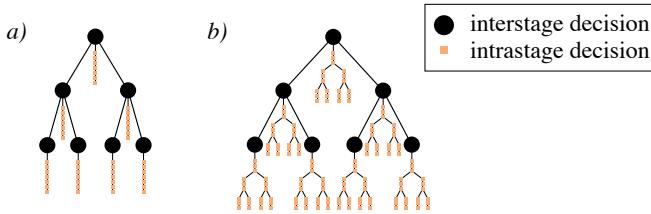


Figure 6.1: Two examples of multi-horizon decision trees. *a)* The uncertain data is revealed at once at the beginning of the intrastage problem. Therefore, the intrastage problem is deterministic. *b)* The uncertain data in the intrastage problem is revealed every three hours for the next three hours, thus, it is a stochastic problem.

Note that the intrastage subproblem itself is a stochastic multistage problem. Depending on the application, the disclosure of uncertain inflows and prices can be modeled differently.

The nested decision structures, see figure 6.1 for two examples, can be depicted as *multi-horizon decision trees*. Thus, the modeling based on such a structure, is proposed to be called *multi-horizon modeling*.

Notes from the modeling point of view

The proposed model formulation mimics the way operators typically think about a MTHP problem, which is water release or filling of the storage reservoirs under unknown optimal short-term operation. Therefore, the data processes which have to be modeled, are typically already available. This involves daily/weekly water inflows for the storage reservoirs and hourly inflows for the balancing reservoirs and hourly electricity market prices. So there is no need for aggregation or extending the data.

Similarly, the output of the optimization, the daily or weekly operating policies, can be used directly in the further optimization of the HPP. So no smoothing of policies is needed, which would be necessary e.g. for hourly policies, since no operator would change its strategy on an hourly scale.

Because of the decomposition of the problem, the water balance for the

balancing reservoirs is not respected from one interstage to the other. That is, the fillings at the beginning and end of each intrastage subproblem are given and fixed beforehand. Thus, the model neglects that with balancing reservoirs water from one day/week to the next one could be stored. Since there is a strong autocorrelation in the daily pattern of water inflows and market prices, this seems acceptable.

Notes from the computational point of view

SDP or SDDP methods can be applied to solve the master problem. The problem is decomposed in time and, for SDP, the storage reservoirs and the amount of water release from these reservoirs are discretized.

The multi-stage stochastic program in the intrastage problem, in contrast, is not decomposed. It is formulated as its deterministic equivalent and a commercial solver is used to solve it.

The proposed model, therefore, allows the exploitation of the strengths of DP and efficient mathematical solvers. That is, the curse of dimensionality in time is avoided by the decomposition in time. But, this is done only to daily/weekly time stages because the deterministic equivalent of a stochastic problem with hourly stages and up to daily or weekly time horizon is usually manageable by mathematical solvers very efficiently.¹

6.2.2 Mathematical model and solution methodologies

In order to formulate the mathematical model, the state variables $z(t)$, which is the filling of the reservoirs $v(t)$, are divided into storage $v^{stor}(t)$ and balancing $v^{bal}(t)$ reservoir state variables. Then, the recursive optimality equation is:

$$Q_t(v_{t-1}^{stor}) = \max_{W_t} \frac{1}{N_t} \sum_{j=1}^{N_t} \theta_t^j(W_t) + Q_{t+1}(v_{t-1}^{stor} - W_t) \quad (6.1)$$

$$\theta_t^j(W_t) = \max_{u,p,o,v,m} \sum_{\tau=1}^{\mathcal{T}} \mathbb{E}_{\mathcal{A}_\tau \in \Lambda_\tau^j} [c(\tau)m(\tau)]$$

¹See also the discussion about computational complexity on page 45.

$$\text{s.t.:} \left\{ \begin{array}{l} v_{\mathcal{B}_\tau}^{bal} = v_{\mathcal{A}_{\tau-1}}^{bal} - o_{\mathcal{B}_\tau} - f_u(u_{\mathcal{B}_\tau}) + f_p(p_{\mathcal{B}_\tau}) + a_{\mathcal{B}_\tau}, \dots \\ \forall \mathcal{A}_{\tau-1} \in \Lambda_{\tau-1}^j, \forall \mathcal{B}_\tau \in U(\mathcal{A}_{\tau-1}), \forall \tau \quad (6.1a) \\ \sum_{\tau=1}^{\mathcal{T}} [f_u(u_\tau^s) - f_p(p_\tau^s) - a_\tau^s] + o_t = W_t, \forall s \in \mathcal{S} \quad (6.1b) \\ m_{\mathcal{A}_\tau} = u_{\mathcal{A}_\tau} - p_{\mathcal{A}_\tau}, \forall \mathcal{A}_\tau \in \Lambda_\tau^j, \forall \tau \quad (6.1c) \\ 0 \leq u, p, o, v^{stor}, v^{bal} \leq ub_t, lb_t \leq W_t, m \leq ub_t \quad (6.1d) \end{array} \right.$$

The objective function, the intrastage subproblem, and how such models are actually solved are discussed next in more detail.

Objective function

The water release decision W_t is modeled as a here-and-now decision. There are N_t number of possible intrastage subproblems. Note that even if $N_t = 1$, the intrastage subproblem θ_t^j can still involve multiple scenarios since it is a stochastic problem.

The filling of the storage reservoirs at the end of the time period is given by $v_t^{stor} = v_{t-1}^{stor} - W_t$, thus, the profit-to-go depending on it, $Q_{t+1}(v_t^{stor})$, can be calculated outside and independently of the intrastage subproblem. Note that the water release W_t can also be negative. This means that the filling of a storage reservoir would increase which can happen due to water inflows or charges from other reservoirs.

Intrastage subproblem

The optimal value of the intrastage subproblem θ_t^j can be seen as an estimation of the short-term revenue. It is formulated with the help of scenario trees with the intrastage time stage $\tau \in \{1, \dots, \mathcal{T}\}$, where the time duration is one hour.

The objective of this problem is maximized revenue, which is the expected product of prices $c(\tau)$ and market positions $m(\tau)$ over all bundles \mathcal{A}_τ . The market position m is defined in the constraints of the intrastage subproblem in (6.1c) as the difference of electricity generation and pumping. It is the amount of energy which is sold or bought back at the day-ahead spot market. This variable would not be needed for the considered application, but it makes the formulation more streamlined, especially for extensions of the model.

The intrastage subproblem is subject to the water balance constraints. For the balancing reservoirs, these constraints have to hold for each hour τ , which is specified in (6.1a). For the storage reservoirs, however, the water balance is maintained only for the interstage t in (6.1b). Therefore, the sum over the hourly in- and outflows of the storage reservoirs for each scenario s has to be equal to the predefined water release W_t . Finally, all variables are positive except for the market position m and the water release W_t . The market position can be bounded in order to introduce some crude form of risk control. The water release W_t in the algorithm is discretized adaptively to reasonable points. The points depend on current water inflows, production capabilities, and filling of the storage reservoirs.

Solution methodologies

The problem (6.1) can be solved with SDP with the algorithm 2, where the subproblem in line 5 itself is a stochastic problem, formulated as the deterministic equivalent.

The resulting here-and-now decisions W_t are actually not used, but only the profit-to-go function $Q(v^{stor})$ and water values derived from it define the optimal policy. Nevertheless, a forward simulation like the one in algorithm 3 can deliver optimal operational decisions if needed.

The size of the variable vectors in the intrastage problem depend on the sum of the number of bundles for each time step $\sum_{\tau} |\Lambda_{\tau}|$. For example, consider a daily interstage problem with the intrastage subtree given in figure 6.1 a). Each intrastage vector (for each reservoir, turbine etc.) then has 24 entries. For the stochastic subtree given in figure 6.1 b) it would require $3^1 + 3^2 + \dots + 3^{24/3} = 9840$ entries. In both model versions, the same number of subproblems would have to be solved, however, the sizes of these subproblems differ.

Multi-horizon models can also be solved by SDDP. This makes particularly sense if the model consists of more than a few storage reservoirs. In SDDP, the profit-to-go is given by its approximation with hyperplanes. Therefore, the trick applied for SDP, where the introduction of the water release W_t allowed the independent calculation of the profit-to-go, is of no value here. Therefore, the model shown in (6.1) is adapted as

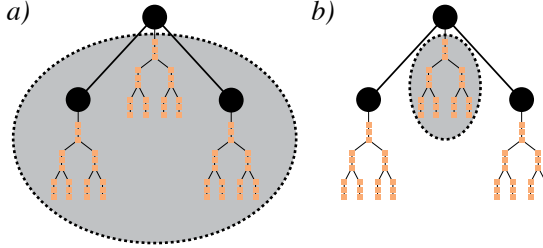


Figure 6.2: Two variants of risk measures in multi-horizon decision trees for the root node. a) The measure is based on the whole subtree. b) The measure is based only on the respective intrastage subtree.

follows:

$$Q_t(v_t^{stor}) = \frac{1}{N_t} \sum_{j=1}^{N_t} \left(\max_{u,p,o,v,m} \sum_{\tau=1}^{\mathcal{T}} \mathbb{E}_{\mathcal{A}_\tau \in \Lambda_\tau^j} [c(\tau)m(\tau)] + Q_{t+1}(v_t^{stor,j}) \right) \quad (6.2)$$

$$\text{s.t.} : \begin{cases} v_{\mathcal{B}_\tau}^{bal} = v_{\mathcal{A}_{\tau-1}}^{bal} - o_{\mathcal{B}_\tau} - f_u(u_{\mathcal{B}_\tau}) + f_p(p_{\mathcal{B}_\tau}) + a_{\mathcal{B}_\tau}, \dots & (6.2a) \\ \forall \mathcal{A}_{\tau-1} \in \Lambda_{\tau-1}^j, \forall \mathcal{B}_\tau \in U(\mathcal{A}_{\tau-1}), \forall \tau & (6.2a) \\ v_t^{stor,j} = v_{t-1}^{stor,j} - \sum_{\tau=1}^{\mathcal{T}} [f_u(u_\tau^s) - f_p(p_\tau^s) - a_\tau^s] - o_t^j, \forall s \in \mathcal{S}^j & (6.2b) \\ m_{\mathcal{A}_\tau} = u_{\mathcal{A}_\tau} - p_{\mathcal{A}_\tau}, \forall \mathcal{A}_\tau \in \Lambda_\tau^j, \forall \tau & (6.2c) \\ 0 \leq u, p, o, v^{stor}, v^{bal} \leq ub_t, lb_t \leq m \leq ub_t & (6.2d) \end{cases}$$

Note that no here-and-now decisions and, therefore, also no interstage decisions are present anymore. Note also that the filling of the storage reservoirs $v_t^{stor,j}$ at the end of an interstage can vary depending on the scenario j . Further, the water balance for the storage reservoirs is now formulated explicitly in (6.2b). With these modifications, SDDP can be applied as it was done in algorithm 4.

Whereas the formulation does not match exactly the one used for SDP, it is still as reasonable as before. Therefore, the results from both formulations will be very similar.

6.2.3 Risk functionals

Typically, a profit risk aware optimization provides more realistic and robust policies. In order to introduce this for multi-horizon models, two variants are proposed. Either the risk measure is based on the whole decision subtree or only on the intrastage scenario tree (see also figure 6.2). Both ways have their advantages and either one can be more reasonable for a specific application.

Risk measures based on interstage subtree

The first formulation of the risk aware multi-horizon problem corresponds to (5.5) if SDP or to (5.6) if SDDP is applied respectively.

From the modeling point of view, the profit risk associated to the short-term intrastage decisions and the future risk adjusted profit-to-go for the storage reservoirs are considered. Such a construction is very close to how operators perceive these profit risks.

But, in order to calculate the risk measure, a reasonable number of scenarios N_t have to be considered. For stochastic intrastage subproblems, this can be a problem because of the computational complexity and modeling effort. Therefore, such risk measures are recommended to be used primarily if deterministic intrastage subproblems are present.

Risk measures based on intrastage subtree

In the second formulation, the risk measure is calculated only based on the current intrastage subtree. In such a case, the objective function in (6.1) changes to the following one:

$$Q_t(v_{t-1}^{stor}) = \max_{W_t} \frac{1}{N_t} \sum_{j=1}^{N_t} \theta_t^j(W_t) + Q_{t+1}(v_{t-1}^{stor} - W_t) \quad (6.3)$$

$$\theta_t^j(W_t) = \max_{u,p,o,v,m} (1 - \lambda) \sum_{\tau=1}^{\mathcal{T}} \mathbb{E}_{\mathcal{A}_\tau \in \Lambda_\tau^j} [c(\tau)m(\tau)] +$$

$$\lambda \text{AV@R}_\alpha \left[\sum_{\tau=1}^{\mathcal{T}} c_\tau^s m_\tau^s \right]$$

Only the objective function of the intrastage subproblem is changed. $\sum_{\tau=1}^{\mathcal{T}} c_\tau^s m_\tau^s$ calculates the revenue per intrastage scenario s . Therefore,

instead of maximizing only the average revenue, the tail of the revenue distribution is weighted more.

Note that this risk measure treats the profit-to-go risk-neutrally. From the modeling point of view, only the short-term decisions are regarded as “risky” decisions, whereas the interstage decisions, the water release for the storage reservoirs, are not. This can make sense for particular applications where the intrastage uncertainty is much more important to consider than the interstage one. For instance, this could be applications, where it is important to consider the profit risk in intraday markets, whereas for a day-ahead market a risk-neutral view is sufficient.

The risk measure is not a dynamic one as defined in (5.4). Therefore, the question arises if this risk measure is time-consistent. The overall formulation is dynamic and risk-neutral. Therefore, time consistency of the found policies, which is the interstage decisions as well as the profit-to-go function, is not an issue.

For the intrastage decisions, on the other hand, the proposed risk measure could lead to time inconsistencies within an intrastage subproblem. So the question here is not about time consistency of the overall model, but rather, if the intrastage subproblem model is still realistic enough. However, remember that the overall objective is to find medium-term operation strategies with time horizons of more than one year. From this perspective, a possible time inconsistency for a few hourly operation decisions seems to be tolerable.

From the formulation of (6.3), it is not clear how to solve the intrastage subproblem. A discretization of the intrastage decisions would be possible but not easily realizable. A rather pragmatic approach is proposed here. It is assumed that scenarios with low inflows and prices will also relate to least revenue. The scenarios are, therefore, first sorted according to their inflows and prices and then corresponding weights are introduced. Such a procedure does not increase the computational complexity and can be implemented in a straightforward way for both SDP and SDDP solution methodologies.

6.2.4 Provision of ancillary services

As described in chapter 2, ancillary services markets, and in particular the market for provision of secondary frequency control (SFC), are very

reasonable to consider in a MTHP optimization. In order to include a SFC market within a multi-horizon model, some simplifications are made. The considered profit out of the SFC market is only the remuneration for holding the capacity, whereas payments for energy delivery are neglected. Therefore, it is assumed that the demanded request is symmetric within the tender period, such that the energy delivery is balanced out which is a valid assumption in a medium-term perspective.

Further on, the bidding process is simplified. The bid capacity is assumed to be a multiple of 10 MW per qualified turbine. Then, the highest acceptable price for the capacity provision is estimated beforehand out of historical data, where in this thesis a conservative value of 20 CHF/MWh is assumed.

These assumptions may be seen as questionable. But, remember that such a MTHP optimization is not used for direct decision support about SFC bidding, but to estimate realistic water values which incorporates the opportunities in SFC markets for a time horizon of a few years. Hence, predefined bid prices seem to be reasonable. Note also that with such a construct the simplifications of the market are similar to the one typically done in energy markets with an hourly priced forward curve.

With these assumptions, the only additional decision to take is how much SFC shall be provided by each qualified turbine. This decision is modeled as a wait-and-see decision within the intrastage subproblem. In order not to make the mathematical formulation unnecessarily complicated, assume now that the interstage time period corresponds to the tender period duration. Then, the optimality equation is as follows:

$$Q_t(v_{t-1}^{stor}) = \max_{W_t} \frac{1}{N_t} \sum_{j=1}^{N_t} \theta_t^j(W_t) + Q_{t+1}(v_{t-1}^{stor} - W_t) \quad (6.4)$$

$$\theta_t^j(W_t) = \max_{u,p,o,v,m,q} \sum_{\tau=1}^{\mathcal{T}} \mathbb{E}_{\mathcal{A}_\tau \in \Lambda_\tau^j} [c(\tau)m(\tau)] + c_t^q \sum q_t$$

$$\begin{aligned}
\text{s.t.:} \left\{ \begin{array}{l}
v_{\mathcal{B}_\tau}^{bal} = v_{\mathcal{A}_{\tau-1}}^{bal} - o_{\mathcal{B}_\tau} - f_u(u_{\mathcal{B}_\tau}) + f_p(p_{\mathcal{B}_\tau}) + a_{\mathcal{B}_\tau}, \dots \\
\forall \mathcal{A}_{\tau-1} \in \Lambda_{\tau-1}^j, \forall \mathcal{B}_\tau \in U(\mathcal{A}_{\tau-1}), \forall \tau \quad (6.4a) \\
\sum_{\tau=1}^T [f_u(u_\tau^s) - f_p(p_\tau^s) - a_\tau^s] + o_t = W_t, \forall s \in \mathcal{S} \quad (6.4b) \\
m_{\mathcal{A}_\tau} = u_{\mathcal{A}_\tau} - p_{\mathcal{A}_\tau}, \forall \mathcal{A}_\tau \in \Lambda_\tau^j, \forall \tau \quad (6.4c) \\
(q_t^{active})^T (q^{min} + q^{max}) \leq u_\tau \leq \bar{u} - (q_t^{active})^T q^{max} \quad (6.4d) \\
0 \leq u, p, o, v^{stor}, v^{bal}, q \leq ub_t, lb_t \leq W_t, m \leq ub_t \quad (6.4e) \\
q_t \in \mathbb{Z}^n, q_t^{active} \in \{0,1\}^n \quad (6.4f)
\end{array} \right.
\end{aligned}$$

Compared with the formulation (6.1) without the provision of SFC, there is an additional integer wait-and-see decision $q(t)$, which describes how much SFC capacity a turbine shall provide. The sum of these capacities over all qualified turbines $\sum q$, times the prefixed remuneration $c^q(t)$, contributes to the intrastage revenue.

If a turbine provides SFC, then the binary variable q^{active} for it is 1. In such a case, its limits are adapted in equation (6.4d) in order to indeed provide the correct capacity for the tender period.

The formulation for an application of SDDP follows in the same way from (6.2) and, therefore, is not shown here.

Because of the integer variable $q(t)$ as well as the binary variable q^{active} , the intrastage subproblem is now a MILP. For the application of SDP this is not an issue. However, traditional SDDP is not applicable anymore since the value function is non-concave. There, extended methods like the dualized SDDP as described in chapter 4 have to be applied.

Such a construction is only possible as long as the tender period of the ancillary services is shorter or equal to the interstage time duration. Since in the near future this period is supposed to be one or less than one day this approach is applicable. Note that if multiple tender periods have to be considered within an intrastage problem, the formulation has to be adapted only slightly.

Similarly, the formulation can be adapted if ancillary services are provided in a more complicated way, e.g. with a turbine in combination with a pump.

6.3 Evaluation

The model for the Kraftwerke Oberhasli AG (KWO) power plant consists of four storage reservoirs and six balancing reservoirs. An application of

SDP to such a model is possible, but computationally very demanding. Therefore, SDDP is used here. In the evaluation, standard modeling approaches are compared with the multi-horizon approach.

The model for the Kraftwerke Mattmark AG (KWM) power plant on the other hand is built up out of only one storage reservoir and one balancing reservoir. This relatively undemanding model is solved primarily with SDP. In the evaluation, it is analyzed, how interweekly dynamics, the intrastage subproblem, can be modeled. Further, KWM serves as a worst case example for the motivation of multi-horizon models.

The different models are compared threefold. First, the models themselves are discussed conceptually. Then, their required computational efforts are analyzed. Finally, the results of the models, the water values, are applied in an operation simulation. The simulation is based on the application of operation decisions from sequential hourly optimizations. In the hourly optimizations, the water values are used as water opportunity costs and a MILP is formulated in order to find the optimal operation.

6.3.1 Multi-horizon modeling against traditional procedures

As already described in the motivation of the proposed MTHP model, there are two obvious modeling alternatives when dealing with storage and balancing reservoirs. The first alternative is to model an hourly time step, which allows considering a detailed HPP model. The second alternative is to aggregate balancing reservoirs in a way that a longer time step is sufficient and thus less computational effort is needed.

Both alternatives are used in academia and practice. They are explained next and then compared with multi-horizon modeling. The models are applied on KWO and SDDP is used to solve them.

MTHP models with hourly time steps

Hourly time steps seem to be the natural choice for modeling a MTHP problem with balancing reservoirs. However, a closer look reveals many difficulties of such an approach depending on which HPP type is considered. Therefore, first a suitable model with hourly time steps is discussed in general, before it is applied to KWO and compared with the multi-horizon modeling approach.

Table 6.1: Options to model a HPP for a MTHP problem and to solve it.

time step	states	SDP	SDDP	model	complexity
Traditional modeling:					
hourly	stor./bal.	+	-	+	-
hourly	storage	++	+	--	+
daily	storage	++	++	-	++
Multi-horizon:					
daily/hourly	stor./bal.	++	++	++	+

A model with hourly time steps and only storage reservoirs as state variables is not reasonable. Because in such a case the HPP model would have to be aggregated a detailed analysis of it does not make sense. Therefore, the only choice is to consider the balancing reservoirs as well.

For a model with hourly consideration of both storage and balancing reservoirs the application of SDDP will be troublesome. The reason for this is that the fillings of balancing reservoirs are very volatile (in a MTHP perspective) and, therefore, difficult to predict. In such a setting, the convergence of SDDP will be poor and an application of SDP is more meaningful.

Table 6.1 shows a comparison of such modeling options. As a conclusion one can say that a model with hourly time steps only make sense if the balancing reservoirs are also considered as states and if the model is solved with SDP.

However, also SDP is troublesome to use in this setting. The more states that are considered the more SDP suffers from the curse of dimensionality. Additionally, the discretization of the states has to be relatively fine in order to be able to estimate profit-to-go functions properly for hourly time steps. Therefore, such an approach is still applicable for HPPs like KWM, whereas for KWO the required computational effort is too big.

Model from commercial software package for KWO

The company operating KWO uses as decision support a commercial software tool, which models the HPP hourly and applies SDP. The huge computational burden for such a solution is eased by a sparse discretiza-

Table 6.2: Computational complexity for different modeling options, applying SDP on KWO with one processor core.

modeling approach	nr. of subproblems	solving time
multi-horizon	33 Mio.	38 days
hourly, bal. res. 1 discrete point	788 Mio.	91 days
hourly, bal. res. 2 discrete points	50432 Mio.	5824 days

tion of the balancing reservoirs to one point. The filling of those reservoirs, therefore, stays constant. Nevertheless, the model requires access to high-performance computing in order to be solvable.

Table 6.2 compares the computational complexity on KWO, if the balancing reservoirs are discretized with one or two points. Since there are six balancing reservoirs the number of subproblems and, therefore, also the solution time would increase by $2^6 = 64$. The calculation is based on the solving time of one subproblem on one processor core. Note that, in parallel computing, the numbers would decrease almost proportionally with the number of available processor cores.

A subproblem in a multi-horizon setting, in comparison, is more difficult to solve, but fewer of them have to be computed. Note that here all reservoir types are taken into account and that the balancing reservoirs are considered continuously.

The results from the commercial software package were compared with the ones from the multi-horizon method. The detailed results are subject to confidentiality, nevertheless, some interesting findings with regard to the modeling can be shown.

Figure 6.3 compares the sum of the water overflow in all basins in KWO for the two modeling approaches. These overflows happen within the optimization and are not the result of applying the optimized policies in a simulation. Therefore, this plot gives some insights in how the different optimizations model the HPP.

It is clearly visible that the multi-horizon model leads to much less overflow (more than 80 %). The reasons for that are threefold. The most influence has the poor representation of the balancing reservoirs in the commercial model. Since their filling stays always constant, storing of water in it is not possible. This leads to high water overflow in periods with high water inflows.

In order to analyze this further, the multi-horizon optimization was adapted to neglect storing of water in the balancing reservoirs as well.

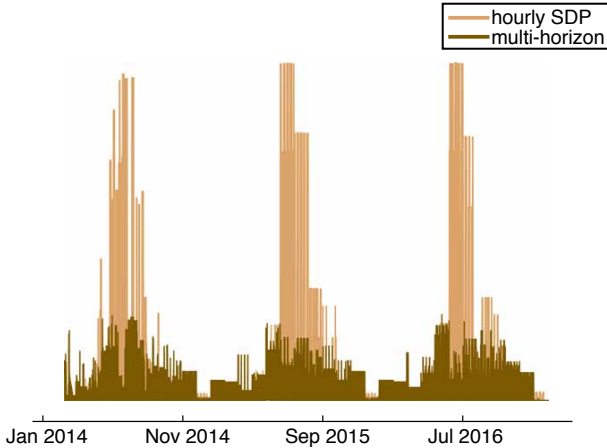


Figure 6.3: Comparison of water overflows between multi-horizon approach and modeling solution based on hourly SDP with one point discretization of balancing reservoirs.

This optimization showed an increase in spillage, but still considerably less than from the hourly SDP optimization. This is due to the second reason, which is more subtle and happens if hourly time steps are considered in a MTHP. Since in DP an independent Markov process is assumed, there is only the knowledge of the expected future. That means that the hourly operation decisions depend on the expected future revenue. Since the water balance in the reservoirs do not change much in an hourly scale, it can be difficult in practice to find correct operation decisions. Since the policy of the hourly SDP was based on the discrete state of the storage reservoirs, its discretization would have to be very fine, which was however not the case in order to avoid further computational complexity.

Finally, the last reason is due to the deterministic modeling of the intrastage subproblem in the multi-horizon approach. So, since the water inflow then can be anticipated within each day, less overflow results.

As a conclusion one can note that a reasonable modeling with hourly time steps with application of SDP for HPPs like KWO is computationally very demanding and, therefore, is not considered anymore from now on.

MTHP model with hourly time steps and application of SDDP on KWO

As described in the previous sections, for models with hourly time steps for complex HPPs like KWO, SDDP can not be applied directly because of convergence issues. Further, it was shown that also SDP is not well suited because of the curse of dimensionality. Finally, a sparse discretization of the balancing reservoirs eased this curse of dimensionality at the price of an inaccurate model. A similar idea can now be applied here so that SDDP is usable.

If the fillings of the balancing reservoirs are not considered as state variables, then the convergence of SDDP is less of an issue. This is because even an hourly filling of the storage reservoirs is not volatile and can be predicted better.

Note that also the one-point discretization of the balancing reservoirs in the commercial solution implies that the balancing reservoirs are not part of the state variables anymore. Therefore, such a simplification leads to the same modeling issues as already described in the previous section. So the balancing reservoirs are considered in the optimization, but their fillings stay constant.

The application of SDDP outperforms the commercial solution for KWO because a state space of four storage reservoirs can be handled much more efficiently. Further, the formulation of an hourly subproblem as a mathematical program prevents the discretization of production decisions.

The recursive optimality equation looks as follows:

$$Q_t(v_{t-1}^{stor}) = \frac{1}{N_t} \sum_{j=1}^{N_t} \left(\max_{u,p,o,v,m} c_t^j m_t^j + Q_{t+1}(v_t^{stor,j}) \right) \quad (6.5)$$

$$\text{s.t.} : \left\{ \begin{array}{l} v_t^{stor,bal,j} = v_{t-1}^{stor,bal,j} - o_t - f_u(u_t) + f_p(p_t) + a_t \end{array} \right. \quad (6.5a)$$

$$v_t^{bal,j} = v_{t-1}^{bal,j} = v_0^{bal} \quad (6.5b)$$

$$m_t = u_t - p_t \quad (6.5c)$$

$$0 \leq u, p, o, v^{stor,j}, v^{bal,j} \leq ub_t, lb_t \leq m \leq ub_t \quad (6.5d)$$

In comparison to the problem (6.2), t has a time duration of one hour. Therefore, the subproblem for a scenario j is very small and consists of one decision per pump, turbine, etc. Both balancing and storage reservoir are treated in the same way, which is in contrast to (6.2). In (6.5a), the respective water balances are assured. In the course of the

algorithm, the filling of the balancing reservoirs is not tracked, since they are not part of the state space. Therefore, their filling is kept constant in (6.5b), e.g. equal to zero.

The implementation of such an algorithm is quite similar to implementing a multi-horizon approach. The key difference is that there are more time steps but the subproblems are smaller. There are also similar requirements for the input data. However, hourly water inflows into the storage reservoirs are needed. This data is mostly not available in practice. Since the correct hourly values of these are not important, this data can be easily constructed out of daily inflows by simply distributing them uniformly across the hours.

MTHP model with daily time steps

As shown in table 6.1, another option to model the MTHP problem for a complex HPP like KWO would be a temporal aggregation leading to daily time steps. For a time horizon in MTHP problems of typically a few years, this aggregation seems to be reasonable at first. Some models in literature even work with weekly or monthly time steps. But an aggregated time step leads also to the need of aggregating the market and the HPP model.

Aggregating an hourly electricity market to a daily one or even further can be meaningful, since the uncertainty in an hourly market a few years ahead is quite large. Possibilities of how to do that are shown in section 6.3.2.

In contrast to the market model, the implications on the HPP model can be more awkward. A daily view on balancing reservoirs is not reasonable. Such reservoirs are then often aggregated which mostly also implies an aggregation of turbines and pumps which is troublesome.

Out of these reasons, a similar approach to the one with hourly time steps is chosen here: The balancing reservoirs are considered, but they are not modeled as state variables. Therefore, the filling of these balancing reservoirs stays the same, but no aggregations of turbines and pumps are necessary.

However, since a daily time step is modeled, the production and also the electricity market are considered with their daily average values.

With these modifications the mathematical model is the same as (6.5) with the time step t modeling a duration of one day. Whereas previously

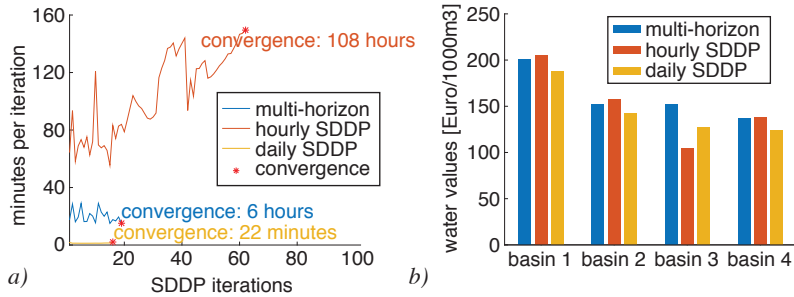


Figure 6.4: *a)* Time duration per SDDP iteration until convergence (gap lower than 5%). *b)* Comparison of water values for the first stage at the beginning filling of the seasonal reservoirs.

the constant filling of the balancing reservoirs let to model inaccuracies, this issue is here less prominent since the detailed operation is not considered explicitly. On the other hand, the performance of the HPP is underestimated considerably with a daily view.

Comparison of computational complexity

Table 6.2 already compared the computational complexity of some (hypothetical) models of KWO, solved by SDP. Now the complexity is compared for the models which are solvable by SDDP, which is more convenient for HPPs like KWO.

To keep the computational burden low as well as to achieve a meaningful comparison, the optimizations were performed without considering different scenarios of water inflows and market prices. The models were formulated risk-neutrally without consideration of a market for ancillary services for a time horizon of one year. Dualized SDDP algorithms were used with a cutting plane selection method based on level one dominance and maturity of the cuts.

Figure 6.4 *a)* compares the time duration per SDDP iteration for the different methods until convergence. The criterion for convergence was chosen to be an optimization gap between upper and lower bound of the algorithm of lower than 5%. Note that because the uncertain data is not sampled in every step, the lower bound is also a deterministic one. The multi-horizon formulation needed around 20 minutes per SDDP it-

eration and convergence was reached after 19 iterations. The optimization, therefore, needed 6 hours.

The hourly SDDP method has smaller subproblems but many more of them to solve. As a consequence, one iteration required around one hour of computing time which increases due to the larger set of hyperplanes.² Since hourly interstage time steps are modeled, the convergence rate is lower and 62 iterations were necessary. Another notable issue with the hourly SDDP formulation is the data handling. Although the hourly hypercuts are very similar, they have to be stored individually. This leads to a roughly 24 times higher requirement of memory and data storage, compared to the multi-horizon and daily formulation.

In the daily formulation, the subproblems to solve are of similar size as the ones of the hourly formulation, but there are 24 times less of them. Compared to the multi-horizon approach, the number of subproblems are similar, but they are easier to solve and the convergence of the algorithm should be slightly better. These findings were qualitatively confirmed. The solution time per SDDP iteration was of around 1.5 minutes and convergence was found after 16 iterations, resulting in an overall required computing time of 22 minutes.

Figure 6.4 *b*) depicts the water values for the very first time period. In practice, the MTHP optimization is often repeated every day with a moving time horizon and, therefore, these water values are especially important.

The water values for the three methods are similar. They reveal the different elevations of the basins with the tendency that the higher the relative elevation of a basin, the higher also the water value. Then, the low water values from the daily approach demonstrate that in the approximated model short-term flexibility is not exploited. But note that only the values for one specific hour are shown and, therefore, this result has to be taken with caution.

The hourly SDDP provides an hourly set of these water values, whereas the other two approaches have only daily ones. However, it can be shown that the hourly water values do not change much from one hour to the other as expected.

²The variability in computing time came from occupation of the server by other simulations.

Table 6.3: Performance of operation policies. The lower bound should be as close as possible to the simulated profit.

	multi-horizon	hourly SDDP	daily SDDP
lower bound [Mio €]:	119	111	96
profit simulation [Mio €]:	116	99	111
deviation:	2.3 %	12 %	13 %

Comparison of operation policies

Even a small deviation of the water values will have a great impact on how the HPP is operated. In order to analyze the quality of the different proposed water values, they are now applied in the operation simulation.

For a proper operation simulation study the simulation would have to be repeated for different sampled uncertain data. Nevertheless, here are presented only the results from one simulation run. Further, the chosen sample of uncertain data is the one which was already used in the optimization. The reason for performing such an in-sample simulation is to be able to compare how well the approaches approximate the original problem. Not discussed here are the questions, if a close representation of the model is necessary and if its policies are then indeed good in practice.

Table 6.3 compares the achieved profits from the different methods. The lower bound is the operation value estimated by the respective SDDP forward step. This value should be as realistic as possible and, therefore, be similar to the profit achieved in the operation simulation. This would indicate that the method models the problem realistically.

Note that a higher profit in the simulation does not necessarily mean that a policy is better. It only means that it is better for this specific scenario, which could be also due to overfitting. Interesting to note is that in all simulations a comparable small amount of spillage is present. So the reason for higher simulation profits is primarily because of a different use of the water in time.

The shown values again confirm the findings given earlier. The multi-horizon approach overestimates the profit but only slightly. This indicates a close representation of the problem. The daily method underestimates the profit, where its poor model leads to a difference of more than 10 %.

Interestingly, the policy from the hourly SDDP approach leads to a simulation performance more than 10% lower than its lower bound. This can be explained by the unnecessary detailed optimization of a relatively simple model.

Concluding remarks

For relatively simple HPPs like KWM, SDP is a good choice as a solution method since it can handle a detailed model. For more complex HPPs, where KWO was analyzed as an extreme example, SDP can be troublesome to use because of the curse of dimensionality.

This problem can be eased by a sparse discretization of balancing reservoirs which leads in return to model inaccuracies. Similar model inaccuracies are present if the problem is solved with SDDP for traditional models. A daily aggregated model performed reasonably well, but it neglects hourly flexibility leading to an underestimation of the operation opportunities. But an extension of it to hourly time steps was even more troublesome, which could be explained by the issue of optimizing a poor model in a more detailed way. Therefore, considering the very high computational requirement of the hourly method, the daily one is much more reasonable.

Multi-horizon models, on the other hand, showed a couple of advantages in this respect. They are more realistic, have less modeling issues since no aggregation is necessary and have a computational complexity which is almost as good as the one from aggregated models. Therefore, properly constructed multi-horizon models solved with SDDP are applicable for complex MTHP problems solvable in less than a day and simultaneously allow many modeling details.

6.3.2 Modeling of the intrastage subproblem

The second part of the evaluation of multi-horizon models analyses how the intrastage subproblem of it can be modeled. In order to demonstrate the findings expressively, an interstage time duration of one week is considered. The different approaches are shown on KWM and SDP is used to solve the models. Additionally, an ancillary services market is taken into account.

From the modeling perspective, two issues are important to consider, first, how to model the electricity market and second, how to model

hourly operation. Four different approaches are presented, which model the weekly intrastage subproblem differently:

- method 1: weekly peak and off-peak prices;
- method 2: price duration curves;
- method 3: deterministic subproblems; and
- method 4: stochastic subproblems.

The first two methods are approaches which can be found in academic literature whereas methods three and four are different alternatives of multi-horizon models.

The methods are first described conceptually and mathematically and then compared in the evaluation part.

Method 1: weekly peak and off-peak prices

The first method neglects hourly flexibility. Water inflows and market prices are estimated and aggregated as weekly expected values. Two different prices are assumed: energy for peak prices $c^{peak}(t)$ and pumping for off-peak prices $c^{off-peak}(t)$. The balancing basins are neglected and turbines and pumps are, therefore, aggregated accordingly. For KWM, this results in a model with a single basin, turbine, and pump.

Mathematically the model is as follows:

$$Q_t(v_{t-1}^{stor}) = \max_{W_t} \frac{1}{N_t} \sum_{j=1}^{N_t} \theta_t^j(W_t) + Q_{t+1}(v_{t-1}^{stor} - W_t) \quad (6.6)$$

$$\theta_t^j(W_t) = \max_{u,p,o,v,q} c_t^{j,peak} u_t - c_t^{j,off-peak} p_t + c_t^q q_t^T q^{max}$$

$$\text{s.t.:} \left\{ \begin{array}{l} f_u(u_t) - f_p(p_t) - a_t^j + o_t = W_t \quad (6.6a) \\ q_t^T (q^{min} + q^{max}) \leq u_t \leq \bar{u} - q_t^T q^{max} \quad (6.6b) \\ 0 \leq u, p, o, v^{stor}, v^{bal} \leq ub_t, lb_t \leq W_t \leq ub_t, q_t \in \{0,1\} \quad (6.6c) \end{array} \right.$$

Note that random data consists of peak and off-peak prices and water inflows a_t . The intrastage decision vector consists of only one entry per generation, pumping, etc. Therefore, the intrastage problem is a small

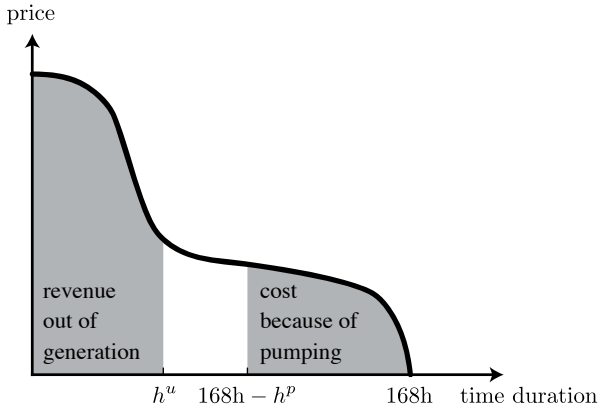


Figure 6.5: Method 2: Schematic example of a price duration curve. Revenue out of generation as well as costs because of pumping are shown. Note that since the overall water discharge is fixed, with more pumping more generation is possible.

MILP.

Alternatively, generation and pumping could have been discretized, which would avoid the use of a mathematical solver. This is a standard approach and is applied often in practice.

Method 2: price duration curves

To have only two prices to describe the energy market for a week, seems a too coarse approximation, especially, for volatile markets with a high penetration of fluctuating renewables. However, one may also argue that the actual price process is not so important for estimating short-term profit in the perspective of a MTHP optimization. In between, there is the concept of price duration curves which is also called occupation times of prices.

A price duration curve (example in Fig. 6.5) is constructed out of the proportion of hourly prices below a certain price for some time duration. Since the revenue depends on price times quantity of sold energy, it can be estimated by an integration of the price duration curve.

In [107], such curves are multiplied by quantity-price offers and integrated with respect to prices. Here, another approach is followed, where

the sum of the water discharge for the next week is discretized. Then, for a given water discharge, an optimization problem is formulated with the objective to find the time durations of pumping h^p and generating h^u . The expected short-term profit can then be derived.

It is assumed that the HPP either generates or pumps fully or not for each hour. Random data involves again prices and water inflows. To estimate random price duration curves pdc^j , the hourly price process itself is sampled and the price duration curve is constructed out of it. The problem can now be formulated as follows:

$$Q_t(v_{t-1}^{stor}) = \max_{W_t} \frac{1}{N_t} \sum_{j=1}^{N_t} \theta_t^j(W_t) + Q_{t+1}(v_{t-1}^{stor} - W_t) \quad (6.7)$$

$$\begin{aligned} \theta_t^j(W_t) = & \max_{h^u, h^p, o, q} u_t \cdot \int_0^{h_t^u} \text{pdc}_t^j(\tau) d\tau - \bar{p} \cdot \int_{168h-h_t^p}^{168h} \text{pdc}_t(\tau) d\tau \dots \\ & + q_t^T \cdot (168h(q^{max} + q^{min})c_t + q^{max}c_t^q) \end{aligned}$$

$$\text{s.t.} : \begin{cases} h_t^u f_u(u_t) - h_t^p f_p(\bar{p}) + 168h \cdot q_t^T f_u(q^{max} + q^{min}) - a_t^j + o_t = W_t & (6.7a) \\ u_t = \bar{u} - q_t^T (q^{max} + q^{min}) & (6.7b) \\ 0 \leq h^u, h^p, o \leq ub_t, lb_t \leq W_t \leq ub_t, q_t \in \{0, 1\} & (6.7c) \end{cases}$$

Note that in this formulation u_t is not the physical utilization of the turbines, because it does not consist of the generation due to capacity provision. Therefore, it is given in (6.7b) differently than before.

Similarly, the water balance (6.7a) is changed accordingly. Since there are no balancing reservoirs to consider, only the water discharge from the storage ones is regarded.

In the objective function, the maximal additional production u_t and the maximal pumping \bar{p} are weighted with the average lowest or highest prices for some hours h_t^u and h_t^p respectively, which are the primary decision variables. Additionally, the profit out of the sold production because of the capacity provision has to be considered with the average market price c_t since the capacity is provided continuously. Finally, the remuneration for this provision contributes also to the intrastage revenue.

The problem turns out to be challenging to solve. Therefore, the price duration curves are assumed to be piece-wise linear, which approximates the problem to a quadratic mixed-integer problem.

From the modeling point of view, there are several approximations with this formulation. The most severe is that, similar to the first method, timing is not respected at all. This means that when and in which order the decisions are taken within a week is not considered.³

The advantage with this formulation is the consideration of a reasonable representation of the opportunities in the hourly day-ahead market.

Method 3: deterministic intrastage subproblems

The third method is based on the multi-horizon approach. The intrastage subproblems are modeled deterministically. That means that the intrastage operation is done with the knowledge of the water inflows and market prices. Therefore, only a fan of scenarios is considered per interstage.

The mathematical formulation is the one already introduced in (6.4). However, since per scenario j no additional uncertainty is present, the set of bundles at each intrastage Λ_τ consists only of one bundle, which is one path of the fan. This simplifies the problem considerably.

Note that the HPP, as opposed to the first and second method, is considered fully with all turbines, pumps, and basins present as well as in hourly resolution.

Approximations made are first that the random data are assumed to be known one week in advance. Further, the fillings of the balancing reservoirs are neglected in the calculation of the profit-to-go functions as well as their water balances are not respected between consecutive weeks. This results in empty fillings of the balancing reservoirs at the beginning and end of each week.

Method 4: stochastic intrastage subproblem

The model from method 4 considers stochastic intrastage subproblems. Whereas in the third method the random data is disclosed at the beginning of a week, the day ahead prices are now revealed daily for one day. So the market prices are known only one day in advance. The water inflows are still revealed weekly to keep the computational burden low and since their influence in this study is less important.

³It may happen that e.g. high market prices occur all at the beginning of a week where the reservoirs may be empty and generation not possible. Such cases are not taken care of with the consideration of price duration curves.

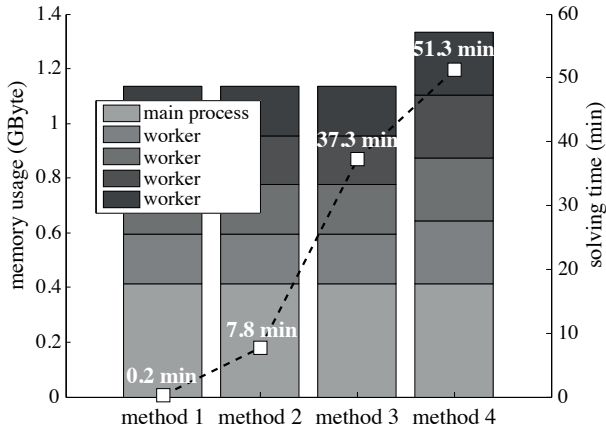


Figure 6.6: Time duration needed for solving the different optimization methods and memory requirement of the optimizations.

The mathematical formulation was already given in (6.4). Compared with deterministic intrastage subproblems, the method 4 is much more realistic from the modeling point of view. Computationally, the same number of subproblems have to be solved, however, the sizes of these subproblems differ. This results in that one stochastically formulated intrastage subproblem was constructed and solved in 0.41 s, whereas the deterministic variant required only 0.12 s.

Evaluation of the four intrastage modeling approaches

The performance of the four different methods are now compared first, regarding their computational burden and then, secondly, regarding the quality of their proposed operation policies.

In figure 6.6, the time durations as well as the memory requirements needed for the different methods are depicted. The first method needs 15 seconds, which is 30 to 200 times faster than the other methods. This would be a clear advantage in daily use. Method four has higher memory requirements than the other ones. Memory usage of this method (as well as solving time) will further increase exponentially if the number of intrastage time periods, stochasticity or HPP complexity (number of

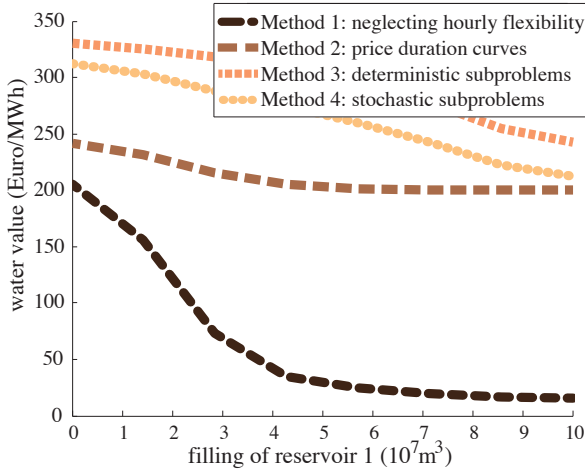


Figure 6.7: Water values for the different methods for the first time stage. Note that the first two methods undervalue relatively the short-term revenue whereas the third one overvalues it.

state variables) is increased.

The results of the optimization methods, the water values, are shown and compared in figure 6.7 for the first time stage. Notable is that the more opportunities in the hourly operations are considered, the higher the water value is. However, since the third method assumes perfect weekly knowledge, it overvalues the water value. Therefore, method 4 should give a more realistic estimation. The water values for methods 3 and 4 are similar not only for the first time stage but also for the others, with roughly 80% of them having a difference of less than 10%. This gives already a hint that the additional effort of a more realistic data process may not have much influence on its performance.

A Monte Carlo operation simulation study is performed in order to analyze the application of the different proposed water values. Because of lack of a sufficient amount of historical data, distributions are estimated out of the available data. Water inflows and market prices are sampled out of it for 100 scenarios.

Table 6.4 shows the expected profit, the relative standard deviation, and the mean profit for the 10% worst scenarios ($AV@R_{10\%}$). The values are shown with and without consideration of provision of SFC.

Table 6.4: Comparison of optimization methods without / with provision of SFC reserves.

	exp. profit [Mio €]	rel. stand. dev.	AV@R _{10%} [Mio €]
Method 1	34.24 / 34.99	2.19% / 2.53%	32.66 / 33.27
Method 2	29.13 / 30.44	1.95% / 2.47%	27.90 / 29.03
Method 3	34.80 / 35.85	2.36% / 3.99%	33.27 / 33.47
Method 4	33.40 / 39.14	2.57% / 4.10%	31.82 / 36.28

Method 1 leads to unexpectedly good results. However, the performance evaluation was based on market data, where peak and off-peak price periods were clearly present, which will or already is not anymore the case.

Method 2 performs worse than expected. An explanation for it could be that although the price process is considered in a detailed way, the HPP itself is simplified considerably. This leads to using non-existing resources more efficiently which may result in less effective policies.

Method 3 outperforms method 4 for optimizations without the consideration of SFC provision. Interesting is also the increased robustness compared with method 1: the AV@R_{10%} is considerably higher whereas the relative standard deviation, as an alternative risk measure, would indicate slightly more risk.

Finally, the proposed method 4 outperforms the other methods only if SFC reserves provision is considered. But, in this case, the increase of both expected profit and AV@R_{10%} is substantially by around 10%.

Concluding remarks

In this section, a comparison of different approaches to model the intrastage subproblems were performed. For the evaluation, the relatively undemanding HPP KWM was chosen which can be seen as a worst case example to motivate multi-horizon modeling approaches.

The results indeed suggests that a simple modeling of peak and off-peak prices can deliver good results at a very low computational burden. But, already the introduction of an ancillary services market reveals the advantage of a more detailed modeling, where both deterministic and stochastic intrastage subproblems performed substantially better.

However, it should be noted that the performance of methods with stochastic intrastage subproblems in practice would require good mod-

eling skills of the random data processes. Therefore, as a good compromise, one could also recommend deterministic subproblems which is relatively undemanding from the modeling point of view, but still delivers reliable results for a reasonable computational effort.

As a final conclusion, it can be stated that either stochastic or deterministic intrastage subproblems are meaningful depending on the specific application. On the other hand, it was shown that even for simple HPPs like KWM a multi-horizon modeling approach is meaningful.

6.4 Application examples

As seen in the evaluation part of this chapter, the meaningful modeling of multi-horizon based approaches has to be adapted depending on the application. Given next are two examples which apply multi-horizon models to problems where different markets are considered. In both examples the modeling flexibility of multi-horizon models is exploited differently.

6.4.1 Long-term valuation

Apart from MTHP optimizations, multi-horizon models can also be used in long-term valuations of pumped HPPs.

Large HPP investments are very capital intensive and need to be evaluated for a long time horizon. This evaluation is subject to many different uncertainties and is, therefore, very challenging to perform reasonably. Especially the market framework could change considerably and is difficult to predict for a time horizon of dozens of years.

However, the added values of HPPs, which are energy provision and short-term flexibility, are subject to less changes. The idea is now to model the economical return on these added values as the profit from the participation in two generic markets, a day-ahead market and an intra-day market.

The day-ahead market provides the platform for energy provision and the intra-day market for exploiting the short-term flexibility, where e.g. also remuneration out of provision of spinning reserves can be part of it. A day-ahead market price, therefore, depends on the marginal price of available power plant technologies in the future and on the load.

The intra-day market price correlates to the day-ahead price but with some additional variability. So, if some short-term upward regulation is needed, this can be modeled as a higher intra-day price compared to the day-ahead price and vice versa.

Multi-horizon model

Given day-ahead and intra-day market price scenarios, a multi-horizon model can be formulated. Because of the enormous uncertainties, the model should be as streamlined as possible without neglecting the optionalities of the HPP.

The evaluation is based on an optimization with a time horizon of one year, which defines the operation strategy for one year. This optimization is repeated every year for the whole valuation time horizon.

As case study, the HPP KWM was regarded and, therefore, SDP was used as solution algorithm. The model can be described by an extension of (6.1):

$$Q_t(v_{t-1}^{stor}) = \max_{W_t} \frac{1}{N_t} \sum_{j=1}^{N_t} \theta_t^j(W_t) + Q_{t+1}(v_{t-1}^{stor} - W_t) \quad (6.8)$$

$$\theta_t^j(W_t) = \max_{u,p,o,v,m^{\text{DA}},m^{\text{IM}}} (c_t^{\text{DA},j})^T m_t^{\text{DA},j} + \sum_{\tau=1}^{24} \mathbb{E}_{\mathcal{A}_\tau \in \Lambda_\tau^j} [c^{\text{IM}}(\tau) m^{\text{IM}}(\tau)]$$

$$\text{s.t.} : \left\{ \begin{array}{l} v_{\mathcal{B}_\tau}^{\text{bal}} = v_{\mathcal{A}_{\tau-1}}^{\text{bal}} - o_{\mathcal{B}_\tau} - f_u(u_{\mathcal{B}_\tau}) + f_p(p_{\mathcal{B}_\tau}) + a_{\mathcal{B}_\tau}, \dots \\ \forall \mathcal{A}_{\tau-1} \in \Lambda_{\tau-1}^j, \forall \mathcal{B}_\tau \in U(\mathcal{A}_{\tau-1}), \forall \tau \quad (6.8a) \\ \sum_{\tau=1}^{24} [f_u(u_\tau^s) - f_p(p_\tau^s) - a_\tau^s] + o_t = W_t, \forall s \in \mathcal{S}^{\text{IM}} \quad (6.8b) \\ m_t^{\text{DA}} + m_{\mathcal{A}_\tau}^{\text{IM}} = u_{\mathcal{A}_\tau} - p_{\mathcal{A}_\tau}, \forall \mathcal{A}_\tau \in \Lambda_\tau^j, \forall \tau \quad (6.8c) \\ 0 \leq u, p, o, v^{\text{stor}}, v^{\text{bal}} \leq ub_t, lb_t \leq W_t, m \leq ub_t \quad (6.8d) \end{array} \right.$$

As interstage time period t , one day is chosen. The bidding into the day-ahead market is modeled as an additional here-and-now decision in the intrastage problem and, therefore, is included in the objective function of it. This decision is performed for each day t for its 24 hours simultaneously. Therefore, the decision vector $m_t^{\text{DA},j}$ for a day-ahead price scenario j consists of 24 entries and is subject to intrastage uncertainty.

The participation into the intra-day market is modeled as wait-and-see

decisions. The intrastage subproblems are modeled stochastically, with the intra-day prices known for the next block of four hours. There are two different price scenarios per block and, therefore, there are $2^6 = 64$ intra-day price scenarios $s \in \mathcal{S}^{\text{IM}}$ per day-ahead price scenario j .

Discussion of the model

Also some alternatives and extensions to the model (6.8) were evaluated but, eventually, rejected.

Evaluated were the introduction of day-ahead price cluster states. These states would model a certain expectation of the next day-ahead price scenarios and would allow an adapted policy based on it. This extension was rejected due to increased computational complexity as well as modeling issues.

Secondly, the actual bidding process could be modeled in more detail similarly to the short-term bidding model in [108]. For fixed offer volumes, optimal bided prices can be found with a MILP formulation. Downside of such a method is the need for many modeling decisions. Further, non-anticipaty of day-ahead prices would require more scenarios of them than the number of fixed volume points, which would increase the computational complexity substantially. Although such a method models the actual bidding process, the difference to the proposed model at the end would be only that the bid volume would be discrete instead of continuous.

Finally, the introduction of a price response in the intra-day market was analyzed similarly to what is shown in the next section. Again this idea was rejected because meaningful price responses are difficult to motivate for such a long-term view.

The model as presented in (6.8) has one severe drawback. Since no price response and a risk-neutral formulation is chosen, the expected values of day-ahead and intra-day prices have to be equal in order to be arbitrage free. On the other hand, if they are made hourly arbitrage free, then there is no reason for the two markets and one of them would be enough.

Whereas in a MTHP problem this issue would need closer analysis here a rather pragmatic approach was used. The positions in both day-ahead and intra-day markets are limited to a number comparable to maximum generation capacity of KWM. The reasoning behind this idea is to introduce a market depth or risk constraint and by that limit

arbitrage possibilities. However, note that the results of the evaluation critically depends on these limits.

Results of the valuation

The model presented here was implemented and tested. Then, it was used in the evaluation part of a long-term evaluation study of KWM. The prediction of day-ahead and intra-day market prices as well as the analysis of the results were done by other people. The study was leaded by a PhD-student at the Ecole polytechnique fédérale de Lausanne (EPFL) and its results are described in his thesis [121] and, thus, are omitted here.

The main conclusions of the study were that, depending on fuel and CO₂ prices, payback times of the HPP investment would be around 30 years with an internal rate of return of 4-8%.

6.4.2 Medium-term hydro power planning planning for a price-maker

The MTHP problem is becoming more complicated if the influence of generation companies (GenCos) on market prices is studied. This influence can be important for markets with limited liquidity, which can be the case in oligopolistic markets like forward, intra-day, and ancillary services market as well as sometimes even spot markets.

Such problems are typically solved by considering a market clearing process, where each market participant either bid their marginal costs or act strategically (see also section 9.3). Apart from modeling issues, this solution concept becomes quickly computationally intractable for time horizons up to a few years.

Therefore, here suggested is an alternative, where by assuming a linear dependency between amount and price of the commodities, price influences are modeled.

The proposed method is both simple and powerful and can be included in multi-horizon models. In this example, the model is applied to a MTHP problem for a price-maker GenCo with the consideration of a forward and a day-ahead market.

Forward contract markets for hydro GenCos

By nature hydro GenCos are in long positions and they try to hedge this position by forward contracts. Most prominent reasons for this is first, due to limited liquidity of market products and second, because of their price influence on it.

Note that another reason for trading forward products can be that a risk premium is present. That is that consumers of electricity are willing to pay a premium in order to reduce their future cost variability and downside risk. This argument can also be applied from the producers side. In literature, mostly indications for the former case is found and, therefore, forward prices would be overestimated with respect to the spot price.

The available and liquid forward products have different delivery periods of days, weeks, months up to a few years for both peak and off-peak products. Therefore, the GenCos first estimate prospective production with a MTHP planning and, then, they hedge this physical long position with the available forward products. The portfolio is constantly updated, especially if forward products with shorter delivery periods get available.

Given that the delivery period of the forward products is different than the actual physical hourly production and pool market, the hedging of it can be non-trivial. Usually some kind of risk is calculated for open positions and the task of traders is then to hedge the position most profitably within given risk limits.

Multi-horizon model for a price-maker in forward and day-ahead markets

In order to give decision support for the bidding in forward markets, multi-horizon models can be used. The idea is that the bidding process is simplified to two steps: the bidding into the forward market and its fulfillment with physical production or bidding in the day-ahead market. The limited liquidity in both forward and day-ahead markets is modeled by a linear price response giving an incentive for the model to take part in the forward market.

In a multi-horizon setting, the bidding into the forward market can be modeled as a here-and-now decision, whereas the intrastage decisions consists of day-ahead bidding as well as the physical operation of the

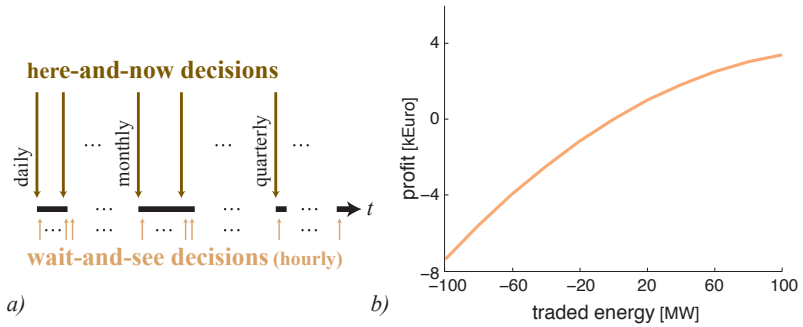


Figure 6.8: *a)* Multi-horizon model with varying intrastage time durations to model the bidding process. *b)* Linear dependent forward product prices results to a quadratically depending profit on traded amount.

HPP (see also figure 6.8 *a)*).

It is important to note that the interstage time durations in such a model are not constant, but are adapted to the available forward products. Therefore, the interstage time periods are first short but are getting longer moving forward.

The model is tested on KWM and SDP is used to solve the model. Only forward peak products are considered since KWM is a peak HPP. These products are traded over-the-counter, but their prices are mostly based on standard future contracts with financial settlement. Therefore, EEX Phelix German peak future prices are taken as estimation of a forward contract bid price.

The day-ahead price is estimated by an hourly priced forward curve which is constructed by the operator of KWM. It is arbitrage-free to the future products. Note that if a risk premium in the forward prices shall be modeled, the estimation of the day-ahead price would have to be adapted accordingly.

Mathematical model

The linear price response leads to a quadratic problem. Consider m_t^{fut} as the volume of a bided forward product at interstage t and c_t^{fut} as its current price. For a linear price response κ^{fut} the return out of trading

this product is therefore:

$$\text{profit}_t^{fut} = (\kappa^{fut} m_t^{fut} + c_t^{fut}) m_t^{fut} = \kappa^{fut} (m_t^{fut})^2 + c_t^{fut} m_t^{fut} \quad (6.9)$$

The same derivation is also valid for the price response in the day-ahead market.

The price response κ can be estimated either by experience and tacit knowledge or out of historical data e.g. from the market stack curve. A qualitative example of it is given in figure 6.8 b). For the case study the price responses were made constant although it would make sense to adapt it.

The mathematical formulation of the multi-horizon model now follows from (6.1) but with a deterministic intrastage subproblem, with the forward market bidding as additional here-and-now decisions as well as with quadratic objective functions (6.9) in both interstage and intrastage problems:

$$Q_t(v_{t-1}^{stor}) = \max_{W_t, m_t^{fut}} \kappa^{fut} (m_t^{fut})^2 + c_t^{fut} m_t^{fut} \dots \\ + \frac{1}{N_t} \sum_{j=1}^{N_t} \theta_t^j(W_t) + Q_{t+1}(v_{t-1}^{stor} - W_t) \quad (6.10)$$

$$\theta_t^j(W_t) = \max_{u, p, o, v, m} (m^{\text{DA}}(\tau))^T \kappa^{\text{DA}} m^{\text{DA}}(\tau) + c^{\text{DA}}(\tau) m^{\text{DA}}(\tau)$$

$$\text{s.t.:} \left\{ \begin{array}{l} v_\tau^{bal} = v_{\tau-1}^{bal} - o_\tau - f_u(u_\tau) + f_p(p_\tau) + a_\tau, \quad \forall \tau \quad (6.10a) \\ \sum_{\tau=1}^{\mathcal{T}} [f_u(u_\tau) - f_p(p_\tau) - a_\tau] + o_t = W_t \quad (6.10b) \\ m_\tau^{pos} + m_\tau^{\text{DA}} + m_t^{fut} = u_\tau - p_\tau, \quad \forall \tau \in \mathcal{T}_{\text{peak}} \quad (6.10c) \\ m_\tau^{pos} + m_\tau^{\text{DA}} = u_\tau - p_\tau, \quad \forall \tau \in \mathcal{T}_{\text{off-peak}} \quad (6.10d) \\ 0 \leq u, p, o, v^{stor}, v^{bal} \leq ub_t, \quad lb_t \leq W_t, m \leq ub_t, \quad m^{fut} \in \mathbb{Z} \quad (6.10e) \end{array} \right.$$

Note that $m^{\text{DA}}(\tau)$ is a vector denoting the day-ahead bid volumes for all interstage hours τ . Subsequently the objective function in the intrastage problem is written in matrix form.

Note also that the financial balance in (6.10c) and (6.10d) is valid on either peak $\mathcal{T}_{\text{peak}}$ or off-peak hours $\mathcal{T}_{\text{off-peak}}$. In there, m_τ^{pos} denotes the portfolio positions from previous trades which is given beforehand and is not part of the decision vector.

The intrastage problem is a quadratic linear problem for τ stages. The forward product volume m^{fut} is discretized whereas the day-ahead market positions m^{DA} remain continuous.

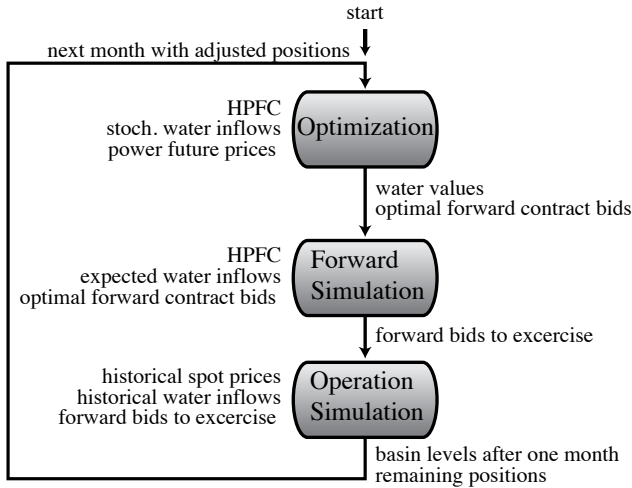


Figure 6.9: Bidding process simulation: For every month one optimization is done with updated data. The forward simulation is for determining forward bids to exercise whereas the operation simulation mimics the actual operation of the HPP.

Notes about the evaluation of the model

The model for the optimization (6.10) considers only one snapshot of the whole bidding problem, which is what would be the most optimal bidding strategy presently without taken into account that this static strategy could be changed afterwards if new market information gets available. Nevertheless, the amount of bid forward contracts could be used as a decision support for the trading group in a GenCo.

The optimization depends on the previously traded forward products i.e. on the actual portfolio positions m^{pos} . In order to evaluate how the application of such a bidding strategy performs over time, it is tested in an operation simulation of the bidding process (figure 6.9). In the course of the simulation, the optimization begins with zero portfolio positions, which are then gradually built up in time with the following procedure:

A respective forward simulation of the optimization (6.10) delivers the bidding strategy that is the number of forward contracts to trade. These trades are then exercised and, therefore, the portfolio positions are

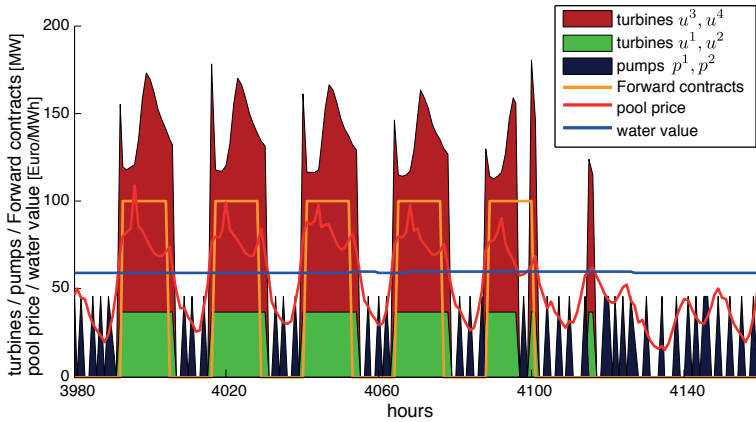


Figure 6.10: Results of the forward simulation for one randomly selected week. Production depends on current water value and pool price. But the balancing reservoir limits generation and pumping capabilities by a large extend.

changed. Afterwards an operation simulation is performed, which mimics operators decisions regarding HPP commitment for one month. This procedure is repeated monthly with a receding horizon until end of the time horizon is reached.

The simulation requires a large amount of data because each optimization and simulation requires different data sets of either predictions or actual realizations of inflows and prices of the day-ahead market and the available forward contracts. As a result the optimization starts at 1st April 2009 in order to have enough historical data available.

Results of the simulation of the bidding process

Given next are results of the operation simulation of the bidding process (figure 6.9). Note here that the computation time for the whole simulation was a modest 35 minutes, which is also due to the undemanding model of KWM.

Figure 6.10 illustrates the forward simulation for a randomly selected week. For better readability not all results are shown and some turbines

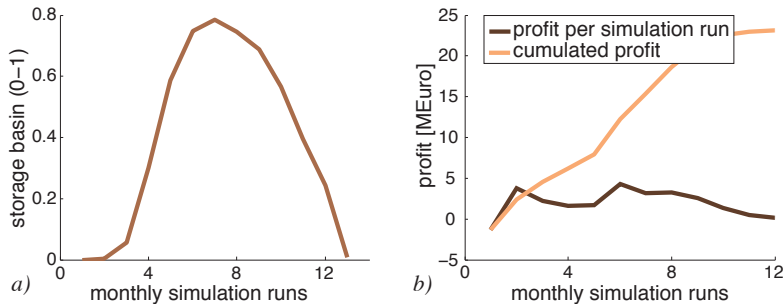


Figure 6.11: Results of bidding process simulation: *a)* Filling of storage reservoir: The basin is used up to 80% of its maximum capacity. *b)* (Cumulated) profit for each simulation run.

and pumps are combined.

For the shown week 100 MW of peak forward contracts were sold. As one can see, the full deployment of turbines and pumps is not always possible. One reason for this is the balancing basin, which is relatively small and, therefore, reduces turbinning and pumping capabilities. Interesting is also the fifth day, where the obligation of forward contracts are not met and, therefore, relatively cheap energy are bought back in the day-ahead market.

Figure 6.11 shows the overall results of the operation simulation. On one side the filling of the storage basin is shown normalized to its capacity. Such a filling pattern was expected, since the basin is empty at the beginning and inflows occur only in the first months. So the basin acts as seasonal storage in order to be able to produce energy throughout the whole year.

Figure 6.11 *b)* outlines the profit, which is achieved in the day-ahead market and by trading forward contracts. At the beginning the profit is negative, since forward contracts were bought so the HPP went even more in a long position.⁴ However afterwards, profit is achieved throughout the year with a tendency to more profit in the summer than in winter time which was also expected because of non-storable water inflows.

If no trading in the forward market was allowed (not shown in fig-

⁴Note that financial and operational risks were not considered explicitly but rather implicitly by the price-maker model.

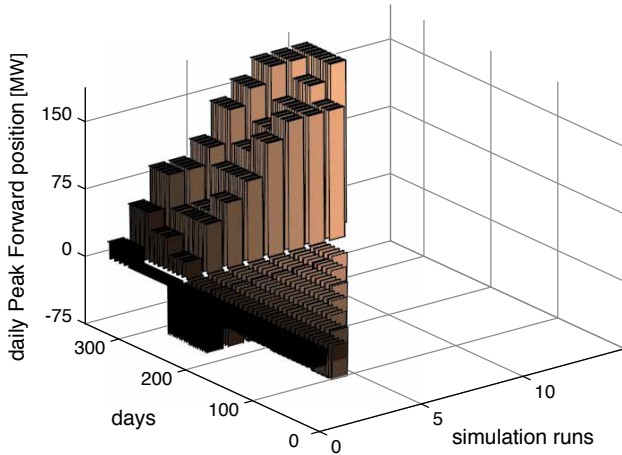


Figure 6.12: Positions out of traded forward contracts. Note that traded contracts are only in the peaks hours and that the higher the simulation run the less hours are able to trade. The positions are gradually increased with even long positions at first.

ures) total profit summed up to 20 Mio. € compared to 24 Mio. € in the former case. So the simulation shows that trading in the forward market results to an increase of profit of about 20%. Note that historical achieved revenue out of sold energy were about 20 to 30 Mio. €, making the simulated ones realistic estimations.

Finally, figure 6.12 depicts the positions of forward contracts. The positions are daily aggregated to get a better overview.

At the beginning, there are even long positions but the less time left until settlement the more the positions are in short. This is reasonable since the storage HPP is long by nature and short positions are needed in order to hedge its operation. Further, the positions are built up gradually because of the price response which is also realistic behavior.

Conclusions

A linear price response is able to model limited liquidity in electricity markets. Such an approach is implementable in multi-horizon models.

The results of such a modeled optimization can be used as a decision support for traders. However, note that the results are sensitive to the modeled price response which has to be treated carefully.

Alternatively, also a risk-averse optimization would be a possibility to model a similar behavior. But, the modeling of stochastic forward and day-ahead market prices would then be very important but also delicate to perform.

6.5 Summary

This chapter presented the multi-horizon modeling approach. The model was extended by risk measures and revenue from the provision of ancillary services. In the evaluation, traditional approaches and alternative modeling methods are compared with the proposed multi-horizon method. Finally, two application examples for long-term valuation and a price-maker were presented to show how multi-horizon models can be applied.

Detailed conclusions were already drawn individually and are not repeated here. The main points were that multi-horizon modeling allows a computationally efficient as well as detailed modeling and outperforms traditional approaches considerably. The modeling concept is very flexible and, therefore, has to be adapted for specific applications.

Chapter 7

Dealing with non-concave value functions

This chapter presents some extensions to dualized stochastic dual dynamic programming in order to deal better with non-concave value functions. Additionally, a measure of the severity of non-concavity applicable in this context is proposed. The chapter is based on the publication [6].

In chapter 4, non-concave value functions and the state-of-the art research were presented. The bottomline was that different methods are applicable depending on the source and severity of non-concavities. So for value functions, which are more than moderately non-concave, ordinary stochastic dynamic programming (SDP) was the appropriate method.

For this class of problems another method is introduced here, which is proposed to be called stochastic dual dynamic programming (SDDP) *with locally valid cutting planes*. The idea of such an approximate dynamic programming (ADP) scheme is to approximate the value function locally with a better quality than a dualized SDDP method would allow. Therefore, the proposed method is also able to solve higher dimensional problems which would be intractable for a SDP method.

7.1 Introduction

The reasons for non-concave value functions in medium-term hydro power planning (MTHP) optimizations can be manifold. The most prominent are:

- consideration of ancillary services markets,
- power plant operation rules,
- varying water head considerations, and
- modeling of market bidding.

The provision of ancillary services introduces non-concavities because of forbidden operating zones in turbines. Similarly, physical properties of the power plant often require the introduction of forbidden operation zones or other operation rules. For instance, pumping is very often only permitted fully or not and thus, if not simplified, is introducing discontinuous value functions and therefore non-concavities.

Also the consideration of a varying water head makes a scheduling problem non-concave since it is depending in a non-linear way on different factors. Finally, if as decisions in a bidding process both volumes and prices have to be decided, then the value function is based on the multiplication of it and thus, non-concave.¹

The two last phenomenas are not considered in this thesis since they are not that important for MTHP problems, particularly not for Swiss power plants.

Apart from concavitation techniques, SDDP can be applied to problems with non-concave value functions with the dualized SDDP method as described in chapter 4. It is not difficult to show that this method doesn't work for value functions which are considerably non-concave. However, it is not clear a priori, what "considerably non-concave" in this respects means.

Therefore, proposed next is a measure of non-concavity. The measure is applied to Kraftwerke Oberhasli AG (KWO) which also introduces

¹Similarly if price states are present, then the value function is also non-concave in respect to these. However, such a problem can be well approached by multi-cut SDDP as presented in section 4.3.

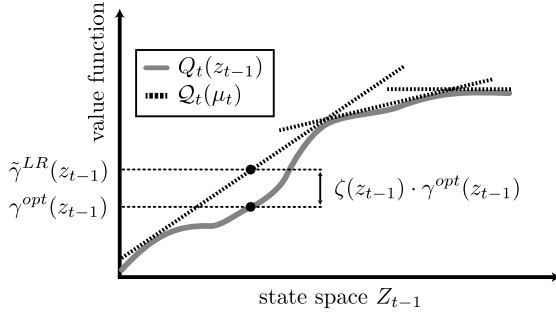


Figure 7.1: Measure of non-concavity $\zeta_t(z_{t-1})$ for state z_{t-1} as the relative difference of the objective values of the Lagrangian relaxed and the original problem.

some opportunities to speed up the computation time of the optimization. Then, the SDDP-method with locally valid cutting planes is introduced. Its performance is compared to the one with dualized SDDP on Kraftwerke Mattmark AG (KWM).

7.2 Measure of non-concavity

The idea of a measure of non-concavity is to quantify the impact it has on the correct representation of the value function by cutting planes. In literature, many different mathematical concepts of measures of non-concavity already exist. Widely discussed is for example the convexity number, which states the minimum number of convex subsets that covers the original set.

Here a measure is introduced specifically for dualized SDDP, which can be constructed cheaply but, nevertheless, gives an insight on the severity of the non-concavity.

Recall the discussion in section 4.3 about dualized SDDP, especially figure 4.1, which is shown again with the proposed measure $\zeta_t(z_{t-1})$ in figure 7.1. With dualized SDDP it is possible to find valid cutting planes for non-concave value functions. The minimum of the finally found collection $\mathcal{Q}_t(\mu_t)$, which is denoted as $\tilde{\gamma}^{LR}$, does possibly not represent the value function perfectly, but a difference can remain due to the

non-concave value function.

The relative value of this difference is proposed to be the measure of non-concavity:

$$\zeta_t(z_{t-1}) = \frac{\tilde{\gamma}^{LR}(z_{t-1}) - \gamma^{opt}(z_{t-1})}{\gamma^{opt}(z_{t-1})} \quad (7.1)$$

In a dualized SDDP algorithm the values needed to compute this measure are already available and the calculation of it, therefore, does not complicate the problem further.

Note that the objective value of the original problem $\gamma^{opt}(z)$ corresponds only to the minimum of the *approximated* value function $\tilde{Q}^i(z)$. Further note that the measure is only valid for a certain state. Out of these reasons the measure is only a qualitative one for the true value function $Q(z)$.

It is also important to mention that $\tilde{\gamma}^{LR}(z_{t-1})$ depends on the actual optimized Lagrange multiplier μ_t . If the stopping criterion in the algorithm 5 in line 5 is solely based on the difference ϵ of $\tilde{\gamma}^{LR}$ with the objective value from the original problem γ^{opt} , then the proposed measure of non-concavity ζ either will stay always lower than ϵ or the optimization of the Lagrange multiplier will not converge at all.

Therefore, in this thesis apart from an ϵ of 0.5%, two additional stopping criteria were used. One is a limit on the maximum multiplier optimization iteration of 100. The other one is a criterion based on the progress of the optimization. So if the multiplier optimization does not result anymore to closer cutting planes, it is stopped.

The first stopping criteria was experienced to be triggered mostly in all simulations. The second criterion on the other hand was never active, since at most around 50 iterations were needed. But in these cases with many iterations, typically, the third criterion terminated the optimizations.

7.2.1 Application to Kraftwerke Oberhasli AG

Figure 7.2 a) shows the measure of non-concavity for the optimization of KWO. The measure is calculated for every cutting plane in each daily time stage.

In KWO there are 44 decision variables modeled, where 9 of them are

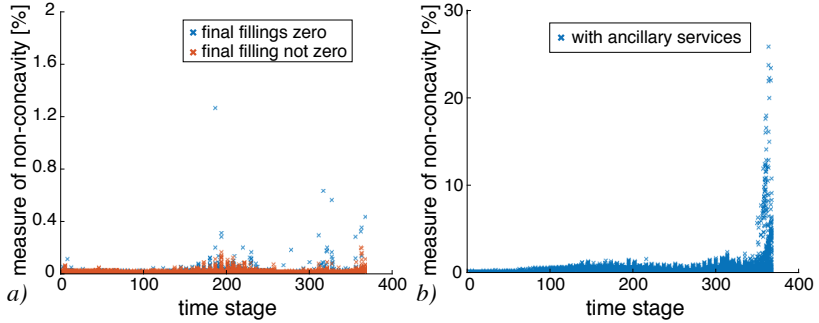


Figure 7.2: Measure of non-concavity ζ of the value function of KWO per time stage: *a)* for final fillings constrained to be zero and non-zero and *b)* for an optimization with consideration of a SFC market. Note the different scaling of the y-axes.

binary decisions. Nevertheless, the measure doesn't show any considerable non-concavity.

Since the measure in figure 7.2 *a)* was not exactly zero, dualized SDDP could possibly still require some iterations until adequate Lagrange multipliers are found. However, it can be shown that for this optimization the Lagrange multipliers were not needed to be optimized because the estimated multipliers from the locally convexified problem resulted directly to a objective value $\tilde{\gamma}^{LR}(z_{t-1})$ close to the original one.

If such behavior is experienced then the optimizations of the Lagrange multiplier can be left out resulting in solving only the original problem as well as the convexified one. This results of a speedup factor of 1.5-2 for the backward step of SDDP.

Figure 7.2 *b)* shows the measure for the same optimization of KWO, but with the consideration of a market for the provision of SFC. This leads to three additional binaries as well as integer variables. In this case, the measure is roughly ten times as large and dualized SDDP is necessary, at least for half of the time stages.

7.2.2 MTHP problems with constrained state values

Generation companies (GenCos) sometimes want to guarantee a certain basin filling at some point in time. It turns out that such constraints

are troublesome if SDDP-type algorithms are used.

Feasibility of a policy can be checked in the forward step of the algorithm and then based on it additional penalties or feasibility cuts can be introduced. However, it is very difficult to find appropriate penalties or feasibility cuts.

In this thesis, only the final value of the basins are sometimes required to be met. For such a case there is also another simpler approach in order to apply SDDP. The value function at terminal time can be adjusted so that fillings lower than the given one have a negative profit-to-go value and, therefore, are prevented.

Such an approach will amplify the non-concavity because of the non-smooth value function at terminal time (figure 7.2 *a*). Note that the reason for the non-concavity is still the non-linear model of the power plant.

7.2.3 Conclusions

With the proposed measure ζ , a qualitative insight on the non-concavity of value functions is possible. This information can be used to further tune the SDDP algorithm.

If a low measure is experienced only the locally convexified problem has to be solved additionally. Otherwise, the Lagrange multipliers are optimized but only for the time stages with higher values of the measure. Depending on the application this heuristic will lead to a speedup of the algorithm up to a factor of 2.

7.3 Locally valid cutting planes

For problems, which have a value function with a high value of non-concavity, dualized SDDP will provide valid but not necessarily close cutting planes to the actual value function.

Even worse, the outcome of MTHP optimizations are water values as gradients of the cutting planes. Those gradients are difficult to estimate correctly by dualized SDDP. As an illustrative example consider the one-dimensional case in figure 7.3. The slopes of the dotted lines in the collection $Q_t(\mu_t)$ do not correspond well to the actual water values, i.e. the slopes of the grey line Q_t .

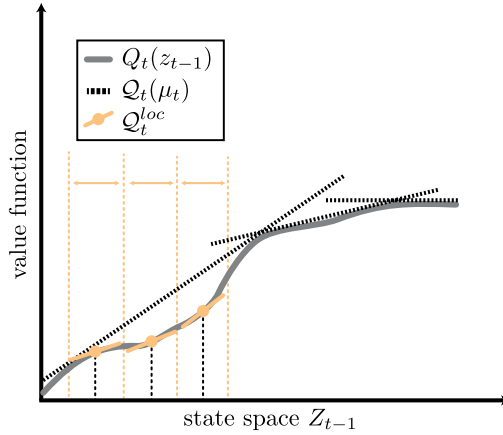


Figure 7.3: Construction of locally valid cutting planes for an illustrative one-dimensional case.

7.3.1 Proposed method

The idea is now to construct cutting planes, which approximate the value function closer but are valid only locally at a certain region. Therefore, the subgradients of them should be more accurate for an estimation of the water values. In figure 7.3 these cuts would be represented by the yellow lines in the collection Q_t^{loc} .

The cutting planes can be constructed out of the objective value of the original non-convex problem and the dual variables of the coupling constraints of the locally convexified problem. Therefore, one additional linear program (LP) has to be solved in order to get this information. A locally valid cutting plane also needs the state it was constructed for in order to specify the valid region of it. The width of this region is proposed to depend both on the SDDP algorithm iteration number i as well as on the experienced measure of non-concavity ζ . It is adapted in the course of the algorithm with a smaller validity region for a higher iteration number and non-concavity measure.

The usage of locally valid cutting planes complicates the calculation of a subproblem in the backward pass. This is because at time point t the state z_t is unknown and therefore the valid cuts for it can not be

determined.

In order to solve this issue, a heuristic procedure is proposed. The subproblems are first solved with an actual trial state \tilde{z}_t and then iterated until the state converges to a value. Depending on the application, this procedure requires the calculation of multiple problems per subproblem resulting in a considerable increase of computational complexity.²

Dualized SDDP and the technique with locally valid cutting planes complete each other. The proposed methodology is, therefore, as follows: For the first SDDP-algorithm iterations, the dualized method is used to quickly find a first approximation of the value function. Afterwards, if a relatively high non-concavity measure is experienced, the technique with locally valid cutting planes is employed to refine the approximation locally and to get to more accurate water values.

7.3.2 Evaluation

In order to evaluate the proposed method it is applied to KWM. If the provision of SFC is considered, the problem gets more than moderately non-concave. Additionally, KWM also allows the usage of SDP as solution algorithm and, therefore, a benchmark can be constructed.

First, the water values proposed by SDP are compared with the water values from the dualized SDDP approach with and without the extension of locally valid cutting planes. Afterwards, the water values are applied in a Monte Carlo operation simulation study.

The number of backward/forward pass iterations for both SDDP methods were limited in order to be comparable: 10 algorithm iterations for the dualized SDDP method were used whereas for the second method there were first 5 iterations of dualized SDDP and then again 5 iterations with locally valid cutting planes. Note that in both cases the upper bounds would have been also stabilized.

Water value comparison

Figure 7.4 shows the water values for the benchmark method SDP as well as for the other two SDDP methods. Note that only values for state

²For the application to KWM the solving of around four additional mixed-integer linear programs (MILPs) were necessary.

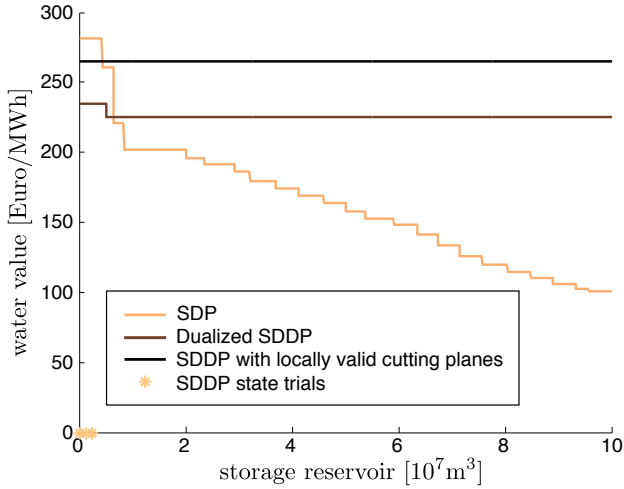


Figure 7.4: Comparison of water values proposed by SDP, dualized SDDP, and with locally valid cutting planes. The water values for SDDP methods are only meaningful for state trials, here for almost empty fillings.

Table 7.1: Monte Carlo operation simulation

	mean profit [M€]
perfect information:	53.04
SDP:	44.70
Dualized SDDP:	43.54
SDDP with locally cutting planes:	44.32

trials are meaningful for the SDDP methods whereas SDP estimates for all filling levels accurate water values.

Qualitatively, one can argue that the values from SDDP with the extension of locally valid cutting planes are closer to the ones from SDP, which could show the benefit of this enhancement.

Performance of the optimization methods

In a Monte Carlo simulation, the hydro power plant operation is mimicked over one year for 100 samples of water inflows and market prices in order to evaluate the quality of the different proposed water values. Table 7.1 shows the resulting average profit for all methods and for perfect information, when the unknown data is known in advance. The enhancement of SDDP with locally valid cutting planes results in slightly higher profits compared with SDDP without the enhancement, which is indicating the better quality of the calculated water values. Note that these results may change for different problem set ups and scenario constructions.

7.4 Summary

Non-concavities can arise in medium-term hydro power planning (MTHP) problems because of many different reasons. A measure of it can be used to get some insights about the severity of non-concavity. Such a measure can also be used for tuning the stochastic dual dynamic programming (SDDP) algorithm, leading to less computational requirements.

For problems which are more than moderately non-concave and where the application of stochastic dynamic programming (SDP) is not possible due to computational tractability issues, dualized SDDP with the proposed extension with locally valid cutting planes can be used. Especially the estimation of water values can be performed better with such an extension.

Chapter 8

Delta-hedging of cashflows from ancillary services markets

The market for the provision of ancillary services is a relatively new revenue opportunity for generation companies. Therefore, the management of its associated risks are not well understood until now. This chapter gives support in this respect.

8.1 Introduction

The provision of ancillary services can be highly profitable, especially, for storage hydro power plants. However, the acceptance of an offer is insecure and consequently this will result to an uncertain revenue stream from it. Additionally, the provision of ancillary services influences the production schedule significantly. Out of these reasons, bidding in ancillary services increases cash flow and production risk considerably.

The idea is now to give some decision support in order to hedge cash flow risks by adapting positions in the electricity forward market. Whereas the risk in the electricity market due to open positions increases, in return the cash flow risk from the ancillary services market can possibly decrease more, leading to an overall mitigated risk exposure.

8.2 Proposed method: delta-hedging

The proposed method is based on the well known concept of delta-hedging. The delta denotes the sensitivity of the value of a portfolio or option to a change in the value of the underlying product. Delta-hedging means to build up a portfolio which has a delta close to zero. Therefore, its value does not change with varying values of the underlying (as long as the delta stays around zero of course).

Delta-hedging is already applied in a way by generation companies (Gen-Cos) for several years, but typically only for the mitigation of cashflow risks in electricity markets. Decision support is given there by calculating the deltas by scenario analyses of deterministic programs. Alternatively, it can be formulated also as a stochastic multistage program as it was shown in [122]. For the hedging of cashflows resulting from the ancillary services market such an approach has to be adapted.

Consider as V_{spot}^t the value of a power plant for a certain time period t , which is realized as cashflows in the electricity spot market. Consider further F_{anc}^t as the remuneration of ancillary services, which is treated as the underlying. The delta Δ is then as follows:

$$\Delta^t = \frac{\delta V_{spot}^t}{\delta F_{anc}^t} \approx \frac{V_{spot}^{t, F_{anc}^+} - V_{spot}^{t, F_{anc}^-}}{2 \cdot F_{spot}^t} \quad (8.1)$$

V_{spot}^{t, F_{anc}^+} denotes the value of the power plant for an increase of the remuneration for the ancillary services by one unit and with V_{spot}^{t, F_{anc}^-} the value for a respective decrease of it. The difference of it is divided by the average spot price of the period t .

The delta Δ^t as defined in (8.1) specifies the sensitivity of the spot position in terms of energy, if the remuneration of ancillary services changes. The time period for which the delta is constructed can be hours, weeks etc. Also the change in the remuneration can be adapted, e.g. it can be changed for all ancillary services products or for only a few of them. Further, it is possible to analyze the behavior if bid ancillary service products are awarded as expected or not at all.

The values of the power plant V_{spot}^{t, F_{anc}^+} , V_{spot}^{t, F_{anc}^-} can be found by an operation simulation, which can be difficult to perform. It can be based on a pre-constructed policy, for instance water values. Further, it should consider the actual positions in the electricity markets, that is from

trading in the forward markets. Given an estimation of the spot prices, the cashflows can be computed which result from a change in the production schedule due to different provision of ancillary services. With such an operation simulation, various deltas can be calculated and the positions in the forward market can be adjusted accordingly if needed.

8.3 Application to Kraftwerke Oberhasli AG

The proposed framework is now illustrated for an application to Kraftwerke Oberhasli AG (KWO). The power plant value shall be hedged with respect to changing remunerations of ancillary services markets, where a hypothetical daily secondary frequency control (SFC) market is considered.

In order to find the position in the electricity market, first, a medium-term hydro power planning (MTHP) optimization is performed based on dualized stochastic dual dynamic programming and multi-horizon modeling as well as with consideration of the SFC market for a given remuneration. Out of this optimization, the production strategy, i.e. the water values, can be retrieved. They are used in an operation simulation, which will deliver prospective hourly production. This long positions are hedged with forward electricity products.

The values of the power plant for increased and decreased remuneration is found by the same operation simulation, where the influence of the changed price leads to a different operation and, thus, to diverging cashflows. With this information various deltas can be computed.

In figure 8.1 *a*), the daily deltas of one month are shown, where the production was hedged with daily products and the remuneration was adapted for the whole time horizon of three years equally.

Typically, the delta is mostly positive, because with higher remuneration more SFC is provided and, thus, higher production results.

Interestingly, the delta can also be negative, e.g. end of August in this example, where for higher remuneration less of SFC is provided. The reason for this is because the remuneration was changed for the whole time horizon and, therefore, more energy is produced in general than expected. This leads to lower basin fillings and subsequently higher water values, which is then balanced out at some point in time.

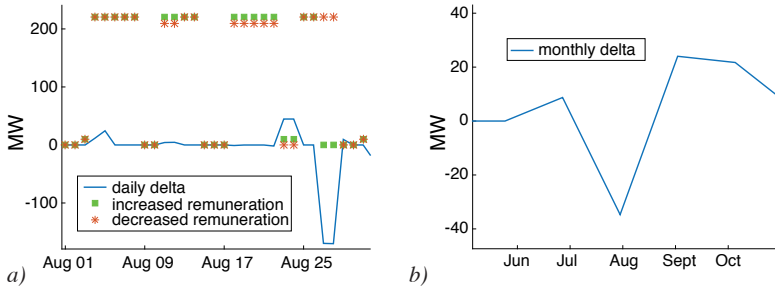


Figure 8.1: Delta as difference of cashflows for increased and decreased remuneration of SFC. *a)* Daily deltas for daily hedged production as well as awarded SFC bids. *b)* Monthly delta for monthly hedged production if in August SFC is not awarded.

Whereas the figure 8.1 *a)* is shown to illustrate the concept, more meaningful are deltas for longer time periods, which corresponds to the available forward contracts. Figure 8.1 *b)* shows an example of it, where the remuneration of SFC was not adapted but instead the offers for the month August were accepted or not. In this case the delta is negative since if the offers are not accepted, the production will be less than assumed and energy would have to be bought back at the spot market. It is not a priori clear, how much less the production will be. The simulation estimates it by around 30 MW for the month August. Therefore, it could be reasonable to not sell all of the prospective production beforehand and stay in a long position. Note also that in the months after August the delta is positive again because of the relatively high filling in the reservoirs, which leads to more energy production than expected.

8.4 Summary

Given the multi-horizon modeling tools and dualized stochastic dual dynamic programming, decision support for hedging of the cashflows from ancillary services markets is possible. Some examples were shown how to achieve this and how a decision support could look like.

The method is based on standard delta-hedging, which is well understood in practice. This is especially important because the proposed

decision support heavily depends on the input data and, therefore, the model itself has to be as transparent as possible.

Chapter 9

Short-term planning

In this chapter the operation strategies calculated earlier are applied to short-term planning. First, a decision support for bidding in electricity markets is presented, where a marginal production cost curve is constructed. Then, an agent-based simulator, giving decision support for strategic bidding in the ancillary services market, is presented. This chapter is based on the publication [4].

9.1 Introduction and bibliography

From the operational point of view the realization of a production schedule can be modeled as a deterministic mixed-integer linear problem. The production function of hydro power plants (HPPs) can there be approximated by a piece-wise linear function incorporating nonlinear turbine efficiencies, forbidden operation zones, head effects, penstock losses etc. as described in [8].

Whereas most problems in short-term scheduling are deterministic ones, there are a few which are reasonable to model stochastically. An example is the provision of secondary frequency control (SFC), where it has to be guaranteed that the called energy can be delivered.

Such and similar problems can be addressed by stochastic model predictive control algorithms where some constraints are formulated as chance constraints and e.g. a scenario approach [102, 123] is used to solve it.

From the market perspective, the self scheduling of a hydro power producer is a bidding problem. Apart from selling of energy, the offering of ancillary services is relevant for this problem.

Water values from a medium-term hydro power planning (MTHP) optimization yield a proxy for marginal costs of hydro generation companies (GenCos). Using them in a bidding problem allows a very detailed modeling, since only the time horizon of the market has to be considered which is typically a day or a week.

There were done many different works in academic literature about such problems. The recent review [124] can be used to dig deeper into literature about bidding problems for hydro generation companies. Also, previously in 6.4.1, there was given an example of a bidding problem where however the bidding decisions itself were not used explicitly.

In this chapter, a rather pragmatic approach is shown how to construct a bid curve for a complex HPP, usable more in a short-term perspective. Further, the strategic bidding of ancillary services is analyzed, which gained not a lot of attention so far in practice and literature.

Bibliography in strategic bidding problems

One method, which is able to model strategic bidding problems, is agent-based models, which were reviewed in [125]. The Q-learning framework, as one of the most commonly used approaches, was introduced in [126] and extended in various papers, e.g. in [127].

In the energy field, the research about agent-based modeling is outlined in [128–131]. Agent-based models are mostly used for analyzing different market structures and not for self-scheduling. Few other works consider thermal or hydrothermal portfolios for an optimal bidding problem. For example in [132], game theory techniques were utilized to locate optimal Nash equilibrium solutions to the electricity market auction. Considered were, apart from the energy market, spinning reserves and reactive power markets with two players with thermal production.

Apart from agent-based modeling, mathematical optimization of estimated residual demand curves with market data can be used to find optimal strategic bidding. For example, the authors in [115] use two estimated residual demand curves of one player and its competition in order to find optimal bidding considering day-ahead and intra-daily energy markets as well as secondary reserve market.

In [124], a recent literature review about energy bidding for hydro GenCos is given. However, bids consisting of several quantity-price pairs

are not considered until now in literature. One reason might be that for standard agent-based modeling the algorithm either quickly gets intractable or the model has to be unrealistically simple, because of the large freedom of quantity-price pairs bids. For residual demand curves it would be possible to include such bids in the optimization. But it is first very difficult to model the demand curve of the competition and secondly, it is even more difficult to model their strategic behavior, thus, resulting again in an unrealistic setting.

If many quantity-price pairs would be present, this could get approximated by a marginal cost curve. In [133] deviated slope and markup of such a marginal cost curve are used to model discrete strategic choices. In the setting here, however, the cost curve is a staircase function and, therefore, a linear approximation of it would be troublesome.

9.2 Bidding in electricity markets

For a price-taker the bidding into markets depends mostly on its marginal costs, which for hydro GenCos are the water values. The water values are given as a result of a MTHP optimization. In order to get to optimal bids the marginal values of turbines and pumps can be constructed: Given the water values of the reservoirs up- and downstream of these machines, the marginal value is the difference of them divided by their production function.

Then, an optimal bid curve consists of the marginal values of the turbines and pumps associated with their capacity.

Whereas this procedure works well for simple HPPs, for more complex ones it is not applicable. Consider for instance the Kraftwerke Oberhasli AG (KWO) power plant. A close analysis of it reveals that water release is possible in multiple ways and therefore “parallel” water streams can occur. Further, operational rules prevent some operation condition that is e.g. if some turbines are running others can not etc.

In such cases the operation of turbines and pumps are linked to other ones. The marginal values are then not independent of the actual operation condition although the water values remain constant. Therefore, it would get quite misleading to use them as decision support for energy bidding.

A way around this issue is proposed here, where a marginal production cost curve of a HPP is constructed by simulation. An operation simula-

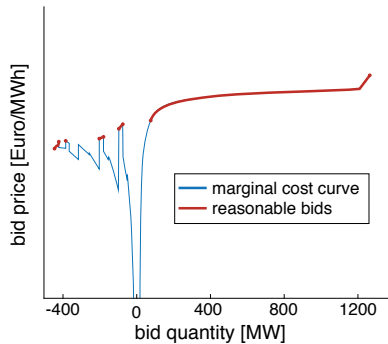


Figure 9.1: Example of a partly confidential marginal production cost curve for KWO. Highlighted are the bids which are reasonable.

tion of the HPP is constructed, which for given water values finds most optimal operation condition. The optimization is repeated for all possible energy amounts that can be produced. The resulting costs define the marginal production cost curve.

Figure 9.1 shows an example of it for KWO for a specific hour. For the production of electric energy the marginal costs increase slowly, since KWO has large basins with a lot of flexibility.

For usage of energy in the pumps the curve is more interesting. Since there are only a few pumps and all of them have forbidden operation zones this results to an discontinuous curve. Often turbines are operating simultaneously with the pumps in order to be able to use a certain amount of energy. Therefore, only the highlighted bids would make sense for usage of energy.

Such a curve can be used directly as decision support for bidding in a spot market. Note that a separate simulation per hour is needed, which requires around 20 seconds of computation time.

9.3 Bidding of ancillary services

The bidding of ancillary services is a difficult task to perform for GenCos. Additionally, in Switzerland such markets are dominated by a few big

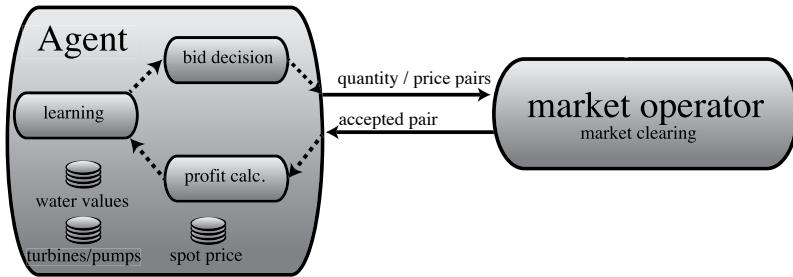


Figure 9.2: Overview of the algorithm with agents deciding on their bids and the market operator who clears the market. The agents are each specified by their water values and production capabilities.

players. So these markets are of oligopolistic nature where theoretically strategic bidding is present and additional profit could be obtained if it is considered.

Therefore decision support about bidding into ancillary services should deliver not only the optimal offer to bid, but should also take into account that strategic bidding could be possible.

Agent-based modeling is one method which is suited to model oligopolistic markets. Here analyzed is the SFC market. For this market in Switzerland only hydro GenCos take part. Therefore, the following model is proposed:

The market players are aggregated into agents, each agent representing one of the bigger market players. Depending on their production capabilities and water values, optimal quantity-price pair bids are found by simulating their strategic behavior, giving a possible decision support to one of the agents.

9.3.1 Proposed model

In agent-based modeling, a notation style is used, which interferes with some of the variables introduced in this thesis so far. Nevertheless, in order to avoid misunderstandings the notation follows the one typically used in such contexts.

In order to simulate strategic behavior, the agents should be able to learn by acquiring knowledge from past actions and decide for upcoming ones based on their experience. The learning here is based on the commonly used Q-learning framework [126].

Figure 9.2 shows an overview of the proposed algorithm. The agents are characterized by their technical production capabilities and water values. For each agent, three tasks have to be done: bid decision, profit estimation for a given accepted bid and memorizing of relevant knowledge. Apart from the agents, also the market clearing has to be simulated. The algorithm is repeated until sufficient learning has occurred and stable results are found. In the following, all of these tasks are explained in more details.

Agent model

The market participants are modeled as agents $i \in I$. The agents are modeled from public available data. Each agent should represent one of the most influencing market participants. Typically, those GenCos have several different HPPs or certain shares of them in their portfolios. In order to get to the relevant information for bidding of SFC, the following procedure is proposed:

1. Calculation of water values per HPP.
2. Determination of SFC power per HPP.
3. Estimation of marginal costs for the provision of SFC per HPP.
4. Clustering of HPPs depending on marginal costs.
5. Determination of power quantity for provision of SFC per agent.

First, water values wv_{hpp} have to be calculated for each HPP in the agents portfolio. This task can be very difficult. In the case study a multi-horizon model is used with the consideration of SFC provision but also other approaches could be used.

Second, for each HPP the maximum quantity of SFC power is determined. Afterwards, marginal costs for the provision of SFC are calculated, that is the smallest remuneration which makes the provision still beneficial. For this calculation a profit estimation, which is explained later, with provision of the SFC bid is compared with estimated profit without it.

The HPPs within a portfolio are then grouped together depending on the marginal costs.

Finally, it is possible to construct for each GenCo a list of SFC power associated with their marginal costs. Note that this list denotes the optimal bidding strategy for the GenCo in a perfect market, with each quantity-marginal costs pair forming one bid. Note also that there are not enough quantity-price pairs in a typical portfolio in order to approximate a marginal cost curve which would be used for alternative formulations.

Bid model

A bid $b \in B$ of some agent is defined as a number of combinations k of the volume offered $q_k \in Q$ and demand charged $s_k \in S$.

The critical question in agent-based modeling is how a bid is adjusted if new knowledge about the performance of it gets available. The idea proposed here is to adjust the list of cumulated power quantities with associated marginal costs depending on certain *characteristics*.

It seems reasonable to bid SFC based on its marginal costs. However, in an oligopolistic environment higher prices for certain quantities could be reasonable. To be able to learn this, bid characteristics are introduced. A bid characteristic could be e.g. a strategy, where the price for the quantity with the highest marginal costs would be further increased. Proposed are ten different characteristics which construct the discrete action set A from which the agents choose their actions $a^i \in A$.

The question remains on how much each price has to be increased. The following assumptions are made:

- each price s_k is at least the corresponding marginal cost, and
- each price s_k is at maximum the prices of the quantities with higher marginal costs s_{k+1}, s_{k+2}, \dots .

The first assumption is obvious whereas the other one is due to the bidding of cumulated quantities. With these assumptions only the *differences* between the marginal prices based on the chosen action a is adjusted. Figure 9.3 a) shows the factors by which each difference is multiplied and added to each marginal cost. Figure 9.3 b) gives an example for the fourth factor how prices increase.

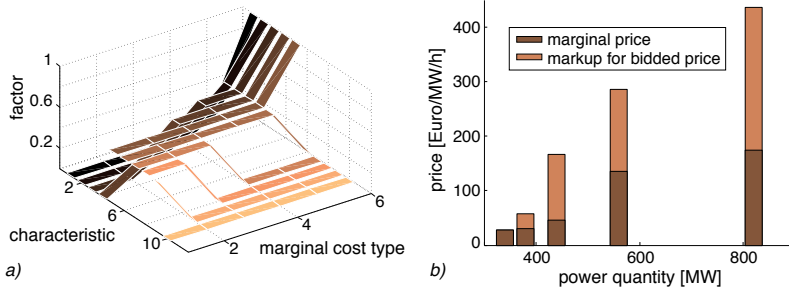


Figure 9.3: *a)* Factors for each characteristic and marginal cost type. *b)* Example: increase of prices by application of characteristic 4.

It should be noted that similar characteristics are close and that the first characteristic is also similar to the last one making the learning process more smooth.

Whereas one bid will consist only of a few different prices, the offered quantity can vary much more because an agent can bid each power quantity per HPP individually or even bid a fraction of it. This makes sense since the demanded power is limited so a bigger quantity is less probable for being accepted. Therefore the corresponding cumulated power quantity for each price (figure 9.3 *b)*) is split into increasing values up to the full amount. This results in many quantity-price pairs for each different marginal costs type.

Bid decision

In each game round the agents have to decide which characteristic to apply on their SFC bid, the respective actions $a^i \in A$. The actions are first sorted, so that similar actions are close together and are given a number $j \in \mathbb{N}^+$ (see also figure 9.3 *a)*). The agent i selects his action based on normal distribution with some standard deviation and the mean μ of the number j_{max} of the action which maximizes its believed reward:

$$\mu = j_{max} = \arg \max_{a \in A} Q_t^i \quad (9.1)$$

The result is then rounded to the nearest integer and the corresponding action is taken.

With this procedure each agent chooses actions which are similar to the one with the best reward. In order to introduce more randomness, actions are drawn for a certain probability out of a uniform distribution.

Profit estimation

Each agent has to estimate the reward r^i for the set of actions for a game round t . From the market operator he gets the accepted quantity-price pair q^{acc}, s^{acc} , if any. The provision of SFC limits the production capabilities. The remaining unconstrained power range can be used for taking part in the pool market.

To model this pool market an estimated hourly priced forward curve is used. The profit from this market depends on the difference between market price and water values wv_{hpp} of the respective HPP. For the sake of simplicity a number of simplifications are made:

- the water values wv_{hpp} remain the same for the whole week,
- water inflows as well as water balance in the basins are neglected, and
- same water values are assumed for HPPs with similar marginal SFC costs.

For bigger basins, where weekly production and water inflows do not influence the filling much, the first two simplifications are reasonable. The third simplification is only valid, if the HPPs are technically similar.

With these simplifications the HPP-portfolio can be clustered. To estimate the profit with this portfolio in the pool market with given accepted quantity-price pair q^{acc}, s^{acc} , a linear program (LP) is formulated. For each HPP in the portfolio there is:

$$\begin{aligned} \max \quad & (\text{HPFC} - wv_{hpp})^T \cdot x & (9.2) \\ \text{s.t.} \quad & \left\{ \begin{array}{l} 0 \leq x \leq \text{unconstrained power}_{hpp}(q^{acc}) \end{array} \right. \end{aligned}$$

x denotes hourly bidding in the pool market, where it is constrained depending on the accepted SFC quantity q^{acc} as well as the technical minimum. The profits of all HPPs in the portfolio together with the remuneration of the accepted quantity-price pairs $(q^{acc})^T \cdot s^{acc}$ leads to the reward r^i for each agent. Note that alternatively to a LP a direct analytical computation would be also possible.

Memory / learning algorithm

In a Q-learning algorithm the agent i keeps in memory a function $Q^i : A \rightarrow \mathbb{R}$, which represents the expected profit previously calculated for action $a^i \in A$. The agent updates his memory after each game round t . This is done as follows:

$$Q^i(a_t^i) \leftarrow Q^i(a_{t-1}^i) + \alpha_t^i (r^i(a_t^1, \dots, a_t^n) - Q^i(a_{t-1}^i))$$

$\alpha_t^i \in [0, 1]$ is known as degree of correction specifying how much new knowledge change the memory. r^i denotes expected reward for agent i if actions a_t^1, \dots, a_t^n are performed with n as the number of agents. So if $\alpha_t^i = 0$ the agent leaves the memory unchanged, if $\alpha_t^i = 1$ the agent doesn't consider past observations at all. Here chosen is the same value of 0.6 for all agents which turned out to be a suitable value.

Market Clearing

The market operator collects the bids from the agents and performs a market clearing. The operator can select at most one of the quantity-price pairs within each bid. Further, the sum of the selected power quantities has to exceed the control demand. This is a typical optimization problem, which can be modeled as a binary LP:

$$\begin{aligned} \min \quad & s^T \cdot x \cdot q & (9.3) \\ \text{s.t.} \quad & \begin{cases} q^T \cdot x \geq \text{control demand} \\ \sum_b x \leq 1 \\ x \in \{0, 1\}, s \in S_t, q \in Q_t, b \in B_t \end{cases} \end{aligned}$$

The binary variable x specifies, which quantity-price pairs get accepted. Within each bid b only one pair can get accepted.

9.3.2 Application to Swiss market system

The proposed model is now applied to the Swiss system in order to give bidding decision support to one of the Swiss GenCos. In Switzerland storage HPPs account roughly for one third of total produced electrical energy. These plants are more than enough to provide the needed amount of SFC power. The three biggest Swiss GenCos and the three

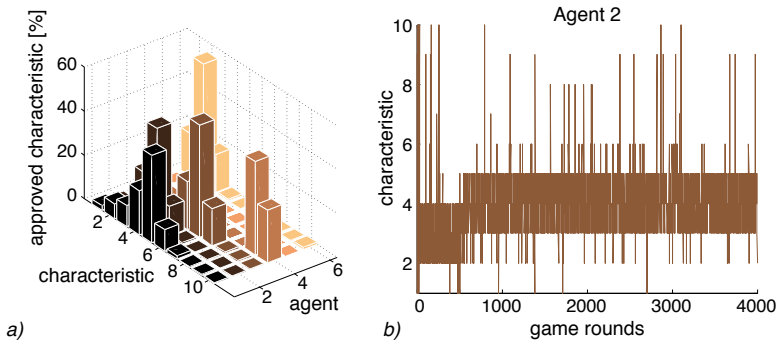


Figure 9.4: *a)* Approved characteristic per agent in % of total number of game rounds. *b)* Bid decision: chosen characteristics for agent 2.

biggest Swiss public utilities own more than 80% of total capacity. So those six entities were chosen for modeling the agents. The following simplifications were made:

- consideration only of hydro storage HPPs with more than 50 MW,
- technical minimum: Francis turbines: 50%, Pelton turbines: 10%, further technical issues were disregarded,
- water values calculated only for six reference HPPs, and
- minimum amount of 20 MW for SFC provision.

Water values are highly depending on the ratio of yearly produced energy to installed capacity. That's why the HPP are clustered based on this ratio and are allocated a water value out of six reference ones. The six reference water values are estimated based on a multi-horizon model optimization of six different typical HPPs in Switzerland for the last week of June. At this time point the storage basins are usually half filled. The taken hourly priced forward curve is also the estimated price curve for this week done some days beforehand. The resulting data is summarized in table B.1 in the appendix.

The algorithm is iterated in parallel, but the knowledge is shared repetitively. The results are shown for 4000 game rounds which needed around 30 seconds of computation time.

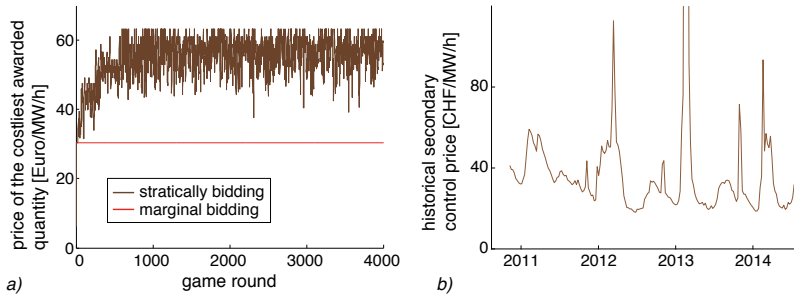


Figure 9.5: a) Simulated prices of costliest awarded quantity. b) Historical values of the average price of the costliest awarded 20 MW.

Results

In figure 9.4 a) for all agents the characteristics are shown, which bids were accepted by the market operator. The bids from agent 5 were seldom accepted which is obvious for the data shown in table B.1. But the other agents also want to get their bids approved and for certain characteristics this is more probable. Except of agent 4 the others bid prices substantially higher than the marginal costs.

In figure 9.4 b) the bid decision for agent 2 is depicted. After around 1000 game rounds the result stabilizes. Note that the spikes in the figure are due to frequent complete randomly chosen characteristics.

Figure 9.5 compares simulated prices for the costliest awarded quantity with historical values. The historical prices are around 20% lower than the simulated ones, which indicates either model inadequateness and/or missing strategic behavior in the real market. If the agents would bid their marginal costs, the costliest awarded quantity would be around 30 €/MW/h which would fit historical ones better.

The results indicate that strategic bidding would be beneficial. Besides it can also be shown that if only one agent is given the opportunity to bid strategically, he will choose to bid marginal costs. So there is a high probability in reality that despite a Nash-equilibrium exists no agent bids strategically. This makes a decision support tool which considers strategic bidding obsolete.

However, there could be special situations, where the SFC amount of-

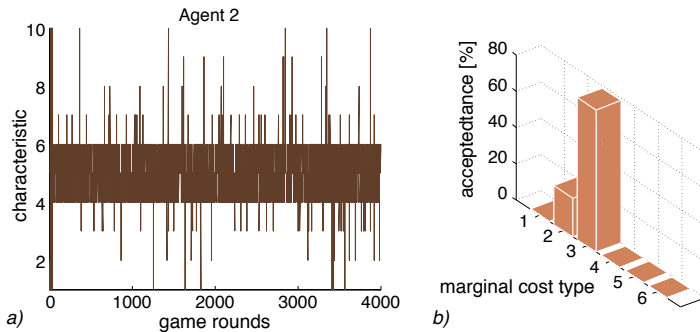


Figure 9.6: Market with reduced amount of SFC quantity and only agent 2 able to act strategically: a) Chosen characteristics. b) Acceptance of marginal cost types for agent 2.

ferred by the agents is reduced (such situations may have resulted in high prices in figure 9.5 b)). Usually this is known before the actual market clearing is performed and an agent could make use of this knowledge. Figure 9.6 shows the results of a simulation, where some HPPs are not available and therefore the amount of bid SFC quantity is reduced. Additionally only agent 2 is learning. In this case agent 2 acts strategically although the other agents bid their marginal costs. He chooses characteristic 5 instead of characteristic 4 as earlier, which means further increased bid prices. Accepted were in this case marginal cost type 3. The same simulation was done without agent 2 given the possibility to act strategically, so all agents bid their marginal costs. The expected return from bidding increases for agent 2 by more than 20% if he bids strategically. If the profit out of energy bidding in the pool market is also considered an increase of more than 10% can be achieved. So it is indeed more beneficial for agent 2 to act strategically in this case.

9.3.3 Summary

An agent-based simulator allows an analysis of the secondary frequency control market as well as decision support to one of the market participants. The simulator was applied to the Swiss system out of public available data. It was shown that strategic behavior would be beneficial but would hardly be applied in practice. A reason for this is also

the oversupply of control reserves in Switzerland. However, in case of special occurrences, it could be indeed beneficial for a generation company to act strategically.

Obviously, the costs for a transmission system operator increases if some market participants act strategically. Therefore, he may prevent this by additional market rules.

Chapter 10

Closure

Summary and conclusions of the thesis

This thesis deals with the self-scheduling of pumped storage hydro generation company in a liberalized market environment. As main contribution, a flexible but very efficient modeling approach, the multi-horizon framework, was introduced, analyzed, evaluated, and applied for realistic test cases.

The thesis started with laying out the challenges which have to be faced in hydro power planning. The combination of some of these challenges makes the problem very difficult to solve, making a decomposition of it into short- and medium-term planning problems necessary.

From the academic point of view also the evaluation of new methods is a challenge since no standard power plant models are available. Therefore, two test power plants were introduced.

Chapters 3 to 5 showed the state-of-the-art methods in hydro power planning. The following methods were explained, analyzed, and compared: deterministic equivalents, stochastic dynamic programming (SDP), stochastic dual dynamic programming (SDDP), multi-cut SDDP, dualized SDDP as well as some exotic variants. Further, some modeling technics were discussed: here-and-now and wait-and-see decisions, stage-wise dependent problems, and risk-averse multistage optimizations.

In the second part of the thesis, the main contributions can be found. Chapter 6 dealt with how to model medium-term planning problems

both reasonably from the modeling point of view and efficiently from the computational point of view. In order to achieve this, the multi-horizon modeling framework was introduced and analyzed. The idea there is to combine dynamic programming with deterministic equivalents and to exploit physically different reservoir types.

The model was then extended by considering risk measures and markets for ancillary services. The model was evaluated against traditional approaches and some alternative formulations were discussed. It was shown that multi-horizon models have a number of advantages for only a modest increase in modeling effort.

Multi-horizon models are very flexible and, therefore, have to be adapted for specific applications. Two examples showed how to do this for a storage plant long-term evaluation and for a price-maker bidding problem.

Chapter 7 extended the dualized SDDP method in order to better cope with problems with non-concave value functions. First, it was shown why non-concavities can arise. A measure of the severity of non-concavity was proposed and based on it the dualized SDDP method was extended by the concept of locally valid cutting planes. It was shown that this extension can lead to better results especially for the estimation of water values.

Chapter 8 presented how to deal with risk associated with ancillary services markets. It was shown how a delta-hedge of its cashflows can provide decision support for a mitigation of its risks.

Finally chapter 9 applied medium-term operation strategies in short-term planning. Decision support for bidding in electricity markets for complex power plants were presented, where a marginal production cost curve was constructed. Then, assistance for bidding into ancillary services were given by an agent-based simulator, which considers strategic bidding. It was shown that for special situations strategic bidding could be beneficial.

As a main conclusion, one can note that despite the mature research field of hydro power planning, modeling approaches and solution methods can still be improved. Whereas academically many results in this thesis could be shown in a similar way also with traditional modeling technics and methods, in practice a more natural and transparent model as well as less computational effort is more valuable.

Outlook

Multi-horizon modeling technics were constructed out of the need for better models for hydro power planning. However, such models could be applied in many other fields, which were not discussed in this thesis, but which would be interesting to explore.

Multi-horizon models are very flexible and it is maybe not always clear how a reasonable set-up could look like. Therefore, a more formal procedure and/or a list of possibilities in how to find meaningful models could be a further playground for research.

The modeling of data processes were not analyzed in detail in this thesis. Therefore, a further direction for research could be to extent multi-horizon models in this respect.

In this thesis, relatively simple production functions for turbines and pumps were used. But multi-horizon models would allow the consideration of much more difficult functions, for instance piece-wise linear ones. Its impact on the results would be interesting to analyze.

As an application example of multi-horizon models a long-term valuation study of a pumped storage hydro power plant was shown. It would be interesting to compare its results with more financial oriented techniques like Monte Carlo regression type algorithms.

Finally, the multi-horizon models were constructed only with at most two different kinds of state variables. This could get relaxed to an indefinite number of it. Whereas computationally it would not introduce any benefits, from the modeling point of view this could make sense, e.g. for power plants with large, medium, and small sized reservoirs.

The improved dualized SDDP as well as the measure of non-concavity probably needs more applications and analyzes in other fields in order to prove if they are indeed applicable in general.

In contrast, delta-hedging of ancillary services is from a theoretical point of view well defined. However in practice, it could be troublesome to use and, therefore, needs more application experience.

Finally, risk-averse optimizations for multi-horizon models were well defined and also applied in this thesis. However, it was not used in bidding problems, which could be a good application.

Appendix A

Case study models

A.1 Kraftwerke Mattmark AG (KWM)

The model of the Kraftwerke Mattmark AG (KWM) power plant is shown in figure A.1. The plant is located in southern Switzerland. It is operated by the Axpo Trading AG but belongs to different shareholders, which can independently use their part of the plant.

The plant is not overly complicated. There are two reservoirs, one seasonal storage v^{stor} with a maximum filling of 100 Mio m³, and a balancing reservoir v^{bal} with a maximum filling of 100000 m³. The balancing reservoir is therefore 1000 times smaller and it is primarily used to catch water inflows. This water is either pumped in the pumps p^1 and p^2 into the seasonal storage or generates energy in the turbines u^3 and u^4 .

Both reservoirs receive a comparable amount of water throughout the year. Most of the inflows originate from glaciers. Therefore the correlation of inflows and temperature is very high and the inflows have a strong seasonality (see also figure A.2). Therefore optimizations on such power plants preferably end in spring time since then the storage basins are mostly empty.

The turbines u^1, u^2 and the pumps p^1, p^2 are build up out of Francis turbines, with each have capacities of 37 MW and 23 MW respectively. The turbines u^3, u^4 are of Pelton type and have each a capacity of 92 MW. The turbines and penstocks are designed in a way, that full production, which is 260 MW, is continuously possible when all turbines operate at

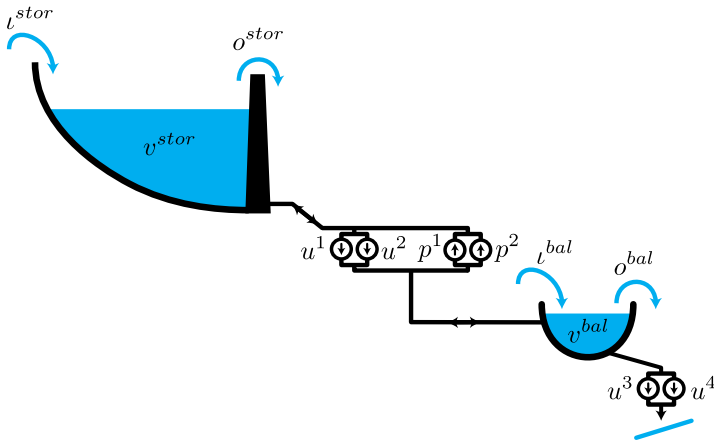


Figure A.1: Model of the Kraftwerke Mattmark AG (KWM) hydro power plant.

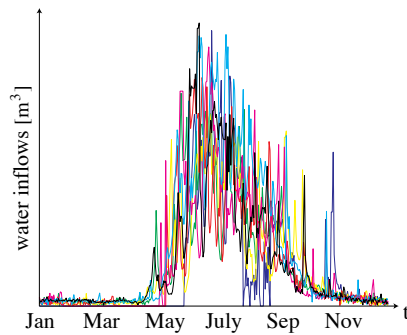


Figure A.2: Example of water inflows into reservoir v^{stor} of KWM for the last few years. Note the strong seasonality.

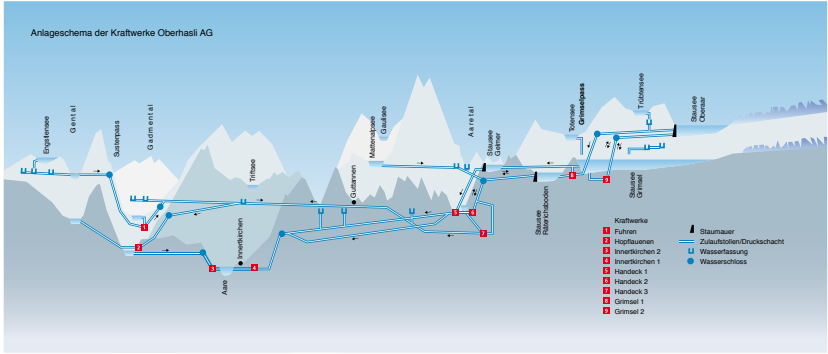


Figure A.3: Overview of the Kraftwerke Oberhasli AG (kwo) hydro power plant.²

their maximum capacity. The capacity of the pumps is approximately half of the one of the turbines u^1, u^2 . Additionally, the turbines u^3, u^4 are prequalified for delivery of secondary frequency control (SFC) with a technical minimum of around 10%.

A.2 Kraftwerke Oberhasli AG (kwo)

An overview of the Kraftwerke Oberhasli AG (kwo) power plant is shown in figure A.3. The plant is located in central Switzerland and belongs and is operated by the BKW Energie AG.

The plant is known as the most complicated one in Switzerland from the operational point of view. In this thesis a model of it is used which was constructed by BKW Energie AG for medium-term optimizations. The confidential model consists of 10 reservoirs with a capacity of 195 Mio m³. This capacity is enough to store around a quarter of the yearly inflows. Four reservoirs can be considered as seasonal storages whereas the other 6 are balancing ones. The turbines and pumps in the model are aggregated to 12 and 8 units with a capacity of 1.125 GW and 424 MW respectively.

The pumps are build up out of Francis turbines. Therefore only a binary on/off operation of them is allowed. The exception is a recently

²Kraftwerke Oberhasli AG, “analageschema-kwo.pdf”, Retrieved from <http://www.kwo.ch>, January 2015.

installed pump which can adjust its operation from 60 – 100% with the help of a frequency converter.

The turbines are of both Francis and Pelton type. Additionally, there are a few operation rules which can be modeled as mixed-integer constraints. Three of the turbines are used for the delivery of SFC.

Appendix B

Swiss system data for chapter 9

Table B.1: Data

Agent (GenCo)	bid power quantity [MW]	marginal cost [€/MW/h]
1: Alpiq	0 100 20 60 20 540	0 30 38 60 69 152
2: BKW	0 20 40 20 80 100	0 38 44 57 74 154
3: Axpo	340 40 60 120 260 0	28 31 46 135 174 349
4: EWZ	0 0 0 120 80 0	0 0 0 31 47 94
5: ewb	0 0 0 0 20 0	0 0 0 0 94 188
6: iwb	0 0 20 20 40 100	0 0 31 57 67 154

Agent (GenCo)	total installed capacity [MW]	tech. minimum [MW]
1: Alpiq	0 359 187 187 132 1260	0 70 43 19 38 126
2: BKW	0 84 260 55 593 253	0 42 59 5 167 25
3: Axpo	967 158 188 506 715 67	193 61 57 234 109 33
4: EWZ	0 0 0 386 330 54	0 0 0 89 77 27
5: ewb	0 0 0 0 204 27	0 0 0 0 59 3
6: iwb	0 0 81 48 256 253	0 0 15 5 73 25

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Curriculum Vitae

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